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THE

PHYSICAL REVIEW

A JOURNAL OF EXPERIMENTAL AND
THEORETICAL PHYSICS

CONDUCTED BY

EDWARD L. NICHOLS, ERNEST MERRITT,
AND FREDERICK BEDELL

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THE
PHYSICAL REVIEW.

THERMODYNAMICS OF THE VOLTAIC CELL.

By Henry S. Carhart.

I. Theoretical.

THE theory, commonly known as Thomson's, that the electrical energy of a reversible Voltaic cell is the equivalent of the chemical energy transformed has been abandoned as an exact expression of the facts since the appearance of Helmholtz's paper on "Die Thermodynamik Chemischer Vorgänge,"[1] followed by the experimental corroboration of Jahn[2] and others. It is now well understood that the electrical energy output of a reversible cell may be either greater or less than the heat equivalent of the chemical reactions. It is greater when the temperature coefficient of the cell is positive, and it is less when it is negative. If such a cell be connected in a circuit with an external resistance so large in comparison with the internal resistance of the cell that the Joule's heat due to the internal resistance is vanishingly small, then heat must be added to the cell to keep its temperature constant when a current passes through it if its temperature coefficient is positive. On the contrary, if its temperature coefficient is negative, it must give up heat to surrounding bodies to remain in thermal equilibrium. In the former case all the potential chemical energy transformed goes over into electrical energy, and in addition some heat undergoes a similar transformation. In the latter case only a part of the transformed chemical energy becomes electrical in form; the remainder appears as heat. The calomel cell devised by Helmholtz is an example of a cell with a positive temperature coefficient and a converter of heat into electrical energy. The Clark standard cell, on the other hand, has a negative coefficient and gives up heat or grows warmer when functioning as a primary cell. In the Daniell cell the electrical energy is almost the exact equivalent of the difference between the heat of combination of the zinc sulphate formed and the copper sulphate decomposed.

---

[1] Sitzungsber. der Akad. der Wiss. zu Berlin, I., 1882, p. 24.

[2] Wied. Ann., Vol. XXVIII., pp. 21 and 491, 1886.

The Helmholtz principle has hitherto been applied to a Voltaic cell in bulk, as it were. In the investigation which I am about to describe I have applied it to the cell in parts—that is, the positive and negative sides of a Voltaic cell are treated separately. In other words, I have considered the thermodynamics of a cell more in detail than has hitherto been done. This treatment may be regarded as an extension of the Helmholtz principle.

A reversible cell is one of which it is at least theoretically true that the electrical energy obtained from its action, when applied inversely so as to reverse the chemical operations, exactly suffices to restore the cell to its initial condition. Imagine the electromotive force of a Daniell cell compensated by a second inverse electromotive force. If the latter is reduced, a current passes through the Daniell in the normal direction, zinc goes into solution and copper is deposited. If the opposing electromotive force is made to exceed that of the Daniell, copper goes into solution and zinc is deposited. When equal quantities of electricity have passed in the two directions, the cell has been restored to its initial condition and the cycle is complete. Hence the laws of thermodynamics may be applied to such a reversible process.

Let T be the absolute temperature and dH the quantity of heat which must be applied to the cell or be taken from it in order to keep its temperature constant during the passage of a quantity of electricity dq. Also let u be the total energy contained in the cell. It is a function of the temperature of both the positive and negative sides of the cell, the cell being constructed so that the two sides may be separately considered. It is also a function of the quantity of electricity q passing through the cell.

Then from the principle of the conservation of energy (indicating the positive side of the cell by p and the negative by n subscript, and letting E be the electromotive force),

$$J \cdot dH = \left(\frac{\partial u}{\partial T}\right)_p dT + \left(\frac{\partial u}{\partial T}\right)_n dT + \left[\frac{\partial u}{\partial q} + E\right] dq. \qquad (1)$$

The change of entropy dS during the flow of dq is

$$dS = J\frac{dH}{T} = \frac{1}{T}\left[\left(\frac{\partial u}{\partial T}\right)_p dT + \left(\frac{\partial u}{\partial T}\right)_n dT\right] + \frac{1}{T}\left[\frac{\partial u}{\partial q} + E\right] dq. \quad (2)$$

Hence

$$\frac{\partial S}{\partial T} = \frac{1}{T}\left[\left(\frac{\partial u}{\partial T}\right)_p + \left(\frac{\partial u}{\partial T}\right)_n\right] \text{ and } \frac{\partial S}{\partial q} = \frac{1}{T}\left[\frac{\partial u}{\partial q} + E\right]. \qquad (3)$$

Differentiating the first of equations (3) with respect to q and the second with respect to T,

$$\frac{\partial^2 S}{\partial T \cdot \partial q} = \frac{1}{T}\left[\left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_p + \left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_n\right]$$

$$= \frac{1}{T}\left[\left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_p + \left(\frac{\partial^2 u}{\partial T \cdot \partial q}\right)_n + \left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] - \left[\frac{\partial u}{\partial q} + E\right]\frac{1}{T^2},$$

or

$$T\left[\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] = \frac{\partial u}{\partial q} + E. \qquad (4)$$

Substituting in equation (1),

$$J \cdot dH = \left(\frac{\partial u}{\partial T}\right)_p dT + \left(\frac{\partial u}{\partial T}\right)_n dT + T\left[\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] dq. \quad (5)$$

This last equation is the heat, expressed mechanically, which must be added to the cell or abstracted from it to keep its temperature constant while the quantity of electricity dq is passing through it. In fact if dT is zero, the equation becomes

$$J \cdot dH = T\left[\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n\right] dq. \qquad (6)$$

$\left(\frac{\partial E}{\partial T}\right)_p$ is the temperature coefficient of the positive side of the cell, the temperature of the negative side remaining constant; $\left(\frac{\partial E}{\partial T}\right)_n$ is the temperature coefficient of the negative side, the temperature of the positive side being maintained constant; the whole bracketed quantity is the temperature coefficient of the cell as a whole, neglecting any small coefficient due to the thermo-electromotive force at the junction of the two halves of the cell.

These positive and negative coefficients are the thermo-electromotive forces at the contacts between the two electrodes and the liquids bathing them respectively. Still neglecting the thermo-electromotive force at the junction between the two sides of the cell, the temperature coefficient of the entire cell is zero only when

$$\left(\frac{\partial E}{\partial T}\right)_p + \left(\frac{\partial E}{\partial T}\right)_n = 0,$$

that is, when the coefficients on the two sides are numerically equal and of opposite sign; or else when both are zero. The first condition is nearly realized in the Daniell cell. For $\mathrm{Cu} \underset{\longleftarrow}{-} \mathrm{CuSO_4}$, $\left(\frac{\partial E}{\partial T}\right)_p$ is 0.00073; for $\mathrm{Zn} \underset{\longleftarrow}{-} \mathrm{ZnSO_4}$, $\left(\frac{\partial E}{\partial T}\right)_n$ is 0.00079. The direction of these electromotive forces is in each case from the liquid to the metal across the heated contact. Hence the former gives to the cell a positive temperature coefficient, and the latter a negative one, while the coefficient of the cell in its entirety is negative and small, as I have shown elsewhere.[1]

A further consideration of equation (6) shows that the maintenance of a constant temperature on both sides of the cell during the flow of the quantity dq, requires that heat represented in mechanical measure by $\left(\frac{\partial E}{\partial T}\right)_p dq$ must be *added* to the positive side of the cell, and $\left(\frac{\partial E}{\partial T}\right)_n dq$ must be *withdrawn* from the negative side; or, in other words, these expressions represent the loss of heat on the positive side, and the gain on the negative respectively. Divide them by the thermal capacity of the two sides of the cell, and we have the *fall* of temperature on the positive side and the *rise* on the negative. These temperature changes are due to the thermal or Peltier electromotive forces at the two contacts. On the negative side of the Daniell cell, for example, the current from the zinc to the liquid flows against the Peltier electromotive force, and heat is generated. On the opposite side the current flows from the copper to the liquid, or with the Peltier electromotive force at the contact, and therefore heat is absorbed and converted into the energy of the current.

---

[1] Primary Batteries, pp. 137–146.

If a reverse current be sent through the cell, all the physical phenomena, except that of the Joule's heat, are reversed. The current on the positive side flows from the copper to the liquid, or against the thermo-electromotive force, and Peltier heat is generated. On the zinc side heat is absorbed. With a reverse current, therefore, the temperature of the positive side of the cell rises faster than that of the negative, while with a direct current the temperature of the negative side rises faster than that of the positive, that is, under practical conditions where the Joule's heat causes a rise of the temperature of the entire cell.

While equation (6) is perfectly general, the preceding conclusions are applicable only when the Peltier electromotive force at the contact of the electrode and the liquid is directed from the latter to the former across the heated junction, both on the positive and the negative side of the cell. There are some exceptions to this general rule. For example, the thermo-electromotive force of $\mathrm{Fe}—\mathrm{FeSO_4}$ is zero. Hence a cell consisting of $\mathrm{Fe}—\mathrm{FeSO_4}—\mathrm{CuSO_4}—\mathrm{Cu}$ has a positive temperature coefficient[1] due to the thermo-electromotive force on the copper side, and the Peltier phenomenon is confined to the positive side of the cell. In the case of the Voltaic series $\mathrm{Ni}\underset{\rightarrow}{—}\mathrm{NiSO_4}—\mathrm{CuSO_4}\underset{\rightarrow}{—}\mathrm{Cu}$, the two thermo-electromotive forces are in the same direction through the cell. The temperature coefficient is large—about 0.8 per cent., and heat is absorbed at both electrodes. Such a cell takes heat from the surroundings on both sides and converts it into electrical energy.

II. Experimental.

The general conclusions arrived at above I have not found it difficult to confirm qualitatively. It was only when quantitative determinations were attempted that serious difficulties were encountered. These I have in a large degree happily overcome. The results are highly interesting.

---

[1] A one-volt standard cell, Am. Jour Sci., Vol XLVI., p. 64.

The Daniell cell was chosen as the most convenient one for the experimental work, particularly because the necessary constants pertaining to it are known with a good degree of accuracy.

The cell was made as follows: Two glass tubes 9.5 cm. long and 3 cm. in diameter were provided near the bottom with side tubes 18 mm. long and 18 mm. wide (Fig. 1). Heavy, tight-fitting rubber collars served to connect the two halves of the cell together, with a plate of fine unglazed porcelain between them as shown. The edges of the porcelain plate were covered with paraffin after the rubber collars had been bound together with fine iron wire. No difficulty was found in making a water-tight joint.

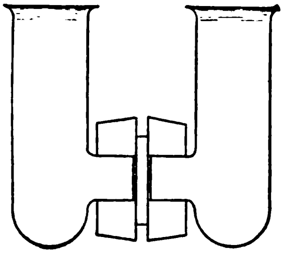

Fig. 1.

In most of the experiments 10 per cent. water-free solutions of zinc and copper sulphates were used. These solutions have relative conductances of 321 to 320.[1] They were chosen with the object of eliminating as far as possible the inequalities of Joule's heat between the two sides of the cell. In every experiment 40 c.c. of each solution were placed in the double cell, which was supported by a stout cord in a large pasteboard box and completely surrounded and covered with eider down.

The earlier observations were made with two Haak thermometers reading directly to fifths, and by means of a telescope to hundredths of a degree. The copper and zinc plates were attached directly to the thermometers by rubber bands, and the liquids were stirred before taking observations by moving the thermometers and the attached plates back and forth. Since the internal resistance of the cell was over 100 ohms, it was necessary to put it in series with a few storage cells and some resistance, with the double object of varying the current and of causing a reverse current to traverse the experimental cell.

---

[1] Kohlrausch and Holborn, Leitvermögen der Elektrolyte, p. 151.

The current was measured as follows: The electromotive force of a Weston standard cell was balanced against the fall of potential over a portion of two resistance boxes containing 10,000 ohms and connected in series with a storage cell. By means of an appropriate switch, consisting of mercury cups in a plate of paraffin, this potential drop in the auxiliary circuit was balanced against the fall of potential over the requisite number of ohms in the main circuit. For example, for 0.03 ampère the fall of potential over 34 ohms was thus balanced indirectly against the electromotive force of the Weston cell, which was very nearly 1.02 volts. The current was not allowed to vary more than about 0.05 per cent.

Fig. 2 shows the results obtained with a current of 0.02 ampère. Curve I, I, I, belongs to the zinc sulphate and II, II, II, to the copper sulphate, both of which in this case were fifteen-per-cent. solutions. These curves are valuable chiefly because they exhibit two facts rather strikingly. The current through the cell was reversed at the points indicated by the two vertical dotted lines. In the first place it is evident that the temperature always rises most rapidly on the side of the cell by which the current enters. The same fact

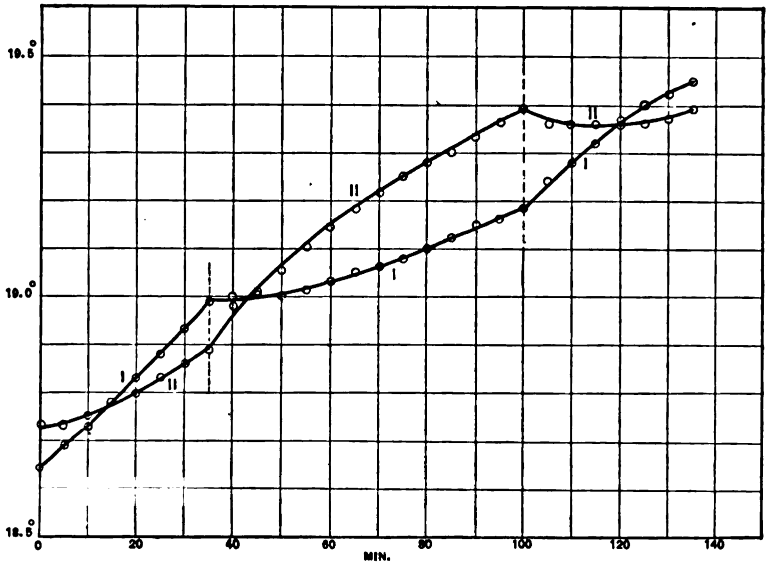

Fig. 2.

appears in every case and in every trial. In the second place the curves show how the phenomenon is complicated by the change in concentration. For this reason the difference in temperature between the two sides of the cell is less before reversal than it would otherwise be, for the concentration and conductance of the zinc sulphate increase while those of the copper sulphate decrease. After reversal this change in concentration accelerates the rise of temperature on the copper side and retards it on the zinc side. After reversing the current an established difference of temperature is reduced to zero in about half the time necessary to produce it. It is therefore obvious that some method had to be devised to eliminate the effect of this change of concentration as well as that of any difference in resistance on the two sides due to the dimensions of the apparatus, position of the plates, etc., if reliable quantitative results were to be obtained. For this purpose the following expedients were adopted: First, ten-per-cent. solutions of practically equal specific conductances were used. Second, the duration of any series of observations in either direction was taken inversely as the current, so that the same quantity of electricity was always transmitted with the same changes in concentration. This quantity was 72 coulombs,—0.03 ampère for 40 min., 0.04 ampère for 30 min., etc.

Third, a curve was constructed from observations showing the equalization of any established difference of temperature between the two sides of the cell as a function of the time. This may be called for convenience a "cooling curve." By means of this curve it is not difficult to correct any series of observations so as to obtain the temperature differences which would have been observed if heat had not flowed from one side to the other during the time covered by the series. Fourth, to eliminate the effect of the change in concentration and the lack of equal resistance on the two sides, a series of observations was first taken with a direct current through the cell, and then after reducing the temperature difference again to about zero without any current, another series of equal duration was taken with the current through the cell reversed. After correcting both series by the "cooling curve," the half sum of the two maximum differences of temperature was taken as the correct value. The difference between the heats generated on the positive and negative sides, with the current first direct and then reversed, may be expressed by the two equations (R' being the resistance of the zinc side and R'' that of the copper),

$$J \cdot H' = I^2R't - I^2dR't + T\left(\frac{\partial E}{\partial T}\right)_n It$$

$$- \left[I^2R''t + I^2dR''t - T\left(\frac{\partial E}{\partial T}\right)_p It \right].$$

$$J \cdot H'' = I^2R't - I^2dR't - T\left(\frac{\partial E}{\partial T}\right)_n It$$

$$- \left[I^2R''t + I^2dR''t + T\left(\frac{\partial E}{\partial T}\right)_p It \right].$$

Half the algebraic differences of these two is

$$T\left(\frac{\partial E}{\partial T}\right)_p It + T\left(\frac{\partial E}{\partial T}\right)_n It = J \cdot dH,$$

and this is the quantity sought, considered arithmetically.

In the later experiments the thermometers were discarded and two thermal couples, consisting of thin iron and "constantan" wires, were substituted for them. The junctions were thickly covered with shellac and were then enclosed in thin glass tubes with enough mercury to cover the soldered junctions to aid in conductivity of heat. One junction was placed in the zinc sulphate side, the other in the copper sulphate. The plates of the cell were attached to the glass tubes containing the thermal junctions, and the tubes in turn were held by rubber bands to two long, thin rods of wood held firmly at their upper ends. The stirring of the liquids was thus easily accomplished by bending the wooden rods.

The galvanometer connected in series with the thermal couples was astatic and sufficiently sensitive, but its period was about 20 sec. The reading telescope was placed at a distance of 2.8 m., and the reduced deflection per degree difference of temperature was 138.3. The zero position was unfortunately a drifting one, so that the cross-wire of the telescope had to be set on it before every reading. The liquids were stirred for fifteen seconds before every reading. Five seconds intervened between the stirring and the observation.

With this arrangement I set out to determine, first, whether the temperature difference per second is proportional to the current, as equation (6) requires; second, how nearly the observed value of any established temperature difference agrees with the calculated value.

Fig. 3 shows the results of one series with a current of 0.05 ampère. Curve I belongs to the series of observations with a direct current, curve II to those with the current reversed, and III is the "cooling curve." Many other observations were taken for curve III, all of them agreeing well with this series.

Time in Min.	Reading, Deflection and Cooling Correction.					
	0.02		0.03		0.04	
0	3.6		4.0	0.2	1.0	
5	4.6		8.5	1.1	3.7	
10	6.0		11.0	1.4	5.0	0.2
15	6.4	0.2	12.5	1.4	7.5	0.6
20	7.3	0.4	14.0	1.6	8.6	0.9
25	7.7	0.5	15.0	1.7	10.0	1.1
30	8.0	0.6	16.2	1.8	11.1	
35	8.3	0.7	16.5	1.9		
40	8.5	0.8	16.8			
45	9.0	0.9				
50	9.1	1.0				
55	9.5	1.1				
60	10.0	6.4		12.8		10.1
Corrected Deflections,		12.6		23.9		12.9
REVERSE CURRENT.						
0	2.8		−0.2		3.3	0.2
5	6.9	0.7	3.0		8.7	1.2
10	10.0	1.2	5.0	0.2	13.2	1.6
15	12.0	1.3	7.7	0.3	16.9	2.1
20	13.3	1.5	9.2	1.1	20.0	2.4
25	14.4	1.6	10.9	1.4	22.8	2.8
30	15.5	1.8	12.0	1.5	24.8	
35	16.8	1.9	13.7	1.6		
40	17.2	2.0	14.6			
45	17.5	2.0				
50	17.8	2.1				
55	18.5	2.1				
60	18.6	15.8		14.8		21.5
Corrected Deflections,		34.0		21.4		31.8
Half Sum,		23.30		22.65		22.35

Time in Min.	Reading, Deflection and Cooling Correction.					
	0.05		0.05		0.06	
0	2.0		3.6		0	
4	5.0		5.8		4.1	
8	7.0	0.2	7.9	0.2	6.8	
12	7.0	0.4	9.6	0.8	10.0	0.7
16	8.9	0.5	12.0	1.0	11.3	0.9
20	9.0	0.7	13.8	1.1	12.7	1.1
24	10.8	1.0	15.4	1.2		
		8.8		11.8		12.7
Corrected Deflections,		11.6		16.1		15.4

REVERSE CURRENT.

Time in Min.						
0	−4.0		4.8		1.0	
4	2.8		9.8	0.3	8.7	
8	8.0		14.1	1.1	13.2	1.0
12	12.5	0.9	18.2	1.4	18.5	1.4
16	16.6	1.3	21.5	1.8	22.8	1.8
20	19.4	1.6	23.3	2.1	26.0	2.3
24	23.0	2.0	26.0	2.3		
		27.0		21.2		25.0
Corrected Deflections,		32.8		30.2		31.5
Half Sum,		22.20		23.15		23.45
Mean of all,				22.85		

The above table contains all the essential data for the several currents used.

A deflection of 22.85 scale parts corresponds to a difference of temperature of 0°.165. If the phenomenon here measured is proportional to the current, then since the quantities transmitted with the above currents were all equal, the resulting corrected deflections should be equal. The results represented by the half sums show good agreement:

Current in Amperes.	Period in Min.	Half Sum.	Variation from Mean.
0.02	60	23.30	+0.45
0.03	40	22.65	−0.20
0.04	30	22.35	−0.50
0.05	24	22.20	−0.65
0.05	24	23.15	+0.30
0.06	20	23.45	+0.60

It is obvious that the heat generated on one side and the heat absorbed on the other are both proportional to the current in the case of a Daniell cell.

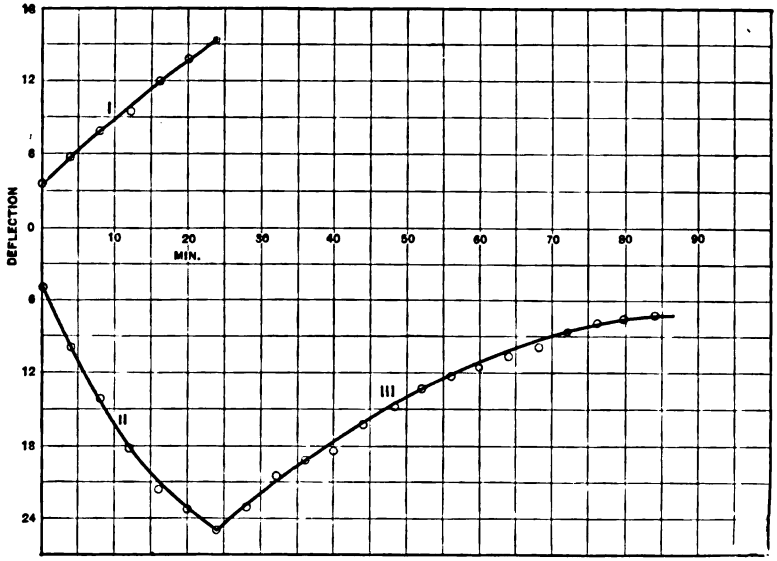

Fig. 3.

To calculate the difference of temperature between the two halves of the cell, it must be remembered that equation (6) expresses only the heat required to keep the temperature of the whole cell constant during the passage of a given quantity of electricity. To calculate the quantity corresponding to the observed mean of 22.85 or 0°.165, we must find the calories generated on one side of the cell and absorbed on the other and divide their sum by the thermal capacity of half the cell. For this purpose we have the following data:

$$1/J = 0.24 \qquad T = 292$$

$$\left(\frac{\partial E}{\partial T}\right)_n = 0.00079 \qquad \left(\frac{\partial E}{\partial T}\right)_p = 0.00073\,^{1}$$

$$dq \text{ or } Q = 72.$$

Hence $H = 0.24 \times 0.00079 \times 292 \times 72 = 3.99$ (zinc side)

and $H = 0.24 \times 0.00073 \times 292 \times 72 = 3.68$ (copper side).

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[1] Primary Batteries, pp. 140, 141.

The specific heat per unit volume of the solutions is 0.99. The mean of two determinations of the thermal capacity of the unfilled cell was 16.5. Hence the thermal capacity of one-half the cell is

$$40 \times 0.99 + 8.25 = 47.85.$$

The temperature difference produced by the passage of 72 coulombs should then be

$$(3.99 + 3.68) \div 47.85 = 0^\circ.160.$$

The mean observed value is 0°.165.

I must reluctantly defer the application of the principles herein discussed to other types of cells for some later opportunity. It is obvious that the same method of investigation may be applied to a silver voltameter, a copper voltameter or a zinc voltameter. The essential fact involved in all is the presence of Peltier or thermo-electromotive forces.

My thanks are due to Professor H. F. Weber for his courtesy in welcoming me to his laboratory and in placing at my disposal the requisite apparatus for the work described in this paper.

ZÜRICH, April 3, 1900.

A NEW FORM OF ELECTRICAL CONDENSER HAVING A CAPACITY CAPABLE OF CONTINUOUS ADJUSTMENT.

LYMAN J. BRIGGS.

ADJUSTABLE condensers may be divided into two classes: The first class consists of condensers in which the capacity is varied by grouping a number of small capacities in series, or in multiple, or as a combination of the two methods, illustrated in the various forms of standard condensers having more than one stated capacity. Such condensers have the advantage of a known capacity for any grouping of the units which may be made. The adjustment is, however, limited to a comparatively few values and the method of adjustment, usually by plugs, is not well adapted for compensating capacity or inductance in a circuit, or for resonance work in general. Willyoung[1] has eliminated the disadvantages of adjustment by plugs in his adjustable condenser, in which a greater or less number of condensers of approximately equal capacity can be quickly thrown in multiple by a sliding switch. In all such arrangements the adjustment is, of course, not continuous. The first class may consequently be summarized as including such forms of condensers as are adjustable by steps.

The second class embraces those condensers which are capable of continuous adjustment, secured by changing the relative positions of the condenser plates. Sir William Thompson's sliding cylindrical condenser is an example of this class.[2] It consists of two conducting cylinders supported concentrically, one being capable of displacement with reference to the other along the common axis. A more familiar example is the classical flat disk air condenser, first used by Wilcke. These condensers leave nothing to be desired, so far as continuous adjustment is concerned. Their large size, however, combined with a comparatively small maximum capacity, does not adapt them to laboratory requirements.

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[1] Condensers and their applications, p. 33. Published by Jas. G. Biddle, Phil., 1898.

[2] Absolute Measurements, Andrew Gray, Vol. I., p. 423.

A stack of alternate conducting and insulating sheets (the conductors being so connected that adjacent sheets are of opposite polarity) gives the maximum capacity with reference to the space occupied. In condensers of this form the conducting and insulating materials used have been chosen heretofore so as to form a close, compact arrangement. If, however, we use an elastic conducting or insulating material which does not favor a compact arrangement of the plates, the capacity of the condenser may be varied at will by simply compressing the system. Such an arrangement combines the advantages of continuous adjustment with a condenser of considerable capacity arranged in the most compact manner, and is the principle employed in the construction of the condenser described in the present paper.

It is, of course, essential that the system of plates return to its original form when the pressure is removed. This is not readily secured through the elasticity of any of the ordinary insulating substances, but it can be accomplished by substituting for the tinfoil usually employed thin sheets of spring brass. To gain the necessary looseness of structure the sheets of brass are curved to conform to the surface of a cylinder of 6 or 8 inches radius. The curvature given to the sheet brass by the manufacturer in rolling it for shipment will usually be found to answer very well. Spring sheet brass, corresponding in thickness to No. 30 B. & S. wire

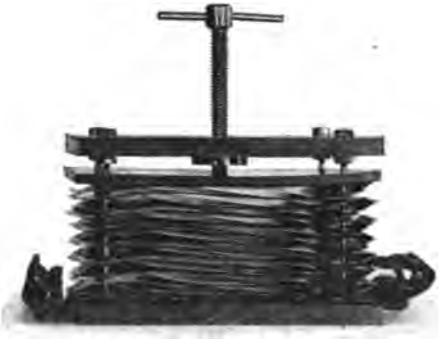

Fig. 1.

gauge, is suitable for condensers of small capacity, the thickness of the sheet depending somewhat upon the size of the condenser and the range desired. Mica is used for insulation.

A small condenser (.05 mf.) of the compression type is illustrated in Fig. 1. The outer end of each brass sheet projects one-fourth of an inch beyond the mica and is cut in the middle into two wings, one of which is bent downwards and the other upwards. The corresponding wings of alternate sheets are bent in opposite directions, so that when the sheets are assembled these wings come into contact, and electrical connection is established throughout the brass sheets of each polarity. The method of making contact between sheets of the same polarity is more clearly shown in Fig. 2. Owing to the movement of the wings when the system is compressed, this form of connection is virtually a rubbing contact, and gives good connection even when the condenser is immersed in oil. All necessity of soldering is thus avoided, and the condenser plates lie perfectly flat and smooth when the system is compressed.

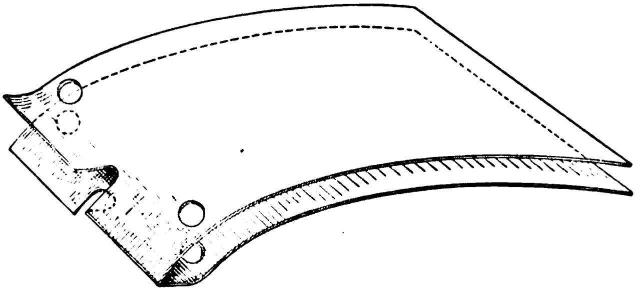

Fig. 2.

Preliminary to assembling the condenser four holes are punched in the respective corners of each mica sheet, a template being used to secure similarity in all the plates. These holes are for the passage of the four guide or retaining rods on the condenser frame. Two similar holes are punched in the outer end of each brass sheet, the length of the brass sheet being so chosen that when the condenser is assembled the inner end of the sheet is at least one centimeter from the guide rods which carry the sheets of opposite polarity. The mica sheets are also two centimeters wider than the brass plates, to prevent leakage at the sides.

The compression frame consists of a base and top of hard rubber connected by the four retaining rods. A compression screw in the top, having a double thread of .1-inch pitch and operating against a compression plate of hard rubber resting on the top of condenser sheets, serves to adjust the capacity.

The assembling of the condenser is very simple, consisting merely in removing the top of the frame and stacking the sheets in their proper order upon the four guide rods. The structure is more open if the sheets representing the two polarities are assembled so as to have the convex surfaces face in opposite directions. Two mica sheets are used between each pair of plates.

The range in capacity which can be obtained depends upon the curvature, thickness and number of plates. In one condenser constructed the capacity when compressed was eight times the capacity when the pressure was released. The small condenser shown in Fig. 1, which is made up of thirteen plates on each side 5 by 8 cm. in effective dimensions, has a capacity in air of .055 of a microfarad when compressed, and about .013 microfarad when released. The maximum capacity of this condenser is thus over four times the minimum capacity. This ratio was found to be considerably greater in some of the other condensers constructed, depending upon the curvature and rigidity of the brass sheets.

The range of capacity may be greatly increased by dividing the condenser into three or four sections, so arranged as to permit their being thrown in multiple by a suitable switch, conveniently located on the top of the condenser frame. By dividing the condenser into three sections, the first of which contains three-fourths of the condenser sheets, the second three-fourths of the remainder or three-sixteenths of the whole, and the third section the remainder of the sheets, the range in capacity can be increased sixteen times, or the minimum capacity obtainable will be one sixty-fourth that of the maximum. The adjustment, moreover, will be perfectly continuous throughout this range. The capacity of the two smaller sections compressed will slightly exceed the capacity of all three sections released, while the capacity of the smallest section compressed will be greater than the capacity of the two smaller sections released. There will thus be no position between the limits through which a continuous variation in capacity can not be obtained. The condenser terminals in this arrangement should of course be connected with the smallest section.

In constructing the condenser so as to permit the cutting out of a part of the plates to increase the range, it is necessary that the guide-rods should be insulated, as otherwise they may establish connection between the plates. This may be secured either by constructing them of hard rubber or by insulating metal guide-rods with thin-walled glass tubing of suitable size.

The range of the condenser in air can easily be increased temporarily by inserting a strip of mica between the wings of the condenser plates at any desired point. This method will often be found very convenient for increasing the range of the condenser when a switch for dividing the condenser into sections has not been provided.

The accompanying table gives the range in capacity which may be obtained with the small condenser shown in Fig. 1, by simply compressing the system of plates, none of the plates being cut out. The measurements were made by the direct deflection method, with an E. M. F. of 20 volts, comparing the deflections with those obtained from a standard condenser of approximately equal capacity.

TABLE I.

Change in Capacity with Expansion of the System of Condenser Plates.

Amount of Expansion Inches.	Capacity Microfarads.
0.00 (Compressed)	.0549
.05	.0497
.15	.0312
.25	.0258
.35	.0221
.45	.0205
.55	.0192
.75	.0175
.95	.0159
1.15	.0150
1.35	.0134

The data given in Table I. are expressed graphically in Fig. 3. The change in curvature at the upper end of the curve is due to the springing the condenser frame, giving erroneous values for the amount of compression. It is to be noted that 90 per cent. of the variation in capacity occurs with but half of the total expansion shown, so that practically double the capacity could be used in a condenser frame of the same dimensions (10 by 7 by 5 cm.) without diminishing the range of adjustment to any extent.

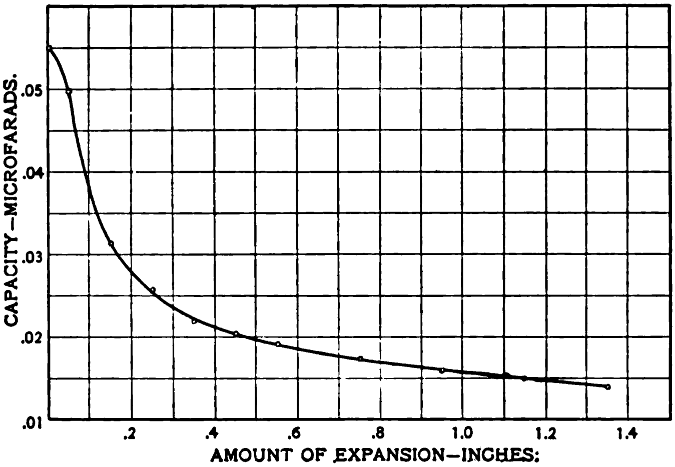

Fig. 3.

The results given in Table I. are the means of two series of determinations, one made while the stack of plates was being compressed and the other during its release. The two series of determinations corresponded very well, indicating an approximately constant capacity for any given position of the compression screw. The arbitrary setting of the condenser at a given capacity without comparison with a standard must necessarily be approximate, to some degree at least, owing to the liability of a slight re-arrangement of the condenser plates taking place, and is subsidiary to the real function of this form of condenser.

The capacity of the condenser is not simply a function of the distance between the plates, as would be the case if a homogeneous dielectric were used, but involves also the change in the specific inductive capacity of the combined dielectric—mica and air or mica and petroleum. The dielectric constants of air, petroleum and mica are approximately as 1, 2 and 6. Since the thickness of the mica remains constant, the inductive capacity of the compound dielectric changes very rapidly in the first stages of the expansion of the stack of condenser plates. Consequently the change in capacity at this stage is much greater than that arising from the separation of the condenser plates alone. This is well illustrated in Fig. 3, which shows the change in capacity with expansion to be a function of the second degree, while with a homogeneous dielectric, the capacity is a linear function of the distance between the plates.[1] The greatest range of adjustment will consequently be attained by combining with mica the dielectric having the lowest specific inductive capacity.

When the condenser plates are entirely released the dielectric consists mainly of air or petroleum. The capacity of the condenser in this position would, therefore, be about twice as great in petroleum as in air and the range of adjustment in petroleum would consequently be only about one-half as much as in air. Experiments with the condenser in the two dielectrics confirmed these conclusions.

Fortunately the insulation resistance in air can be made sufficiently high so that an oil insulation is not necessary for ordinary voltages. The condenser when compressed in air successfully stood the continuous application of a direct E. M. F. of 225 volts, the highest E. M. F. available in the laboratory. The insulation resistance of the condenser when compressed in air was found to be about 5,500 megohms, determined by leakage. The insulation resistance is thus sufficiently high to permit the leakage being neglected in alternating current work.

An idea of the convenience and compactness of this condenser over other forms capable of continuous adjustment may be gained by comparing it with a parallel disk condenser of equal capacity. Assuming the mean distance between the disks to be 1 mm. the disks would have to be 275 cm. in diameter for a maximum capacity equal to that of the small condenser shown in Fig. 1 (.055 mf.).

This form of adjustable condenser was originally devised for the purpose of eliminating polarization capacity of the electrodes in an electrical method of investigating the water content of soils *in situ*.[1] It has since proved convenient in compensating polarization capacity in electrolytic measurements in general. It is also adapted for use in other electrical measurements with alternating currents, such as correcting the displacement of the wave by induction, or in compensating capacity or induction in a bridge circuit.

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[1] Absolute Measurements, Gray, Vol. I., p. 57.

PHYSICAL LABORATORY, DIVISION OF SOILS,
U. S. DEPARTMENT OF AGRICULTURE.

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[1] Electrical instruments for investigating the moisture, temperature and soluble salt content of soils. Briggs, Bulletin No. 15, Division of Soils, U. S. Department of Agriculture, 1899.

This method of investigating the water content of soils depends upon the variation in the electrical resistance with changing moisture content. The soil resistance is measured between two carbon electrodes permanently buried in the soil at any desired depth. In order to secure a well-defined balance in the bridge it is necessary to counteract the capacity of the electrodes by shunting an equal capacity across that arm of the bridge which has a terminal in common with the soil electrode and the telephone receiver, so as to throw the two capacities on opposite sides of the bridge with respect to the receiver. Since the capacity of the bridge-arm embracing the soil resistance varied with different pairs of electrodes and with different conditions of the soil as regards moisture, it was necessary to have the compensating capacity adjustable and desirable to have the adjustment continuous throughout the required range.

ON THE THEORY OF THE COHERER.

By Karl E. Guthe and Augustus Trowbridge.

Introduction.

No satisfactory explanation has as yet been advanced of the phenomenon of the lowering of the resistance of a loose contact when electromagnetic waves are set up in the surrounding medium, in spite of the numerous investigations made thereon. Of the existing theories, those of Branly, Lodge and Auerbach are the most important. Branly[1] supposes that the medium between the conducting particles of the coherer undergoes a modification under the influence of the electromagnetic waves and that thus the whole becomes conducting.

Lodge[2] tries to explain the phenomenon chiefly on electrostatic principles. He supposes the particles to be separated by films—for instance, oxide films. These films he calculates to be under enormous pressures, due to electrostatic attraction. Under these circumstances the surfaces may be welded together "especially if the electric stimulus simultaneously acted in any way as a flux, by reducing the infinitesimal tarnish of oxide or other compound which must be supposed normally to cover them."

The fact that Lodge compares the coherer phenomena to the welding of metallic spark gaps due to lightning discharges of leyden jars has led to an interpretation of his theory by later observers, that the spark plays the all-important rôle in producing an intimate contact between the particles, by the formation of metallic bridges.

Auerbach[3] advances what he calls a mechanical theory. He showed that by periodic vibrations of a mechanical nature a lowering of the resistance of a coherer takes place. He explains this as follows:

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[1] C. R., 111, p. 785, 1890; 125, p. 939 and 1163, 1897.

[2] Phil. Mag., Vol. 37, p. 94, 1894; Electrician, Vol. 40, p. 87, 1897.

[3] Wied. Ann., Vol. 64, p. 611, 1898.

A disturbance is produced in the coherer of such a nature that each particle is displaced from its old position of equilibrium so that it comes into more intimate contact with its neighbor—it does not return to its former position, after the mechanical vibration, which caused its displacement, has ceased because of adhesion between the particles now in intimate contact.

He proposes to extend this explanation to coherer action in general.

Thus far very few experimental data are at hand to test any of the three theories outlined above. That of Branly can hardly be subjected to a crucial experimental test.

While Auerbach's theory seems probable enough for the rather strong mechanical vibrations he used, it can hardly be assumed that since the effect produced (viz: the lowering of the resistance) is the same in the mechanical and electrical case, the cause must be identical.[1]

A number of experimental proofs of Lodge's theory have been reported, notably those of Arons,[2] Tommasina[3] and Sundorph.[4]

The same criticism as the one advanced against Auerbach's theory holds here. It can very well be conceived that if sparks really do pass, a fusion of particles may take place, and the resistance be lowered—but the question still remains, "Is this the action of the coherer made use of in wireless telegraphy?"

If the lowering of the resistance be due to sparking, then we can only expect a very irregular behavior of the coherer. To quote Aschkinass who discusses Arons' and his own results in the following words:

"A lowering of the resistance takes place always when the excitation is weak, while no sparking, fusion or mechanical motion can be observed at the contacts. On the other hand, with stronger excitation, sparks, etc., can be seen, while the resistance is influenced in an irregular manner."[5]

He further states that when sparks of considerable intensity were apparent, the resistance was sometimes lowered, sometimes increased, sometimes the changes were not permanent; often sparks could be seen without any change in resistance taking place at all.

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[1] See also Drude, Wied. Ann., 65, p. 486, 1898, and Aschkinass, ibid., 66, p. 306.

[2] Wied. Ann., 65, p. 567, 1898.

[3] C. R., 128, p. 1092, 1899; 127, p. 1014, 1898; 129, p. 40, 1899.

[4] Wied. Ann., 68, p. 594, 1899.

[5] Aschkinass, loc. cit., p. 290.

From this it would appear that the phenomenon of coherer action can be best studied only when sparking is avoided. This point was thoroughly established by the quantitative experiments made by one of us a year ago[1] in which a remarkable regularity in the behavior of the coherer was observed, undoubtedly due to the fact that the above condition was fulfilled.

In making the experiments here reported, we were careful to avoid all disturbing influences, and here also the results are very concordant.

DESCRIPTION OF APPARATUS.

The lowering of the resistance of the coherer was produced by the closing of a circuit which consisted of a high E. M. F., which could be varied at will, a variable resistance *R*, and the coherer, in series.

The resistance of the coherer was measured by observing simultaneously the current through, and the potential difference at the terminals of, the coherer.

In order to work always under the same conditions, the apparatus was so arranged that the above measurements weretaken always after the same very short time interval had elapsed since the circuit was closed. This could best be accomplished by the use of a pendulum which in short succession actuates four circuit keys.

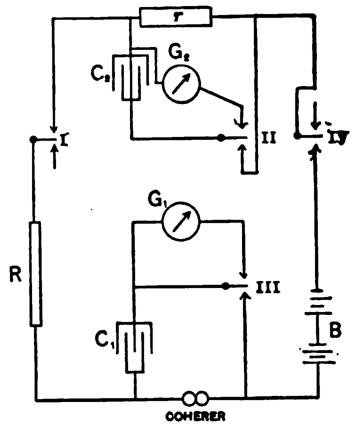

Fig. 1.

I., II., III. and IV. are those keys. When the keys are set before the pendulum is released, they make contact on their under sides. The pendulum in swinging from left to right releases them so that they make contact on their upper sides.

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[1] Trowbridge, Am. Jour. of Science, Sept., 1899.

As soon as the pendulum in swinging releases key I., the current flows through the coherer and the condensers C_1 and C_2 are charged. Then keys II. and III. are released simultaneously, which discharges the condensers C_1 and C_2 through the galvanometer G_1 and G_2 respectively; key IV. opens the circuit again. In general key I. was kept always in the same position relative to keys II. and III. However this was not necessary, for, as will be seen later, the time element does not enter into the problem. The galvanometers with their condensers, were calibrated beforehand so that the deflection of G_1 enabled us to determine the potential difference at the terminals of the coherer, that of G_2 the current flowing in the coherer circuit.

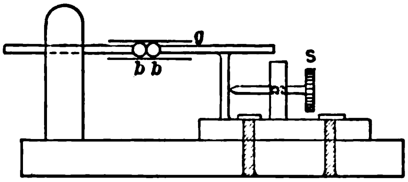

Fig. 2.

The coherers investigated by us were all ball coherers. The following form was found very convenient to work with.

The balls *bb* were soldered to brass rods, one of which was in turn soldered to a stout metal post firmly embedded in the non-conducting base of the instrument. The other was supported by a brass spring, soldered to a plate which could be clamped to the base in any desired position. Against this spring a screw S could be pressed, allowing a fine adjustment of the contact.

In the investigation of a coherer of several contacts a glass tube g could be slipped over the balls to hold them in position.

In making observations, the method of procedure was the following: The coherer was adjusted by means of the screw so that it had a very high finite resistance. This resistance was roughly measured by means of an auxiliary circuit through the coherer, consisting of a battery of low electromotive force, a resistance and a galvanometer calibrated to indicate the current flowing in the auxiliary circuit.

This method was found to be preferable to one originally adopted by us, which was to set the contact by means of a microscope. By this method it was found difficult to get always an original high finite resistance, as sometimes when under the microscope the balls seemed to be in contact no coherer action took place on closing the main circuit. By using the method of adjustment with the auxiliary circuit, coherer action always took place on working the pendulum apparatus.

In each series of experiments the applied voltage of the battery B was kept constant and with a given resistance $(R + r)$ in series with the coherer the pendulum was released, and the readings of galvanometers I. and II. were noted: the contact was then readjusted to its original high resistance, and the observations repeated from 10 to 15 times.

Next $(R + r)$ was varied and a similar set of observations taken.

The range of the values of $(R + r)$ was so chosen that the current through the coherer was varied from 0.002 ampère to 0.7 ampère.

For each coherer, several such series of observations were obtained with different applied voltages at B.

RESULTS.

A. *Experiments on coherers of a single contact.*

1. *Steel.*—Balls of three different diameters were used, 4.75 mm., 6.35 mm. and 9.5 mm.

In the following tables i denotes the current through the coherer observed with the galvanometer G_2, p the potential difference between the terminals of the coherer observed with the galvanometer G_1 and the resistance of the coherer calculated from p and i.

TABLE I.

Steel: Diameter = 4.75 *mm. Applied voltage* 6.5 *volts.*

i	p	ρ	i	p	ρ
0.0027	0.0457	16.59	0.0521	0.234	4.48
0.0054	0.0894	16.62	0.124	0.229	1.84
0.0106	0.174	16.40	0.254	0.237	0.94
0.0273	0.208	7.63	0.492	0.228	0.47

TABLE II.

Steel: Diameter = 4.75 *mm. Applied voltage* 14.0 *volts.*

i	p	ρ	i	p	ρ
0.0026	0.0218	8.42	0.124	0.224	1.84
0.0064	0.0451	7.10	0.212	0.229	1.08
0.0127	0.0904	7.12	0.309	0.228	0.74
0.0256	0.160	7.07	0.604	0.235	0.39
0.0621	0.211	3.40			

TABLE III.

Steel: Diameter = 4.75 *mm. Applied voltage* 60 *volts.*

i	p	ρ	i	p	ρ
0.0051	0.0103	2.01	0.0989	0.141	1.42
0.0124	0.0218	1.77	0.247	0.196	0.78
0.0247	0.0458	1.94	0.494	0.232	0.47
0.0494	0.0832	1.79	0.617	0.226	0.37

TABLE IV.

Steel: Diameter = 4.75 *mm. Applied voltage* 220 *volts.*

i	p	ρ	i	p	ρ
0.0056	0.0021	0.38	0.221	0.0873	0.39
0.0223	0.0114	0.51	0.313	0.144	0.45
0.0551	0.0255	0.46	0.438	0.224	0.51
0.0749	0.0374	0.50	0.722	0.218	0.30
0.111	0.0534	0.48			

TABLE V.

Steel: Diameter = 6.35 *mm. Applied voltage* = 51 *volts.*

i	p	ρ	i	p	ρ
0.0118	0.024	2.04	0.162	0.186	1.09
0.0236	0.058	2.46	0.219	0.224	1.02
0.0477	0.104	2.20	0.479	0.239	0.50
0.0954	0.160	1.68	0.784	0.240	0.37

TABLE VI.

Steel: Diameter = 6.35 *mm. Applied voltage* = 220 *volts.*

i	p	ρ	i	p	ρ
0.0208	0.0134	0.64	0.297	0.121	0.41
0.0512	0.0250	0.49	0.420	0.178	0.42
0.103	0.0450	0.44	0.523	0.190	0.36
0.210	0.0760	0.36	0.696	0.230	0.33

TABLE VII.

Steel: Diameter = 9.5 *mm. Applied voltage* = 51.5 *volts.*

i	p	ρ	i	p	ρ
0.0106	0.0260	2.46	0.171	0.213	1.25
0.0257	0.0686	2.66	0.255	0.225	0.88
0.0512	0.0935	1.83	0.510	0.216	0.42
0.0727	0.160	1.72	0.836	0.224	0.27
0.105	0.181	1.68			

2. *Copper.*—Copper plated steel balls, diameter 9.5 mm.

TABLE VIII.

Copper: Diameter = 9.5 *mm. Voltage applied* = 50 *volts.*

i	p	ρ	i	p	ρ
0.0101	0.004	0.41	0.512	0.171	0.33
0.051	0.041	0.80	0.819	0.174	0.21
0.102	0.081	0.78	1.252	0.171	0.15
0.256	0.154	0.60			

3. *Lead.*—Diameter 3.0 mm.

TABLE IX.

Lead: Diameter = 3.0 *mm. Applied voltage* = 14.2 *volts.*

i	p	ρ	i	p	ρ
0.012	0.020	1.67	0.118	0.120	1.06
0.024	0.040	1.67	0.291	0.121	0.42
0.059	0.081	1.53	0.573	0.123	0.22

TABLE X.

Lead: Diameter = 3.0 *mm. Applied voltage* = 39.5 *volts.*

i	p	ρ	i	p	ρ
0.009	0.009	1.00	0.120	0.107	0.90
0.018	0.020	1.11	0.182	0.131	0.72
0.036	0.036	1.00	0.361	0.129	0.36
0.073	0.069	0.94	0.591	0.131	0.22
0.092	0.087	0.94			

TABLE XI.

Lead: Diameter = 3.0 *mm. Applied voltage* = 220 *volts.*

i	p	ρ	i	p	ρ
0.021	0.008	0.39	0.433	0.085	0.19
0.051	0.016	0.31	0.512	0.101	0.19
0.102	0.036	0.35	0.683	0.129	0.17
0.209	0.060	0.29	1.024	0.127	0.12

4. *Phosphor Bronze.*—Diameter 3.2 mm.

TABLE XII.

Phosphor Bronze: Diameter = 3.2 *mm. Applied voltage* = 220 *volts.*

i	p	ρ	i	p	ρ
0.022	0.024	1.07	0.317	0.182	0.60
0.055	0.032	0.58	0.442	0.189	0.43
0.110	0.072	0.64	0.727	0.197	0.29
0.222	0.112	0.51			

From the foregoing tables it appears that, for increasing i, p approaches asymptotically a finite maximum value. This value is different for different substances but the same for any one substance whatever the applied electromotive force and, at least in the case of steel, it is practically independent of the size of the balls.

We have plotted the results contained in tables I. to IV. in Fig. 3, those of tables V. and VI. in Fig. 4 and those of tables IX. to XI. in Fig. 5.

In these figures the abscissas are the currents and the ordinates the corresponding values of the terminal potential differences at the coherer.

The simplest differential equation which fulfills the condition that p approaches a finite maximum value P is $dp = k(P - p)pi$ where k is a constant whose dimensions must be an inverse of a current.

This differential equation when integrated gives $p = P(1 - \epsilon^{-ki})$ Eq. 1, and, as will be shown later on, this exponential equation expresses analytically almost all of our results.

It seems perhaps to be more than a mere coincidence that this formula is identical with that which one of us (Guthe, PHYS. REV., Vol. 7, p. 193, 1898) has shown to hold for the polarization of copper in copper sulphate solution.

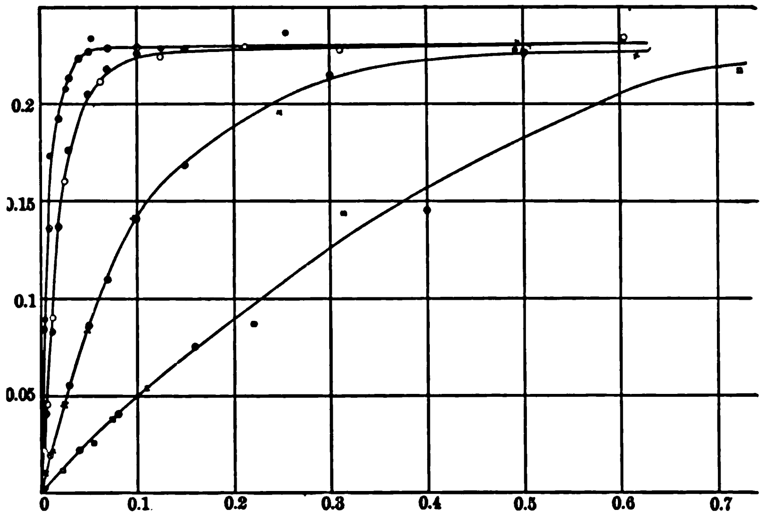

Fig. 3.

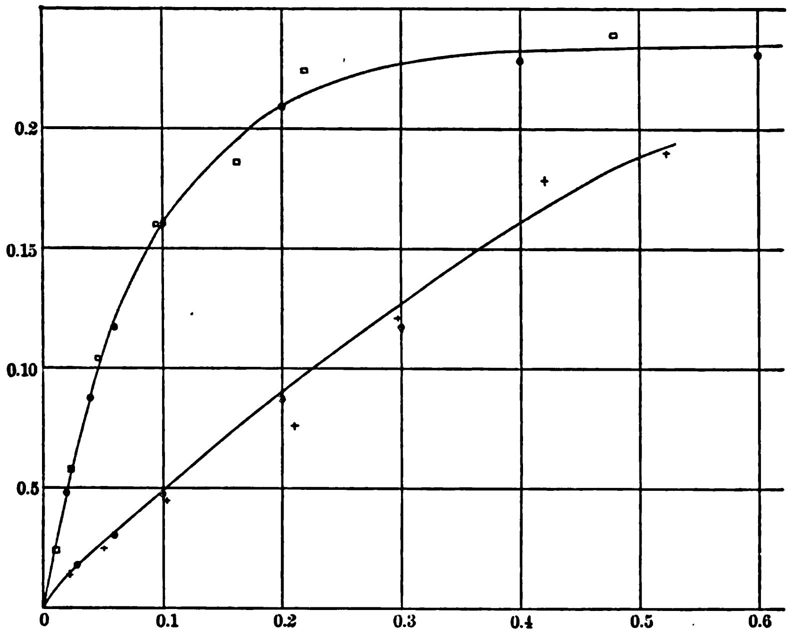

Fig. 4.

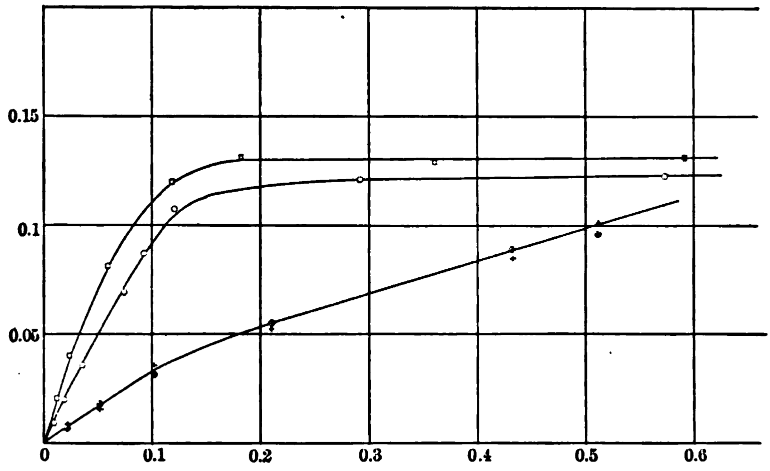

Fig. 5.

May not coherer action be explained in the following way?

On the surface of the metals in loose contact we have a condensed, badly conducting atmosphere, possibly water vapor. Thus before a current flows the resistance is very high.

For simplicity let us consider two spherical conductors in loose contact with these films between them. As soon as potential difference exists between them the films are squeezed together by electrostatic attraction and over a circular area *A* the thickness of the films will be of molecular dimensions.

In Fig. 6, using the same coördinates as in the foregoing figures, the tangent of *OT* would represent the resistance of the film *A* at the moment the voltage is applied. As the current flows ions from the positive electrode break through the film forming metallic contact and thus reducing the resistance.

After the current stops flowing the balls are in intimate contact and held together by cohesion. Therefore the resistance does not increase again after the current ceases to flow.

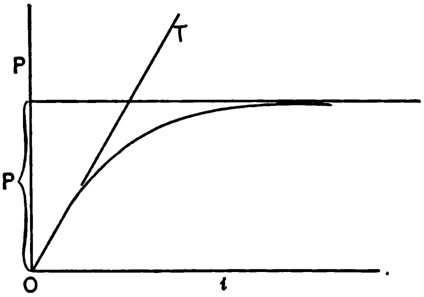

Fig. 6.

That the resistance after lowering is not affected by a current *smaller* than that which produced the lowering was found to be the case by a series of experiments, while if a *larger* current were sent through, the resistance adjusted itself to the value it would have had had we started with a very high original resistance and sent through it this larger current.

In general we found that it makes very little difference what pressure the coherer is under before the current is sent through provided always the pressure is varied within reasonable limits.

As mentioned above the coherer particles under electrostatic pressure are in contact to within molecular distances over a certain area *A*. Now we make the supposition that a definite minimum number of ions is needed to carry any given current *i*.

Let *P* be the difference of potential over a film *A* filled with a number of ions just sufficient to carry the current *i*. Or in other *P* is the potential difference over a molecular layer of ions in which there flows through each ion the maximum current it can carry. If the current be increased beyond this point one of two things might happen. Either the area A can be increased by the ions filling out a space exterior to A and the current be now carried by an area proportional to it or if the area cannot change then heating and perhaps melting might take place.

The area A defined above we will in future call the *critical area* and we will assume that the first of the above possibilities is what takes place in the coherer as it explains the horizontal part of our curves where $P=$ const.

Suppose now in the case of a given current i the area of the film of molecular thickness to be larger than the one which, when filled with ions, would carry this current. A greater than the *critical number* of ions will break through the surface in the first impulsive rush when the voltage is applied. In consequence the difference of potential produced will be less than the critical value P.

If n ions would just carry the current i we would have an ions in the case of a larger surface where a is greater than unity and will increase with the size of the contact surface.

For the critical surface the potential difference would be, according to definition, $P=\frac{r}{n}i$, for a surface larger than the critical $P=\frac{r}{an}i$, *i. e.*, $p<P$. r would be the ohmic resistance of an ion and a can be determined from our differential equation.[1]

This would mean that the nearer filled up the area is the less readily the ions will go over and hence the increase of p with increasing i will become smaller and smaller until it finally becomes zero when P is reached.

A rather striking relationship between the critical voltages P of the substances investigated by us suggests itself.

From the tables above we have P steel 0.23 volts, P copper 0.173 volts, P lead 0.127 volts. Supposing the valency of Fe and

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[1] In case of surfaces of very large dimensions as in ordinary metallic contact then a will approach ∞ and the resistance of the contact which is $-r/an$ would approach zero. Now $dp/di=kri/n(1-1/a)$ and i/n is a constant. If a is so large that $1/a$ can be neglected we have the case of a continuous conductor and our differential equation reduces to Ohm's law.

Cu to be 2 and of Pb to be 4 then the gramme equivalents are respectively 28, 31.75 and 51.75.

Multiplying P/Fe by 28 gives 6.44,
P/Cu by 31.75 gives 5.50,
P/Pb by 51.75 gives 6.57,

or $P \times$ gramme equivalent = const. nearly.

When we come to consider the curves for different applied electromotive forces we have to expect different electrostatic attraction between the balls to begin with.

Let us assume Hooke's law to hold for the coherer substance and that no shearing takes place. Let S be the area on which the pressure π acts, then $d\pi = a'Sdl$ where dl is the displacement produced and a' is a constant. For displacements as small as we must assume them here to be $dS \sim dl$ from which follows $d\pi = aSdS$ or the total pressure $\pi = a/2S^2$, Eq. 2.

Now to the same order of approximation π varies as the square of the potential gradient; therefore, very nearly, S varies directly as E. But the difference of potential necessary to bring the coherer balls into point contact is P (according to the assumption made above) hence if the applied voltage be called ES varies as $(E-P)$.

As shown in Fig. 6 the tangents of the curves at the origin are a measure of the resistance of the areas S. Now since for ($\lim i = 0$) all our curves are straight lines the law of resistances must hold for the films at this point and ρ must be inversely proportional to S.

If this is true then the general form of our differential equation is $dp = c(P-p)1/(E-P)di$, Eq. 3.

The tangent for any curve, $E =$ const. for ($\lim i = 0$) is $CP/(E-P) = kP$. As mentioned above this is the resistance of the film between the balls when the voltage E is applied. $k(E-P)$ should then be constant for one and the same substance. That this is so will be shown in Table XIX. In the tables preceding this one we have compared the numerical values calculated by means of Eq. 1 with the experimentally determined ones from our curves.

The results with phosphor bronze were rather irregular as compared with those for steel and copper.

Table XIII.

Steel: Small Diameter. Applied voltage = 6.5 *volts.* $k = 90$. $P = 0.23\ v$.

i	p from curve.	p calculated.	i	p from curve.	p calculated.
0.005	0.078	0.083	0.04	0.223	0.223
0.01	0.136	0.136	0.05	0.227	0.226
0.02	0.192	0.191	0.07	0.229	0.228
0.03	0.213	0.212	0.10	0.229	0.229

Table XIV.

Steel: Small Diameter. Applied voltage = 14.0 *volts.* $k = 45$. $P = 0.23\ v$.

i	p from curve.	p calculated.	i	p from curve.	p calculated.
0.005	0.040	0.042	0.05	0.205	0.205
0.01	0.080	0.083	0.07	0.218	0.219
0.02	0.137	0.136	0.10	0.223	0.226
0.03	0.170	0.176	0.15	0.228	0.229

Table XV.

Steel: Small Diameter. Applied voltage 60 *volts.* $k = 9.5$. $P = 0.23\ v$.

i	p from curve.	p calculated.	i	p from curve.	p calculated.
0.01	0.020	0.021	0.10	0.141	0.140
0.03	0.056	0.057	0.15	0.169	0.168
0.05	0.086	0.087	0.3	0.214	0.216
0.07	0.110	0.111	0.5	0.227	0.227

Table XVI.

Steel: Small Diameter. Applied voltage 220 *volts.* $k = 2.5$. $P = 0.23\ v$.

i	p from curve.	p calculated.	i	p from curve.	p calculated.
0.04	0.022	0.022	0.4	0.145	0.145
0.08	0.041	0.041	0.8	0.198	0.198
0.16	0.075	0.076			

Table XVII.

Steel: Medium Diameter. Applied voltage = 51 *volts.* $k = 12$. $P = 0.23\ v$.

i	p from curve.	p calculated.	i	p from curve.	p calculated.
0.02	0.048	0.048	0.2	0.210	0.209
0.04	0.087	0.087	0.4	0.232	0.228
0.06	0.120	0.117	0.6	0.235	0.230
0.1	0.160	0.160			

Table XVIII.

Steel: Medium Diameter. Applied voltage $= 220$ *volts.* $k = 2.4.$ $P = 0.23$ *v.*

i	p from curve.	p calculated.	i	p from curve.	p calculated.
0.03	0.018	0.018	0.2	0.090	0.087
0.06	0.030	0.030	0.3	0.125	0.117
0.1	0.048	0.048			

Table XIX.

Summary for Steel Balls.

(*a*) Small Diameter.			(*b*) Medium Diameter.		
E	k	$k(E-P)$	E	k	$k(E-P)$
6.5	90	564	51	12	610
14.0	45	619	220	2.4	528
60.0	9.5	568		Mean	569
220.0	2.5	550			
	Mean	575			

Table XX.

Copper: Applied voltage $= 50$ *v.* $k = 6.$ $P = 0.171$ *v.*

i	p from curve.	p calculated.	i	p from curve.	d calculated.
0.03	0.028	0.028	0.2	0.123	0.120
0.06	0.050	0.051	0.3	0.150	0.156
0.1	0.081	0.080	0.6	0.172	0.171

$$k(E-P) = 298.$$

The curves for lead, while they have the same general form as the others, cannot be represented by our equation 1 except in the case where $E = 220$ volts. Certainly we cannot expect very concordant results with a substance as inelastic as lead. The lead balls on examination after use showed that they had undergone a permanent deformation.

Table XXI.

Lead: Applied voltage $= 220$ *v.* $k = 2.76.$ $P = 0.127$ *v.*

i	p observed.	p calculated.	i	p observed	p calculated.
0.021	0.0081	0.0071	0.433	0.0848	0.885
0.051	0.0161	0.0168	0.512	0.101	0.961
0.102	0.0363	0.0312	1.024	0.127	0.119
0.208	0.0525	0.553			

$$k(E-P) = 607.$$

In the curves given above the calculated values are plotted with the observed and marked by a ⊕ In general the agreement seems to be very satisfactory.

We satisfied ourselves that we have here to do not with a time-rate of heaping up of ions until a maximum is reached ($dp \sim dq$) but rather that the potential difference depends, in the main at least, on the current. This we did by changing the position of key I. (see Fig. 1) thus varying the time during which the current flowed within a considerable range without any appreciable effect on the value of p.

Additional Results.

Besides the above observations on coherers of a single contact we made a test of the effects of the number of contacts on the critical voltage P.

From the foregoing theory we must expect that the formula for a coherer of n contacts our differential equation must be

$$p = nP(1 - e^{-ki})$$

for if we have n contacts our differential equation must be

$$dp = k(nP - p)di$$

or we would expect that the potential difference is additive in character.

Table XXII.

Steel: Small Diameter. 5 contacts. Applied voltage = 15.2 *v.*

i	p	ρ	i	p	ρ
0.00363	0.228	62.8	0.130	1.258	9.7
0.00744	0.499	60.4	0.630	1.216	1.9
0.0142	0.832	58.6			

Table XXIII.

Steel: Small Diameter. 5 contacts. Applied voltage = 51.75 *v.*

i	p	ρ	i	p	ρ
0.0127	0.156	12.3	0.101	0.915	9.1
0.0254	0.339	13.5	0.251	1.210	4.8
0.0508	0.524	10.3	0.499	1.227	2.5

TABLE XXIV.

Steel: Small Diameter. 5 *contacts.* *Applied voltage* = 220 *v.*

i	p	ρ	i	p	ρ
0.00565	0.0166	2.94	0.223	0.478	2.14
0.0223	0.0686	3.08	0.316	0.707	2.23
0.0551	0.145	2.63	0.442	0.998	2.27
0.111	0.305	2.74	0.722	1.112	1.53

TABLE XXV.

Steel: Small Diameter. 10 *contacts.* *Applied voltage* = 220 *v.*

i	p	ρ	i	p	ρ
0.0226	0.143	6.33	0.313	1.684	5.38
0.0551	0.324	5.88	0.438	2.200	5.02
0.110	0.665	6.03	0.727	2.270	3.12
0.221	1.112	5.03			

The above tables are sample ones from the experiments on several contacts. We also investigated lead and phosphor bronze and found them to exhibit the same behavior, *i. e.*, $p_n = np_1$.

It may be mentioned here that in working with coherers it is advisable to have the coherer substance always in the same condition as regards oxidation. This has been pointed out by other observers. We usually cleaned the balls with the finest grade of emery paper and then let them stand for several hours before using. If used directly after cleaning a high initial resistance could not be obtained when the balls were in contact.

After several days' use a marked oxidation was observed to have set in and the critical voltage was found to have become somewhat higher.

The following example brings out this point. A set of balls freshly set up gave the following critical voltages:

One contact, 0.232 volt.
Two contacts, 0.468 volt.
Four contacts, 0.936 volt.
After a few weeks the values were:
Five contacts, 1.43 volts.
Ten contacts, 2.85 volts.
Twenty contacts, 5.84 volts.

The effect of oxidation may be that the badly conducting oxide particles make our hypothetical film between the balls thicker and hence the resistance of the film greater and the terminal potential differences also greater.

It may be of interest to point out the similarity with the behavior of electrolytic cells. Copper in copper sulphate after the copper is slightly oxidized shows a very much higher value of P than when the electrodes are clean.[1]

To satisfy ourselves that the coherers used were sensitive to Hertz radiations we set up a Hertz oscillator in the same room with the apparatus described above. The coherer was connected neither to earth nor to a vertical wire.

The resistance of the coherer was measured before and after the lowering due to the radiation by the method employed throughout this investigation, being careful, however, to use a current so small that it alone would produce no further lowering of the resistance of the coherer after radiation.

The Hertz oscillator was placed at distances varying from 2 m. to 7 m. from the coherer. The lower limits of the resistance were wholly independent of the original high resistance and decreased not quite as fast as the square of the distance.

CONCLUSION.

From the present investigation and from the work of other observers it seems to be justifiable to attribute the high original resistance of the coherer to the film on the surface of the metallic particles.

Dorn has shown that very little or no coherer action takes place between surfaces of the noble metals when they have been heated in a vacuum so as to expel as far as possible the surface film.[2]

He also found a decided decrease of resistance for other metals.

Aschkinass,[3] on the other hand, tried to remove the surface film from the filing particles in a tube by exhausting and heating but did not observe any change in the behavior of the coherer. He admits himself, however, that this can hardly be considered a decisive test.

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[1] See Guthe and Atkins, Proc. A. A. A. S., 1899, p. 109.

[2] Dorn, Wied. Ann., 66.

[3] Loc. cit.

The fact mentioned above that metallic surfaces after thorough cleaning do not show a high contact resistance is also in favor of our supposition and we believe that Auerbach's results can easily be explained by the rubbing away of the films by mechanical action.

The films must form a sort of protecting layer on the metal surface as the following experiment shows:

Two coherers, consisting of one contact each, were put in parallel and the circuit closed through them. The resistance of each was measured before and after lowering and it was found that coherer action took place in one only. After standing a day this one was always oxidized while the other showed a bright surface where the balls had been in contact. The coherer action had apparently destroyed the protecting film of the one in which it took place.

Aschkinass mentions his interesting results with coherers consisting of PbO_2 or CuS particles as a proof against Lodge's theory. They behave in the opposite way from the ordinary coherer.

We would suggest in view of the similarity with polarization that the compounds are in this case broken up into two components, one of which conducts well and the other poorly. No actual metallic contact need then occur and the S or O would play a decided rôle in the increase of the resistance.

It seems that particles of any conducting substance may form a coherer—among others carbon. This suggests an interesting question in connection with the electric arc.

Ayrton and Perry[1] found that the difference of potential between the two carbons of an arc lamp was independent of the current strength provided the distance between them was kept constant, *i. e.*, the *apparent* resistance of the arc was inversely as the current.

Does this correspond to the horizontal line of our curves and does cohesion play any part in the starting of the arc?

We offer the foregoing considerations merely as a tentative explanation of this little-understood phenomenon, being well aware that some of our assumptions are somewhat crude, but hoping nevertheless that they may lead to a fuller understanding of the subject since they seem to furnish a satisfactory hypothesis.

PHYSICAL LABORATORY OF THE UNIVERSITY OF MICHIGAN, June, 1900.

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[1] Phil. Mag., May, 1883.

ELECTRICAL RESISTANCE OF THIN FILMS DEPOSITED BY KATHODE DISCHARGE.

BY A C. LONGDEN.

INTRODUCTION.

IF standard high resistances of great precision and unvarying values could be obtained at low cost, their application in the determination of insulation resistance by the direct deflection method would soon cease to be their sole field of general usefulness. The condenser and ballistic galvanometer would no longer be regarded as important or even desirable in comparing electromotive forces, and a high resistance could even be used with some advantage in place of a condenser, in the condenser method, for measuring internal battery resistance.

The use of numerous shunts in the determination of figure of merit is always regarded as something to be endured rather than desired. With a suitable high resistance in series with the galvanometer and standard cell, the determination of figure of merit becomes absolutely simple. The range of the Wheatstone bridge may also be enormously increased by the use of high resistances as bridge arms, and heavy currents may be measured with delicate galvanometers in series with high resistances, without making the resistance of the shunt through which the main current passes so small that the percentage of error in the calculations shall be large.

The use of carbon as a high resistance material is tolerably satisfactory for some purposes, if we are willing to re-standardize our resistances every time we use them, and to reckon with the enormously high temperature coefficient of the material; even then, the uncertainty of the contacts in most forms of carbon resistances is so great as to condemn such resistances in all cases where anything like careful or accurate work is contemplated.

A number of forms of carbon resistances have been used in connection with the research which furnishes the subject matter of this article. It would not be in place to give detailed descriptions of them here, but it may be well to say in passing, that those which were the most nearly trustworthy consisted of sticks of pipe clay saturated with sugar solutions, of different degrees of concentration —the sugar being subsequently carbonized in the sticks by continued exposure to a red heat, in the absence of oxygen. The resistance of the stick depends upon the degree of concentration of the sugar solution used in preparing it.

While these resistances were very satisfactory for carbon, it must still be said that no carbon resistance can be considered for a moment in comparison with standard wire resistances.

In wire resistances, the use of alloys instead of pure metals is based upon the fact that alloys have, in general, lower temperature coefficients and higher specific resistance than pure metals ; but it must be borne in mind that, so far as high specific resistance is concerned, it is not in itself an advantage, but is only a means to an end. A wire, having a high specific resistance, enables us to obtain a high resistance, having small weight, small bulk and comparatively low cost. If these conditions could be met as well or better in some other way, high specific resistance would be of no importance whatever.

Alloys are certainly inferior to pure metals in some respects. Aside from the molecular rearrangement which may be going on in either the alloy or pure metal, alloys suffer from disintegration and possibly from internal chemical changes which are impossible in pure metals. It is also true that alloys frequently suffer from contact with their surroundings. Manganin, for example, is very easily oxidized, and is even pronounced by some investigators as worthless. Among the pure metals there are several which resist oxidation and other chemical changes admirably. Now if it is possible to obtain a high resistance in the form of a pure metal, and at the same time to retain all the advantages of an alloy, such a resistance ought to soon find favor in the electrical world.

From the results of some work done a few years since by Miss Isabelle Stone,[1] we have reason to believe that metals in the form of thin films upon glass exhibit certain qualities unlike those of the same metals in the ordinary form. Miss Stone reached three conclusions, which may be stated as follows :

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[1] "On the Electrical Resistance of Thin Films," PHYSICAL REVIEW, Vol. 6, pp. 1–16.

1. The electrical resistance of such films as she investigated decreases in value quite rapidly for a short time, then less rapidly for a much longer time.

2. The higher the resistance of the film, the more rapid is the decrease in value.

3. For very thin films, the ratio of the measured resistance to the calculated resistance is high.

Miss Stone's research included silver films only, and these were deposited from aqueous solutions by what is known as the Rochelle salt method ; but her work is suggestive of a very large field of research, if other methods of deposition could be used.

As early as 1877 Professor A. W. Wright[1] of Yale, described a method of depositing thin metallic films upon glass by electrical discharge. Professor Wright produced both opaque and transparent mirrors from a large number of metals, pointed out the difference in rate of deposition of different metals, and suggested that on account of the difficulty of depositing aluminum and magnesium, these metals should be used for electrodes in vacuum tubes in order to avoid the discoloration so common in the neighborhood of the kathode when platinum electrodes are employed.

About two and a half years ago it was my good fortune to learn the practical details of Professor Wright's process in Ryerson Physical Laboratory at the University of Chicago ; and to have the opportunity of preparing a number of mirrors and thin metallic films, with the admirable apparatus designed for this purpose by Professor Stratton and Dr. Mann, of that institution.

DEPOSITION OF FILMS.

The deposition is effected in a vacuum, by a process which may here be included for convenience under the general term kathode discharge,[1] from an electrode consisting of the metal to be deposited.

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[1] "On the Production of Transparent Metallic Films by the Electrical Discharge in Exhausted Tubes," Am. Journal of Science and Arts, Vol. 13, pp. 49–55.

"On a New Process for the Electrical Deposition of Metals and for Constructing Metal-Covered Glass Specula," Am. Journal of Science and Arts, Vol. 14, pp. 169–178.

The necessary apparatus for doing the work advantageously includes a vacuum pump capable of reducing the pressure in the receiver to a few millionths of an atmosphere; an induction coil capable of producing a spark eight or ten centimeters long; an interrupter making a complete break in the circuit, and preferably of high frequency; and a source of electrical energy, capable of furnishing a current of several ampères, at a pressure of not less than fifty volts, if the Wehnelt interrupter is used.

The pump used in the part of the work done in Chicago was a double acting Geissler pump, all glass, and exhausting into a very good secondary vacuum at each end of the stroke, so that the mercury in the main pump was never in contact with the air. This pump is capable of producing a splendid vacuum, but must be rather carefully handled, as the tendency to develop leaks is somewhat aggravated by the fact that the entire apparatus is in continual motion when in use.

In continuing the work at Columbia University, it seemed desirable to construct a pump which should be free from constant danger of developing leaks, and which should be capable of producing the required vacuum rapidly and easily. The Sprengel pump is slow, and the ordinary Geissler pump with its numerous ground joints, valves and stop-cocks, and its rubber connecting tube, is a little uncertain, and at best, somewhat less effective than the necessities of this case require. A Geissler pump was finally decided upon, but not without a determination to eliminate some of its objectionable features.

Figure 1 shows a diagram of the working parts. The right-hand side of the pump, as viewed in the diagram, is in some respects similar to the Bessel-Hagen[2] pump, but simpler. The tube (*A*) leading from the exhaust chamber to the receiver rises to the height of a full meter above the level of the mercury at (*B*), when the reservoir (*C*) is elevated. This feature of the apparatus is used instead of a valve to prevent the flow of mercury from the exhaust chamber (*D*) to the receiver. As the mercury sometimes rises in this tube with considerable momentum, it reaches a point considerably above barometric height, but the bulb (*E*), two or three centimeters in diameter, arrests any unusually active mercury which might otherwise pass around the bend at the top of the tube. The tube (*F*) completes the passage to the receiver, by way of the drying chamber (*G*), which is sealed on with the blow-pipe. There are no ground joints or even mercury seals in any part of the apparatus.

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[1] The nature of the process will be considered more in detail hereafter.

[2] "Ueber eine Neue Form der Toepler'schen Quecksilberluftpumpe und einige mit ihr angestellte Versuche," Wied. Ann., Vol. 12, pp. 425–445.

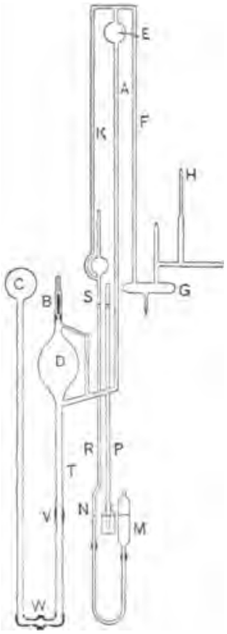

Fig. 1.

The tube (*H*) is a capillary through which air may be gradually admitted when the receiver is to be opened. The tube (*K*) leads to the McLeod gauge (*L*). The reservoir (*M*) is for the purpose of filling the McLeod gauge. Before beginning to exhaust, the mercury in the gauge tube stands at (*N*), at the same level as the mercury in the cistern of an independent barometer (*P*). As the exhaustion proceeds the mercury rises in the tube (*R*) until it stands at (*S*), the level of the mercury in the independent barometer. Of course the tube (*R*) answers the purpose of a gauge until the mercury rises so high that the difference between the two mercury columns is not easily readable. After this the McLeod gauge is used.

The rubber tube which usually connects the reservoir with the exhaust chamber in the Geissler pump is entirely displaced in this case, by an iron pipe with a swinging joint. This improvement originated with Professor Wm. Hallock, of Columbia. The glass tube (*T*) extends downward from the exhaust chamber as far as (*V*), where it is securely cemented into an enlargement on the iron pipe. From this point on to the reservoir (*C*) the mercury passage consists entirely of iron pipe. The swinging joint (*W*) is a carefully selected pipe union with well polished bearing surfaces moving upon a leather washer. The reservoir is raised and lowered by simply swinging the pipe, and the rubber tube is thus entirely eliminated.

This pump has now been in use several months, and its behavior is most thoroughly satisfactory. Entire freedom from danger of leaks is a source of inestimable satisfaction in work of this kind. The results obtained are so uniform that, as long as the same receiver is used, it is easy to tell before beginning an operation, just how many strokes will be necessary to produce a certain degree of attenuation in the receiver. So confidently can this uniformity of results be relied upon that the McLeod gauge has become almost unnecessary.

The exact degree of exhaustion which this pump is capable of producing has not been carefully determined, because it has not been necessary to push it to its limit; but the fact that a pressure of one hundred thousandth of an atmosphere may be reached with extreme ease and certainty is evidence that the limit of usefulness of the pump has not been approached.

Notwithstanding the fact that this pump is single-acting and open to the air at one end of the stroke, the necessary vacuum for the deposition of metals can be produced with it in less than half the time required for the same operation with the double-acting pump already referred to.

The induction coil used in the earlier experiments was a large one, capable of producing a 30-cm. spark in air. The coil used later was not more than half as large; but a coil half the size of either of them would be abundantly large for the purpose.

The current interrupter used in the early part of the work was a mechanical interrupter operated by an electric motor in a separate circuit. The speed of the motor was usually about 1,500 revolutions per minute, with two breaks per revolution. This was quite satisfactory except that the deposition of metal would have been more rapid with an interrupter of greater frequency. The Wehnelt interrupter[1] is so well suited to this work that no other has been used since its advent.

The receiver in which the films are deposited is represented in vertical section in figure 2. (*AA*), figure 2, is a rubber stopper through which a glass tube (*B*) enters the receiver. This tube serves at

[1] Elektrotechnische Zeitschrift, Vol. 20, pp. 76–78.

once as an exhaust tube and as a passage way for the kathode wire (*C*). The tube and stopper are securely cemented into the open top of the receiver, with sealing wax (*DDDD*). The lower end of the kathode wire terminates in a thin aluminum tube, just large enough to drive firmly into the lower end of the glass tube (*B*). The kathode plate (*E*) which consists of the metal to be deposited, is supported by an aluminum rod (*F*), which slides into the thin aluminum tube with just enough friction to hold it in place. (*G*) is a heavy aluminum base plate, which serves as the anode, and (*H*) is an additional aluminum plate, which serves as a support for the glass plate (*L*), upon the upper surface of which the film is to be deposited. The dimensions may be taken from the figure as it is drawn to scale.

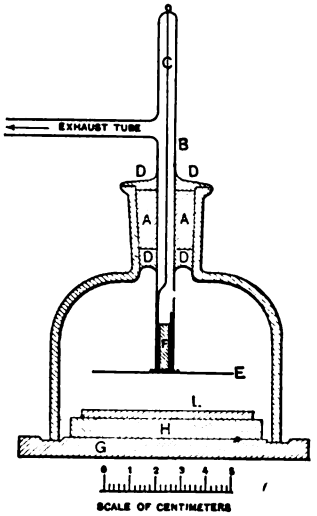

Fig. 2.

When the air is exhausted from the receiver and discharges from the kathode of an induction coil take place from the surface of (*E*), particles of the kathode plate (*E*) are deposited in the form of a brilliant film upon the surface of the glass plate (*L*) and, in fact upon the entire inner surface of the receiver.

The character of the film depends largely upon the rate of deposition, and this in turn depends upon the vacuum, the electromotive force, the current, the frequency of the interrupter and the distance from the kathode to the glass plate upon which the film is to be deposited. If all the conditions are properly adjusted, moderately rapid deposition will produce films of great hardness, density and brilliancy. If the deposition is too rapid, the resulting films will possess these qualities in less degree.

The factors which have been enumerated as affecting the rate of deposition are so intimately related, and so dependent upon each other, that it is quite impossible to discuss them independently. Professor Wright produced beautiful mirrors from a very small kathode in a 2 mm. vacuum at a 3 mm. distance with a primary electro-motive force of perhaps a dozen volts, but in order to obtain an even distribution of metal over any very considerable surface, he found it necessary to keep the kathode moving over the surface during the process of deposition. It is just as easy to obtain an even distribution of metal over a large surface from a stationary kathode by using a correspondingly large kathode and placing it at a large distance from the glass. This, however, involves working in a higher vacuum and using a higher electro-motive force; and in order that the process may be as rapid, it necessitates using either a stronger current or a higher interruption frequency. A glass surface 5 x 6 centimeters may be beautifully platinized by placing it at a distance of 12 or 15 millimeters from a kathode plate of similar dimensions, in a vacuum of from .0001 to .00001 of an atmosphere, and operating with a primary current of 5 or 6 ampères at 110 volts, and with an interrupter frequency of about 300 per second.

The two factors, vacuum and distance, are related to each other in a way which demands a more detailed consideration. With a certain fixed vacuum, if the distance from the kathode to the glass plate is much too great, the film will be soft and spongy; while if the distance is much too small it is almost impossible to get any metal deposited at all. It is found by experiment that the golden mean between these two extreme conditions is attained when the surface of the glass plate is just about in the plane which marks the

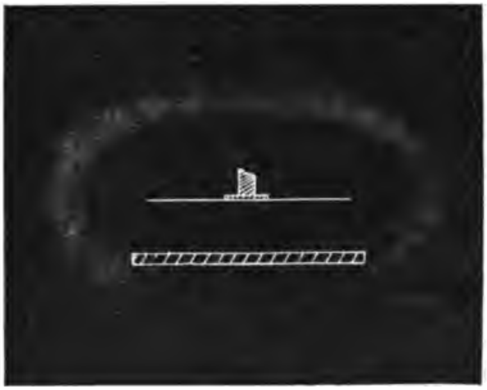

Fig. 3.

boundary between the kathode space and the luminous glow which surrounds it. (See figure 3.) Films deposited at other positions vary very greatly in hardness and density. Some platinum films will scarcely endure the touch of a camel's hair brush, while others can scarcely be removed from the glass by the most vigorous rubbing, or by the action of hot aqua regia.

At the suggestion of Professor Rood an attempt was made to discover the condition of the metal during its transition through the kathode space. A glass plate 5 x 6 centimeters, was placed at the usual distance, about 15 mm. from the kathode. In the center of this plate was placed a small aluminum stand, supporting a small glass plate about 12 millimeters square, within 3 mm. of the kathode, as in Fig. 4. The air was then removed from the receiver until the kathode space just reached the surface of the large glass plate (*A*). A platinum film of considerable thickness was then deposited. When the plates were removed from the receiver, it was found, first that the film on the upper surface of the small plate (*B*) was exceedingly thin; next, that the stand had not cast a distinct shadow, but that the film on the large plate under the stand gradually shaded off to a comparatively thin center, as if the particles of platinum had drifted under the stand in considerable quantities. The third and most surprising fact to be noted was that the unprotected corners of the small plate were quite heavily coated on the *under* side.

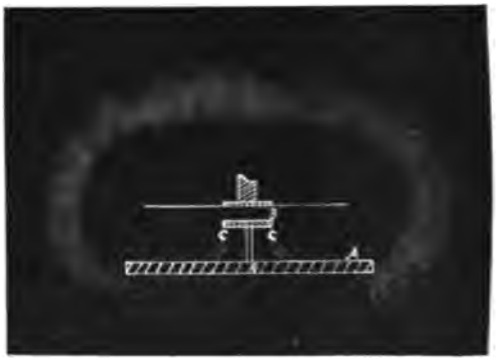

Fig. 4.

In the light of these facts there can scarcely be any doubt in regard to the nature of the process. The surface of the kathode is intensely heated, and particles, probably molecules, possibly smaller particles,[1] are projected into space. These particles radiate from the kathode in the gaseous form until they reach the limit of what is called the kathode space. In other words the kathode space *is* the space in which the metallic matter radiated from the kathode is still in the gaseous state. When the temperature has fallen to a sufficient degree, condensation begins, and we have the visible glow just outside the kathode space—a miniature snow-storm.

The very hot metallic gas near the kathode will not easily adhere to and condense upon the glass, and the comparatively cool "*vapor,*" if we may use the term, condenses in rather a loose, soft, spongy layer. It is on this account that the best mirrors are formed just in the *edge* of the kathode space, as described on page 47.

This view is supported by the fact that the visible glow rises to the edge of the small stand as represented at (*C*), in figure 4. The metallic gas flowing over the surface of the stand is cooled somewhat, and the snow-storm therefore begins at a shorter distance from the kathode in this region than in the free space in the other parts of the receiver. Further evidence is offered in the fact that the glow around the stand is more conspicuous at first than it is after the stand itself has become somewhat heated.

The process seems to be simple distillation, in which the vaporization of the kathode depends *largely* upon its electrification.[2] That the process is not *entirely* dependent upon electrification, however, is evident from the fact that selenium, which *boils* at about 700 degrees, is deposited thousands of times more rapidly than platinum.

It is a noteworthy fact that when a rectangular kathode is used in a cylindrical receiver, the deposit on the sides of the receiver is thickest at small areas opposite the corners of the kathode. This is not because the distance is less, but because the surface density of the charge is greater at these points.

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[1] J. J. Thompson, in Phil. Mag., Dec., 1899, "On the Masses of the Ions in Gases at Low Pressures," says that in case of the stream of negative electrification which constitutes the kathode rays, there are reasons for thinking that the charge on the ion is not greatly different from the electrolytic one, and that in the former case we have to deal with masses *smaller than the atom.*

[2] See Sir WilliamCrookes "On Electrical Evaporation," Electrical Review, Vol. 28, pp. 796–798 and 827, 828.

In conducting this process, it is not an easy matter to keep the vacuum at a fixed value during the first part of the experiment. When the circuit is first closed, the effect is to drive off the residual air and occluded gases. This produces a change of pressure in the receiver, which is quite rapid at first, but less and less rapid as the process continues. The rate at which the first change takes place depends largely upon the nature of the kathode, the condition of the atmosphere to which it has been exposed and the length of the exposure.

Under what might be called ordinary conditions, when a platinum kathode is used, it is not well to allow the current to continue more than a few seconds when first turned on, without stopping to observe the condition of the vacuum. If the vacuum is allowed to fall below .0001 of an atmosphere[1] there is danger of the film being rather soft. The film in this condition will not adhere well to the glass. Hence the importance of being particularly careful about the vacuum at first.

The curves (*A*) (*B*) and (*C*) in figure 5, represent the rate of deterioration of the vacuum during the rapid part of its change. The vertical portions of the curves represent intervals during which the coil is not in operation, but the pump is being used to improve the vacuum. These curves are all three for platinum films. Curve (*A*) represents an extreme case in which, after depositing a film, the receiver had been quickly opened, the film removed, new glass inserted and the receiver again sealed and exhausted—all within a few minutes. Curve (*C*) represents another extreme case in which both the kathode and the inside of the receiver had been exposed to the air for a long time and under very unfavorable conditions. Curve (*B*) may be said to fairly represent the deterioration of the vacuum during the first ten minutes of the process under average conditions. It must be understood that in all these cases, the minutes represented on the axis of abscissæ are not consecutive minutes, but minutes during which the process of deposition is actually going on.

Usually after ten or fifteen minutes of *actual deposition*, the condition of the vacuum does not change much and the deposition may then go on continuously if the strength of the current used is not such as to heat the receiver excessively. After this stage has been reached, if the process is discontinued for half an hour or so, if there has been no perceptible leakage, the vacuum is found to have improved considerably on account of the fact that most of the residual gas in the receiver has been occluded by the kathode and film. If the current be again started, however, the vacuum will soon fall to about its normal value.

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[1] With the distance, voltage, etc., as stated on page 47.

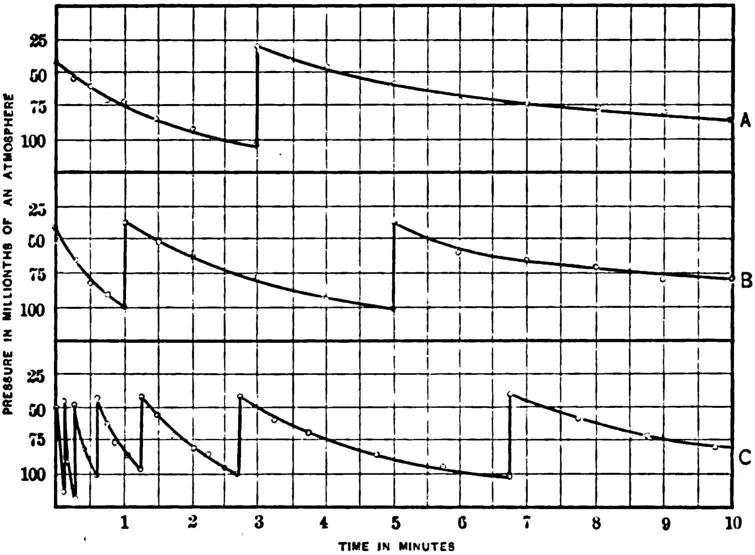

Fig. 5.

Electrical Properties of Films.

Besides possessing splendid reflecting surfaces, such as commend them strongly for all high grade optical work, and besides displaying the colors of the different metals by transmitted light, and the selective absorption of different thicknesses of the same film, metallic films deposited in accordance with the method here described possess certain advantages as electrical resistances.

Without an exact method of determining the thickness of a film it is impossible to make an exact estimate of its specific resistance; but even with only an approximate determination of thickness, the very rapid increase in resistance corresponding to diminishing thickness is so conspicuous as to leave no room for doubt that in *thin* films the ratio of the measured resistance to the calculated resistance is high. For example, a platinum film 5 cm. long, 15 mm. wide and .0002 mm. thick has a resistance of only a few ohms, while a film apparently about one tenth as thick[1] has a resistance of several hundred ohms, and a film probably about one hundredth as thick, has a resistance of hundreds of thousands or even millions of ohms.

There is plenty of room for further investigation in this direction, but even as the matter stands, it seems quite unnecessary to use alloys for the purpose of obtaining high specific resistance.

Quite early in the history of this investigation it was observed that during the heating and cooling of certain resistances, for the purpose of artificial ageing, the resistance changes were not as great as the temperature changes seemed to warrant, and in one noteworthy case the resistance change seemed to be in the wrong direction. Accordingly, the temperature coefficient of this particular film was carefully determined.

This film was deposited April 4, 1898. During the preliminary treatment for bringing it to a condition of stability, its resistance was measured quite frequently. The measurements were made at temperatures differing by a few degrees, and, even during the first few days, while the changes in resistance were quite large, there was at least an indication that the temperature coefficient was probably negative.

On April 21st, the temperature of the film was reduced 19 degrees, and the reduced temperature was kept constant for several hours. This fall of temperature was accompanied by an *increase* of resistance amounting to a little more than 6 ohms, the total resistance of the film being a little less than 2,300, though it had not yet reached its final value. When the temperature of the film was raised to its former value, the resistance *fell* about 4 ohms. The discrepancy between the 6 ohms rise and the 4 ohms fall was due to the fact that the process of artificial ageing was not yet finished, but there was no longer any room to doubt that the temperature coefficient of this film was negative.

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[1] Methods of determining approximate thickness will be discussed later.

On April 27th, when the resistance had become more nearly constant, this film was provided with platinum terminal wires, and sealed into a glass tube from which the air was afterwards exhausted to about .001 of an atmosphere, the tube being then hermetically sealed.

Its record on the last three days of the month was as follows :

Date.	Temperature.	Resistance.
Apr. 28, 1898	23.2 degrees.	2284.1 ohms.
" 29 "	2.0 "	2290.5 "
" 30 "	23.1 "	2284.2 "

The temperature coefficient calculated from these results is — 0.00013.

Even at the date of these measurements, this film was not *perfectly* seasoned, but the subsequent changes in resistance were very slight, and after October 1, 1898, no changes whatever could be detected except such as were in exact harmony with the above named temperature coefficient.

In November, 1898, Mr. F. B. Fawcett's very interesting article "On Standard High Resistances" appeared.[1] In this article Mr. Fawcett points out the early rapid decrease in the resistance of a film and the importance of artificial ageing. He also indicates a method of standardizing the resistances and a method of artificial ageing; but in determining the thickness of the films, he assumes that the specific resistance is the same for all films.

THE ELECTRICAL CONTACTS.

In these experiments for a considerable length of time the contacts between the films and their terminal wires were an unfailing source of annoyance. Clamps were used at first, but they proved to be untrustworthy. A piece of tin foil or silver foil may be clamped to a *thick* film and the contact may be as good as any clamp contact, which is not saying very much. Of course bright metal plates may be very successfully clamped together for temporary connections, but even the best of clamp connections can hardly be considered first class permanent contacts on standard resistances. Furthermore if a resistance having clamped contacts be boiled in oil or paraffin for a number of hours, as in the process of artificial ageing, there is a strong probability that the insulating material will get into the joints, especially if the coefficient of expansion of the clamp is greater than that of either of the materials held together.

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[1] Phil. Mag., 5, Vol. 46, pp. 500–503.

Aside from all this, when we consider making contacts with a *thin* film, there is an additional difficulty. These films are rather delicate and will not endure the rough usage to which *thick* films, wires and metal plates may be subjected. If the slightest crack be produced in the film, by the pressure of the clamp, the crack will expand and contract under the influence of temperature changes, and in this way a variable resistance will be introduced into the circuit. It was for these reasons that a number of attempts were made, several of which resulted in perfectly satisfactory methods of making electrical contacts with even the thinnest of the films.

The first attempt, which was not altogether successful, is represented in figure 6. (*AA*) are strips of tin foil or platinum foil fastened to the glass with shellac varnish and thoroughly baked. The film (*B*) is deposited afterwards over the entire surface of glass and foil, and if thick enough, the continuity between the film on the glass and the film on the foil is perfect. If, however, the film is thin, there is lack of continuity at the edge of the foil. The method may be used even for thin films by covering the edge of the foil with gold leaf before the film is deposited. This makes a joint which is electrically good, but poor mechanically.

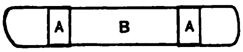

Fig. 6.

The second method is satisfactory in every respect and is applicable to films of any thickness. This method is represented in figure 7. In this case the film is deposited first, and over the entire surface of the glass. The receiver is then opened and the portion (*B*) of the film is covered. The receiver is again closed and exhausted and the deposition of metal is simply continued until the portions (*AA*) are very thick. We then have a continuous film, as thin as we please, for a high resistance, but with *ends* as thick as we please for making connections. The fine copper wires (*CC*) are then wound on and permanently secured by electrolytic deposition of copper on them.

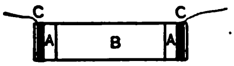

Fig. 7.

The resulting contact leaves absolutely nothing to be desired, but the process is rather tedious. A *thin* film may be produced in a few minutes, after the receiver is exhausted, but these *thick ends* require hours. It was on this account that a third method was devised. Figure 7 represents this method as well as the preceding one. In this case, the ends of the glass plates are first immersed in a silvering solution, and thick films of silver (*AA*) are deposited upon them by any of the well-known methods. As the plate stands in the silvering bath the portion which is below the plane of the surface of the bath becomes heavily coated; but there is a small area just above this plane, which receives silver from only the small amount of liquid which rises above the plane of the surface of the bath by capillary action. For this reason, the film, however thick it may be, always terminates in a very thin edge so that the film (*B*), which is afterwards deposited is perfectly continuous over the entire surface of the glass and silver. The copper wires (*CC*) are secured in this case in the same way as in the preceding case. If the silvering of the ends had to be done one piece at a time, this method would have very little advantage over the preceding one, but, as a large number of plates may be placed in the silvering bath at the same time, the amount of time spent in silvering the ends of the plates is very small.

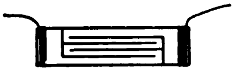
Fig. 8.

The finished film, as it appears in figure 8, may be introduced into a Wheatstone bridge circuit, or into any other electric circuit for which it is suitable, just as if it were a coil of wire. It has, however, the advantages of being non-inductive and practically without capacity.

(To be continued.)

A METHOD OF STUDYING ELECTROSTATIC LINES OF FORCE.

BY F. J. ROGERS.

SINCE the days of Faraday and Maxwell much has been made of electrostatic lines of force. In recent years this method of dealing with electrostatics has been extensively introduced into elementary text-books. Now, without experimental illustration these lines of force represent an extremely abstract conception, arrived at only by processes of reasoning and imagination.

To make the existence and reality of these lines of force evident to the average student, an experimental demonstration is necessary. This is also highly desirable even for those students who are gifted with acute reasoning powers and a well-trained imagination.

I have tried a number of methods for obtaining this result with but partial success. The method described by several writers of suspending an elongated dielectric by a fiber in the field to be studied is very unsatisfactory. The forces are weak because the dielectric constant is always a small number; the case being very different from the analogous one of a piece of soft iron in a magnetic field, where the permeability is large. Furthermore, the dielectric "needle" is likely to be attracted bodily to a charged conductor and stick to it.

I finally tried a very simple method which is quite satisfactory; so much so, that by its aid students may trace out electrostatic lines with almost as great a facility as they can magnetic lines.

The method consists in the use of an "electrostatic needle" mounted on a rigid axis. In order to increase the forces brought into action, each end of the needle is provided with a small gilt pith ball.

I have used two forms of this apparatus. The first consists of an aluminum wire pierced at the center and provided with pith balls at its ends. This "needle" is mounted on a horizontal axis consisting of a pin stuck into an ebonite handle (see Fig. 1). When this needle is brought into a fairly strong electrical field, the two balls acquire, by induction, opposite electrical charges, and the forces acting on these cause the needle to point along the lines of force.

With the aid of this electrostatic needle it very easy to show: (1) That lines of force are normal to the surface of a charged conductor (an elongated cylinder for example); (2) that lines of force are *not* normal to the surface of a hard rubber rod one end of which has been charged by friction; (3) that in the case of two oppositely charged conductors placed near each other the lines of force issue from one conductor, curve over, and end on the other; (4) that in the case of two similarly charged conductors, the lines of force from each leave it normally and pass out into space, none of them passing from one conductor to the other.

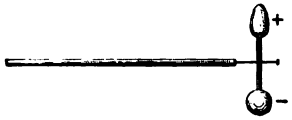

Fig. 1.

A much more useful form of electrostatic needle is constructed as follows: In place of the aluminum wire a small rod of hard rubber is used. It is mounted on a shaft and provided with pith balls as in the first form.

The pith balls are oppositely charged by contact with the poles of an electrical machine. While so charged the electrostatic needle behaves in an electrostatic field just as a magnetic needle does in a magnetic field.

In addition to the experiments already described, this form of electrostatic needle shows the positive direction of lines of force; consequently, in passing a neutral point in the field, the direction in which the needle points is reversed. It is very useful for testing the field about the different elements of an induction machine. For example: Suppose the neutralizing-rod of a Voss machine is removed while the machine is running. Under favorable circumstances the inductors become charged to a high potential, then lose their charges, and acquire an opposite one. This cycle may be continuously repeated every few seconds. To the experienced operator this periodic change is rendered obvious by the change in the appearance and sound of the brush discharge. That the polarity of the machine is being periodically reversed may be made evident to anyone by holding near one of the inductors a polarized electrostatic needle. The needle reverses its direction every time the polarity of the machine is changed.

The pith balls of the electrostatic needle do not remain charged for any great length of time. These charges are lost quite rapidly in the neighborhood of a brush discharge.

NOTES ON THE PREPARATION OF RETICLES.

By David P. Todd.

To the physicist more often perhaps than to the astronomer occurs the problem of preparing a reticle of unusual design. If only straight lines are desired, nothing surpasses the fine web of the spider, which is easy to thread; or if regular intervals are requisite, they may be graduated on glass with a ruling engine.

When, however, a complicated reticle becomes necessary for any purpose, or one with curved lines, letters and figures, the spider cannot help us, and execution of the design on glass becomes very difficult, and too expensive. But a photographic method is still very serviceable.

Three of the instruments devised for the Japan Eclipse Expedition of 1896, described elsewhere,[1] required circular reticles about 0.2 in. and 0.5 in. in diameter, with careful graduation around their circumference.

First, an exact original was draughted on Bristol board with India ink, making the circle about 9 inches in diameter, and including all the lettering, with the finest gradation necessary. A sharp negative of this was then made, reduced to about one inch; and from this the required number of positives were prepared, with a farther reduction suitable in each case to the focal length of the telescope in which it was used. The proper reduction may be found by measuring the focal length, and calculating the linear value corresponding to the arcs intended.

By using the thinnest cover glass, and employing the wet collodion process, a fine grained reticle of high transparency is easy to obtain; one, too, which will bear a high degree of magnification.

Observatory House, Amherst College.

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[1] *Monthly Notices Roy. Astron. Society*, LVIII. (1898), 296; *Proc. Am. Assoc. Adv. Science*, XLVI. (1897), 178.

NEW BOOKS.

Electromagnetic Theory. Vol. II. By OLIVER HEAVISIDE, F.R.S. London, *The Electrician* Printing and Publishing Company, limited, 1899.

The second volume of Mr. Oliver Heaviside's "Electromagnetic Theory" forms a suitable continuation to the first volume of that work as well as to his "Electrical Papers," and this will be welcomed by all students of his writings. The present volume is mainly occupied with detailed mathematical calculations on the propagation of electrical disturbances along circuits of various kinds, a subject which Mr. Heaviside has made particularly his own. Those who wish to gain a general knowledge of the consequences of Maxwell's theory will find them explained with great clearness and with very few mathematical symbols in the first volume of this treatise; these descriptions are not the work of a mere mathematician driving blindly some analytical machine, but of a man possessing a singular insight into the physical processes involved in the propagation of electromagnetic waves.

A university education, and the help which it brings by intercourse with men of like pursuits, are great aids as many mathematicians of the present day would acknowledge; but these things have not come to Mr. Heaviside. Attracted, perhaps, to electrical studies by the practical work of telegraphy, he has gone steadily on without these aids, always keeping the problems of electromagnetism before his mind, and striving with marked success to improve the mathematical methods of attacking these problems. By patient investigation, extending over many years, he has gained a remarkable knowledge of the detailed results of Maxwell's theory and a special skill in the mathematical analysis appropriate to the subject, winning a very high position among the ranks of investigators and awaking a feeling of curiosity among physicists to know something of his work. The average professional physicist spends so much time in teaching that he is not often able to study books requiring very close application, such as those of Mr. Heaviside. This is sad, but it is true. Such men often confess that one of the things they are always intending to do is to make a thorough study of Mr. Heaviside's work; they are sure there is a great deal in it.

The first chapter of this volume is for the most part occupied by a mathematical account of those results in the theory of the diffusion of heat which shed some light upon the age of the earth. The following chapter deals with other diffusion problems, these now referring to the spreading of an electric charge along a cable on the [impossible] assumption that the effect of the inductance (self-induction) of the cable is neglible. Solutions of many problems are obtained, some of the most interesting cases being those in which various arrangements of resistances, capacities and inductances are placed between the cable and the source of electrical energy.

The third chapter is perhaps the most important part of the book, dealing, as it does, with the propagation of electrical waves along a real circuit. The twin wire circuits of long-distance telephone lines have four characteristics of which account must be taken, viz: R the resistance, L the inductance, S the permittance (capacity) and K the leakage conductance from wire to wire, all per unit of length of the circuit. The fundamental problem and its solution may be described as follows: Suppose that at any place on such a circuit, the two poles of a battery are put into connection with the two wires for a very short time τ. Then in either direction there will go out two disturbances, each of thickness $v\tau$, travelling with the velocity $v = (LS)^{-\frac{1}{2}}$, the velocity of light in the medium between the wires. These disturbances shrink as they go, the attenuating factor being $\varepsilon^{-\rho t}$ where $\rho = R/2L + K/2S$. But this is not all. For along the whole of that portion of the circuit which lies between these two waves there is a disturbance which, beginning by being comparatively small, ultimately (in practical cases) drains all the energy out of the two original waves. These separate parts of the disturbance Mr. Heaviside calls the "head" and the "tail." The ratio of the total disturbance in the "tail" to that in the "head" at any time t after the starting of the waves is $\varepsilon^{\sigma t} - 1$, where $\sigma = R/2L - K/2S$; hence σ may be taken as a measure of the distortion of the signals. Thus it is plain that for effective signalling both ρ and σ must be small. With R and K both zero, ρ and σ vanish and the circuit is a perfect one transmitting signals without either attenuation or distortion. But all circuits have wires of finite resistance, and thus this ideal can never be realized in practice. But even though the attenuation cannot be got rid of, it is possible to cause the distortion to vanish by making $R/L = K/S$; in this case there is no "tail" to the wave, and the signals, though shrunken, are just as sharp on their arrival at their destination as when first despatched. But, with the values of R and L obtainable in practice, the shrinking would be so excessive that next to nothing would be left of the waves after they had gone a little way. Though (on this account) useless in practice, this "distortionless" condition of the circuit is very useful in theory, for it forms a sort of half-way house between the perfect circuit, in which both R and K are zero, and the circuit in which all four characteristics have any given values. Now in all practical circuits K/S is less than R/L, and depends simply upon the insulating and dielectric qualities of the medium between the wires, being independent of the size of the wires and of their distance apart. Thus, when this medium has been chosen the only way left for making ρ and σ small, and thus obtaining good signalling, is to make the resistance R small and the inductance L large. It is because it is the magnetic part of the energy which is wasted by the resistance of the wires that it is a help to the signals to increase the magnetic energy by increasing the distance between the wires.

These elementary facts can be deduced by very simple mathematics, being merely examples of the law of compound interest, and thus may quite well be taught to junior students. The detailed examination of the exact form of the "tail" of the wave is a matter of greater difficulty. The result was stated by Mr. Heaviside in 1887 ("Electrical Papers" Vol. II., pp. 150) and proof was given a little later; in the present volume several methods of proof are give. The formula involves Bessel's functions.

This chapter contains a great many of the mathematical weapons necessary for attacking the problem of finding the form of the wave due to any given impressed voltage applied at the end of the circuit; it will be read with great interest by those who are interested in Bexsel's functions. The formulæ are generally complicated; this is perhaps largely due to the fact that the series which make their appearance have never been studied and thus are without any simple abbreviated forms. Even e^{mx} sin nt will appear complicated when expressed in a series of powers of x. This chapter and those parts of the "Electrical Papers" which deal with the propagation of waves along wires are deserving of the closest study.

The last chapter, devoted to a brief discussion on generalized differentiation and divergent series, will, it is hoped, attract the attention of professed mathematicians. An unprofessional mathematician must be content to remark that even if the mathematical methods are largely experimental, they may still be none the worse for that. It has happened before now that things have been discovered by men who have gone out of the beaten track and have made experiments. These subjects were considered by the author in three papers on "Operators in Physical Mathematics" communicated to the Royal Society. Two of these were read and published; the third was not read. Those who are interested in the theory of the motion of electrically charged bodies will regret that this third paper (from whatever cause) made no further progress. For by the methods therein developed Mr. Heaviside has obtained the solution for the motion of a charge at a speed greater than that of light; it is to be hoped that this incident will not permanently deter Mr. Heaviside from publishing the solution.

In one of the appendices Mr. Heaviside makes some interesting remarks on the motion of a charged body at a speed equal to or greater than that of light. The fact that the electromagnetic energy of a charged body in *steady* motion at the speed of light is infinite has led some who have considered the problem to conclude that it is impossible to make a charged body move at a speed greater than that of light. Mr. Heaviside shows that this conclusion is unwarranted. The error has arisen because the manner in which the steady state is reached has not been duly considered. When a charged body has been set in motion at speed u a force must be applied to it to keep its speed constant, the work done by the force appearing as electromagnetic energy in the field. This force continually decreases as the time increases and after an infinite time vanishes, unless u is greater than the speed of light. Mr. Heaviside states that the force does not tend to vanish when the speed is that of light, but a simple calculation shows that in this case when the body—a sphere of radius a carrying a charge Q—has travelled over a distance vt great compared with its dimensions the force required to keep its velocity constant approximates to $\frac{Q^2}{2avt}$. The force is thus (except near the beginning of the motion where this approximation may not apply) inversely proportional to the time which has elapsed since the motion began and hence vanishes after an infinite time.

A whole science of electrical dynamics is waiting for development; some of its elementary problems may be briefly indicated.

1. The force required to maintain a charged body in uniform motion in a straight line.
2. The acceleration of a charged body due to a uniform force.
3. The impulse required to give a charged body any given velocity.
4. The radial and tangential forces required to maintain the uniform motion of a charged body in a fixed circle. [The unpublished researches of the Rev. P. E. Bateman, of Tonbridge School, England, may possibly give some information on this problem.]

These problems, rendered specially interesting at the present time by the experimental work of J. J. Thomson and others on the behavior of the ions in vacuum tubes, are well worthy of the attention of those who have the mathematical skill necessary for their solution.

It is impossible in a review to give any adequate account of the author's work. A close study of the book itself is necessary. If difficulties are encountered which the reader cannot overcome, let him remember that many students have found Mr. Heaviside very ready to help them in their troubles when they have brought them to his notice.

In conclusion it must be said that *The Electrician* deserves very great praise for providing Mr. Heaviside with the opportunity of publishing this work. It is not every journal which will devote large portions of its space to matter which it is sometimes (falsely) imagined that no one reads.

G. F. C. Searle.

Cavendish Laboratory, Cambridge, England.

Electrical Testing for Telegraph Engineers. By J. Elton Young. New York, The D. Van Nostrand Company; London, The Electrician Company (no date). Pp. 249.

In this work the author has well carried out his purpose of supplying the want felt by telegraph engineers (more particularly submarine cable electricians) of some book devoted exclusively to their work, comprising both a more complete exposition of previous knowledge and a fuller account of its latest developments than previously published. Measurements of conductor resistance, insulation resistance and capacity, tests for disconnections, earths and contacts and similar topics receive systematic and satisfactory treatment, with particular view to the practical utility of the methods described. The work will likewise be found instructive for other readers than telegraph engineers, although not so well suited for those not familiar with the technical details of cable testing. For such readers the subject is not so clearly set forth as it might well be, and the use of technical abbreviations detracts from its utility as a work of reference. For example we note such headings as "I. C. Tests during Manufacture," and read that "observations of the I. C. are made along with those of the D. R. and C. R." The pages bristle with these and other abbreviations, as G. P., N. R. F., and E. C., and it commonly occurs that an explanation of the abbreviation is not to be found, at least not in the preceding pages.

The book is of unquestionable value to engineers of submarine cable, telegraph and telephone lines, and will prove of some use to those who have the testing of light and power cables.

F. Bedell.

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THE

PHYSICAL REVIEW.

THE ATTENUATION OF SOUND.

By A. Wilmer Duff.

IN an earlier paper[1] the author stated the results of an experimental method for finding the rate of decay of the intensity of sound with distance. Prefixed to this statement of experimental results was a mathematical investigation, founded on methods developed by Stokes, Kirchoff and Rayleigh, of the parts played by viscosity and conduction and radiation of heat in the attenuation of sound. The general result of this discussion will be here restated and amplified with a view to facilitating the understanding of a second experimental method, the results of which it is the chief purpose of the present paper to describe.

It was found that the radial velocity of a gaseous medium through which spherical waves are passing is proportional to

$$e^{-mr}\left\{\frac{n}{ar}\sin n\left(t-\frac{r}{a}\right)-\frac{1+mr}{r^2}\cos n\left(t-\frac{r}{a}\right)\right\} \tag{1}$$

in which n denotes the vibration frequency × 2π, a denotes the velocity of sound, r denotes the distance from the source, and m consists of the sum of three terms depending on viscosity, conduction and radiation respectively; namely

$$m=\frac{2}{3}\frac{n^2}{a^3}\mu'+\frac{\gamma-1}{\gamma}\frac{n^2}{2a^3}\nu+\frac{\gamma-1}{\gamma}\frac{1}{2a}\lambda \tag{2}$$ [2]

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[1] Phys. Rev., 6, p. 129, 1898.

[2] This expression was incorrectly stated in the original paper, but the error does not affect the subsequent calculations which were made from the correct expression as here stated.

μ' denoting the kinematic coefficient of viscosity of the medium, ν denoting the conductivity of the medium, λ denoting the constant of radiation according to Newton's law, and γ, the ratio of the specific heat at constant pressure to that at constant volume.

From (1) it is seen that the motion at any point may be regarded as the resultant of two simple harmonic motions, differing in phase by a quarter of a period. The relative importance of these terms differs widely for different values of r. For large values of r the second component is negligible, since m is certainly a very small quantity, and the motion is represented by

$$\frac{e^{-mr}}{r}\sin n\left(t-\frac{r}{a}\right). \tag{3}$$

At small distances the term mr may be neglected in comparison with unity and the resultant motion is represented by

$$\frac{e^{-mr}}{r}\sqrt{1+\frac{a^2}{n^2r^2}}\,\sin\left\{n\left(t-\frac{r}{a}\right)-\varphi\right\} \tag{4}$$

in which
$$\varphi=\tan^{-1}\frac{a}{nr}.$$

Moreover, for small values of r, e^{-mr} does not differ appreciably from unity. Hence the law of intensity at small distances is

$$\frac{1}{r^2}\left(1+\frac{a^2}{n^2r^2}\right) \tag{5}$$

while that at large distances is

$$\frac{e^{-2mr}}{r^2}. \tag{6}$$

The method described in the previous paper consisted in observing the distance to which eight similar small whistles blown under a definite pressure were audible and also the smaller distances to which the whistles arranged in pairs were audible. Five different sets of such observations, made under widely different atmospheric conditions (except that in all cases there was no wind), gave consistent results, the mean value of m being 0.000042 per cm. Subtracting from this value of m the parts due theoretically to viscosity and conduction, there remained 0.000035 due to radiation and other causes not taken account of in the mathematical discussion. Of such unconsidered causes the only ones that readily suggested themselves were refraction and internal reflection, and it did not seem probable, in view of the close agreement of results obtained under widely different atmospheric conditions and other circumstances, that these could have largely affected the final result. Hence a value or rather an upper limit of 8.3 was deduced for the hitherto unknown constant of radiation of air in Newton's law of radiation.

In a discussion[1] of the preceding results which Lord Rayleigh has contributed to the *Philosophical Magazine* the ground is taken that the upper limit thus assigned to the constant of radiation must be very much higher than its actual value (so much higher in fact that only an entirely inappreciable part of the rate of decay actually observed could have been due to radiation) and that, if no part of the decay of intensity can be shown by further observations to be due to atmospheric refraction, a further cause must be sought. This cause Lord Rayleigh thinks may be found in the time required for the kinetic energy produced by compression, which at first is purely translational, to become shared between the translational and rotational forms. The suggestion thus made seems to be of such great importance that it would be well if any evidence bearing on it could be obtained.

In the meantime it seemed well to attempt by an independent method to settle whether the rate of decay of intensity is really as high as the previous experiments seemed to show. The results obtained by this second method are still somewhat imperfect, the work having been discontinued owing to unfavorable weather; but, as an opportunity for repeating and extending the work may not present itself for a considerable length of time, it has been thought permissible to publish the results already attained.

The sounds studied by this second method were produced as before by whistles blown under definite pressure, the observations being made at a very quiet part of the River St. John in New Brunswick. To obtain a variable standard with which to compare the sound at different distances, a telephone transmitter was placed near the whistle and a receiver, connected to the transmitter through a line of constant resistance, was held so that its distance could be varied until the intensity of the sound heard directly was equal to that of the sound given by the receiver. At first only a single whistle was employed, it being placed in a heavily padded box that could be alternately opened and closed, the telephone circuit being closed when the box was closed and opened when the box was opened, so that the sounds to be compared were heard alternately. Afterwards it was found better to employ two whistles of the same form and tuned by adjustable plugs to the same pitch. One, mounted in the open air, acted as the source of sound to be studied, the other was enclosed in the box with the transmitter and the two were blown alternately from the same wind chest. The whistles were sounded by an assistant on an isolated pier in the middle of the river at an elevation of about eight meters above the surface of the water. The observer stood in a canoe which was allowed to float down stream from the pier and arrested at any desired point by means of a cord attached to the pier, this cord also affording a means of determining the distance from the pier. The telephone line consisted of 300 meters of twin wire doubly insulated with rubber to protect it from the water, the whole line being always in circuit. Only one ear was used in making the observations, the other being filled with wool and closely covered by a heavy woolen pad. The distance of the receiver from the ear was taken as the distance from the disc to the outline of the skull adjacent to the ear.

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[1] Phil. Mag., XLVII., 308, March, 1899.

In adopting this method it was hoped that the quality of the direct sound and that of the sound from the telephone receiver would not be found to differ appreciably for the high pure tone employed; but in making the observations the ear became more and more sensitive to minute differences of quality between the sounds compared. These differences seemed to vary with the pitch of the sound and the position of the transmitter relatively to the whistle that acted on it. For only one particular pitch did the differences seem to disappear entirely, namely for a pitch of about 4,000. For this particular note, the sound heard directly and that emanating from the receiver were at times so nearly identical as to be entirely indistinguishable when the receiver was held at the proper distance.

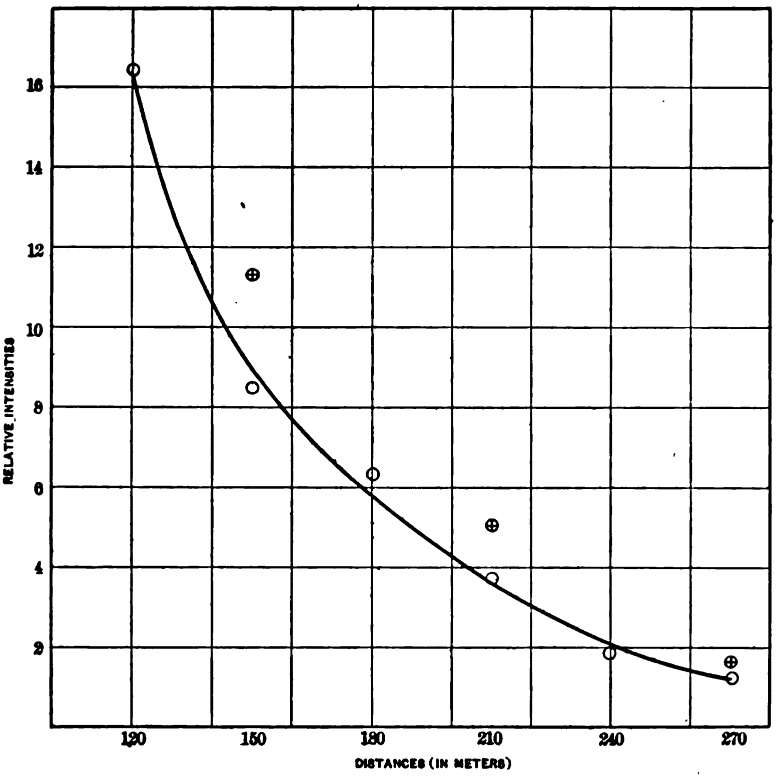
It may be here noted, as a matter of independent interest, that this frequency of 4,000 was afterwards found by calculation to be also that of the free vibration of the iron disc of receiver, not the fundamental vibration, however, but those of the first higher mode. Unfortunately the point was not foreseen and the range of the sounds tried probably did not include that corresponding to the fundamental vibrations of the disc. It seems probable that a telephone (with a carbon transmitter and an iron disc receiver) transmits comparatively unmodified only those notes which synchronize with one of the modes of vibration of the disc of the receiver. I am not aware that the point has been carefully studied.

Several preliminary trials showed the disappointing fact that the ear is curiously incapable of distinguishing small differences of intensity of sounds of appreciably the same pitch and quality. The weakness of the ear in this respect is in marked contrast with its power of discriminating minute shades of pitch and quality (a fact that seems capable of being readily accounted for on the theory of evolution of organs by their usefulness) and in great contrast with the power of the eye in judging equality of illumination in the ordinary photometric method. An attempt to improve the ear in this respect by some practice (and it may be stated that the writer's ear is a fairly good one in the ordinary musical sense) seemed to render it still more acute to minute differences of quality. Hence the only hope of eliminating errors lay in multiplying the observations at each distance. This will explain the numerous readings in the table of observations, made between 10 a. m. and 1 p. m. of an exceptionally clear, quiet day with few clouds, the average temperature being 30°C.

In the table the observations are entered as made, with one exception. The last three readings at 120 m. were made on returning to that distance after the readings at 180 m. and 240 m. had been made and similarly as regards the last four at 150 m. The purpose was to ascertain whether atmospheric conditions had appreciably changed. No change was detected in the first case ; in the second there seemed to be a slight change, but the evidence was uncertain. In one unfortunate respect there was a break in the continuity of the readings that is not indicated in the table. Toward the end of the

Distance from source (in meters).	Setting of receiver (in cm.).	Mean Setting.	Relative Intensity.	Intensity × (distance)^2 + 10^4
120	25.0	24.7	16.4	23.6
	23.8			
	26.3			
	23.7			
	23.8			
	24.5			
	26.2			
180	37.5	40.7	6.33	20.5
	53.7			
	41.3			
	36.9			
	34.3			
240	71.3	74.2	1.84	10.6
	78.7			
	71.2			
	76.3			
	71.3			
	76.2			
150	36.2	29.8	11.3	25.4
	28.8			
	26.7			
	31.3			
	32.5			
	26.2			
	30.0			
	27.5			
	31.2			
	27.5			
210	55.0	44.8	4.98	21.9
	40.0			
	45.0			
	50.2			
	39.5			
	38.8			
	45.0			
270	92.5	78.6	1.62	11.8
	87.5			
	77.5			
	70.3			
	83.7			
	71.3			
	66.5			
	94.2			
	70.0			
	72.5			

first half of the series of readings the ear became sensitive to a slight difference of quality between the direct sound and the telephonic sound. To remedy this, the transmitter was moved slightly further away from the whistle. This, of course, made the instrument less sensitive, and raised the succeeding estimates of the intensity of the direct sound in a constant ratio as compared with the earlier ones. This is shown in the accompanying plot of the results, in which the crossed circles indicate the intensities at 150 m., 210 m. and 270 m. given in the table, while the plain circles immediately below them indicate the values they would have if multiplied by the reducing factor 0.75. With this reduction all the results seem fairly well represented by the curve drawn, the divergence being not greater than would naturally be expected from the difficulties in the method already referred to.



If now we calculate the mean value of m from each set of observations it turns out to be .000034 in the first case, and .000032 in in the second case If we reduce the second set in the constant ratio suggested, and then use the whole, with the mean value of 0.000033 for *m*, for testing the theoretical law, we get the following table:

Distances (in meters).	Observed intensity.	Intensity from curve.	Intensity by Theoretical Law.
120	16.4	16.4	17.9
150	8.5	9.0	9.1
180	6.3	5.8	5.4
210	3.7	3.6	3.2
240	1.8	2.1	2.0
270	1.2	1.2	1.3

A comparison of the last column with the two preceding shows that while the agreement is not wholly satisfactory the differences are not greater than might be expected to result from the difficulties and uncertainties inherent in the method. The range of intensity covered by the above test is considerable—from 16.4 to 1.2. The evidence on the whole is strongly in favor of the theoretical law.

The mean value of *m* given by the preceding is 0.000033. If we subtract the part of *m* due to viscosity and conduction, as calculated from the first two terms of formula (2), we get 0.000031 as the part due to radiation, which is seen by (2) to be independent of pitch, and to any other causes not included in the mathematical discussion. If this result be compared with the earlier one obtained by a wholly different method, viz: 0.000035, it will be seen that the agreement is very satisfactory. The difference is not greater than might be expected from the admissible errors in either or both of the methods. The agreement seems at least to fix the order of the quantity in question with a fair degree of reliability.

Now, according to Lord Rayleigh's conclusion as to the possible magnitude of the constant of radiation, the above rate of decay (subtraction having been made of the parts due theoretically to viscosity and conduction) is hundreds of times greater than can be attributed to radiation. It remains to be considered whether it can be due to refraction. In this connection it is to be noted that, while the whistle and ear were respectively 8 meters and 2 meters above the level of the water, the acoustic ray from the whistle to the ear, when the latter was at its greatest distance, would not, under normal atmospheric conditions, fall at any point more than one fifth of a meter below the direct line;[1] and, for atmospheric conditions differing from the normal, the depression would be proportional to the gradient of temperature. Hence an appreciable effect due to refraction seems very improbable. Moreover it should be noted that the elevation of the whistle in these later experiments was from three to eight times as great as in the earlier experiments, and in the latter it varied considerably on the five different occasions of observation, while the temperature gradient must have varied widely. Hence the agreement of the earlier results among themselves and the agreement of their mean with the later results would both seem inexplicable were refraction an important cause of the decay observed. It would seem to the writer more probable that, while, under ordinary circumstances of observation, refraction may play an important, perhaps a dominating part, under the exceptional circumstances here selected its effects were comparatively slight.

For reasons stated in the earlier paper, it is very improbable that internal reflection played an important part. Thus the evidence points strongly to the existence of some additional cause of decay of intensity, one whose effects are in accordance with an exponential law of decay.

Law of Intensity at Small Distances.[2]

In the preceding it has been assumed that the intensity of a sound, whose vibration frequency is 4,000, emanating from a telephone receiver, varies, at distances of 20 to 80 cm., according to the theoretical law (5), which, for these distances, does not differ appreciably from the law of the inverse square of the distance. Of this it does not seem possible, (the dissipation at such distances being negligible), that there can be much doubt. Nevertheless an experimental test seemed of interest.

The intrinsic intensity of a whistle (vibration frequency = 4,000) was varied by different coverings and its intensity at a convenient distance was estimated (1) by a telephone receiver as explained above and (2) by the same receiver shunted by a wire of suitable resistance. The ratio of the estimates obtained by the two series of comparison should be a constant, provided that the true law of intensity at small distances has been employed.

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[1] Rayleigh's Theory of Sound, II., § 288 (6).

[2] In a résumé of this work printed in the Proceedings of the A. A. A. S. (1899), it was stated that the results of the following method were so inconsistent that nothing positive could be deduced, but a revision of the calculations has shown the presence of a continued error.

In the following table the first column contains the mean distances of the receiver with no shunt, the second those with the shunt, the third the ratio of the intensities according to the law of inverse squares, and the fourth the ratio according to the theoretical law (5) stated in the earlier part of this paper.

d.	d'.	Inverse squares.	Theoretical law.
20.7 cm.	8.75 cm.	5.60	5.70
38.7	16.5	5.50	5.52
57.2	23.5	5.92	5.92
108	43.5	6.17	6.17

The last column is as nearly constant as could be expected when the difficulties of such comparisons, referred to earlier, are considered. Moreover, it is to be noted that the range of distance covered by the test is much wider than that employed on the previous test.

HISTORICAL NOTE.

Since the preceding work was completed, I have discovered, in a paper by Wien,[1] a record of some experiments made, with a view to testing the law of inverse squares. The amplitude of the sound at different distances was studied by means of an aneroid barometer arranged for great sensitiveness. According to the law of inverse squares of the distance, the product of amplitude of vibration by distance should be a constant. The following is the most consistent of the three sets of observations made, r being in meters.

r	23.1	33.2	48.1	61.5	83.7	137.8
Ar	485	478	457	436	377	375?

The author states his belief that his observations establish the law of inverse squares and accounts for the diminution of Ar by friction on turf, motion of air and slight unevenness of the ground. Nevertheless it is not quite evident that these causes should be so much more effective at greater than at smaller distances, and it seems probable that a considerable part of the diminution was due to causes similar to those studied in these papers.

POLYTECHNIC INSTITUTE, WORCESTER, MASS., May 1, 1900.

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[1] Wied. Ann., XXXVI., 1889.

SOME LECTURE ROOM METHODS IN THE ELEMENTARY THEORY OF ELASTICITY.

W. S. Franklin.

NEXT to thermodynamics the elementary theory of elasticity is perhaps the most difficult branch of physics to present to a class. The importance of this subject to the engineer, however, demands a more or less adequate treatment of it in our technical schools and every possible means should be employed to render the details of the theory concrete and to develop the geometric-mental images which form the basis of any understanding of the subject. The object of this paper is to describe some lecture experiments which have been found by the writer to be helpful in the presentation of this abstruse subject in the lecture room.

The elementary theory of elasticity may be conveniently divided into four parts, (*a*) Strains, (*b*) Stresses, (*c*) Stress-strain relations, (*d*) Particular cases such as the bent beam and twisted rod. Part (*c*) is not satisfactorily treated in any of our text-books, as it seems to the writer, and at the end of this paper an outline is given of what seems to be the most satisfactory elementary development of stress-strain relations.

Concerning strains it seems to the writer to be exceedingly important that the student get a clear idea of the character of homogeneous strain, that he understand the significance of the elements of a homogeneous strain—namely three mutually perpendicular stretches, and that he see the applicability of the notion of homogeneous strain to the small parts of a body non-homogeneously strained. In the attainment of these objects the notion of the strain ellipsoid is of the greatest usefulness. Now most students are not sufficiently familiar with solid geometry to get a clear idea of the strain ellipsoid and the writer finds it helpful to use the distortion of an elastic sheet as an illustration in two dimensions, where the relation of the ellipse to the circle gives the geometrical picture of the strain. An extremely thin sheet of rubber, slightly oiled with a non-mineral oil to make it semitransparent, is stretched over a light frame fifteen or twenty centimeters square. This state of the sheet is taken as the normal or unstrained state and small circles (also a large circle) are marked upon it. The small circles may be projected by means of a lantern. The sheet may now be stretched homogeneously, the sheet being fastened to the frame by a number of spring clips, and the circles are seen to be changed to similar and similarly placed ellipses. The student's knowledge of the relation of the ellipse to the circle, gotten perhaps from his work in projection drawing, enables him to realize that the distortion of the sheet at each point consists of two mutually perpendicular stretches, while the similarity of the ellipses shows that the strain is homogeneous. Stretching the sheet non-homogeneously the small circles are changed to dissimilar and dissimilarly placed ellipses. The attention of the student may then be directed to the fact that the large circle seen directly differs sensibly from an ellipse while the small circles *even when greatly magnified on the screen* are sensibly elliptical. It is also instructive to show the character of the distortion of square figures marked upon the sheet.

Concerning stresses, it is very important that the student get a clear idea of homogeneous stress, that he understand the significance of the elements of a homogeneous stress, namely, three mutually perpendicular pulls, and that he see the applicability of the notion of homogeneous stress to the small parts of the body, subjected to a non-homogeneous stress. These things cannot, perhaps, be treated experimentally in the lecture room, and, therefore, it seems advisable to return to the experiments with the rubber sheet after a statement of the fundamental fact of the stress-strain relation for isotropic substances, namely, that the axes of stress and of strain are parallel, the student being led to infer the stresses from the observed strains.

In the discussion of shearing strain, the writer uses a large, square, hinged frame, over which muslin cloth is stretched, the threads of the cloth being parallel to the sides of the frame. The reduction of a shearing strain to two diagonal stretches (mutually perpendicular, of course) is most clearly shown by drawing a large circle upon the undistorted muslin. This circle is changed by the distortion to an ellipse with its axes diagonal to the frame.

In the discussion of the strains (and stresses) in the twisted cylinder, the writer uses the following model: A large tin cylinder is covered with muslin cloth, which is fastened to the cylinder at one end and to a loose circular disk at the other end. Squares and circles marked upon this cloth show very clearly the strains (and stresses) produced when the circular head is slightly turned. In particular, the student can see that the cylindrical shell of muslin is lengthened along a set of helical lines which are inclined at an angle of 45° to the straight elements of the cylindrical surface, and shortened along another set of helical lines, which are everywhere at right angles to the first set. The student can infer the existence of tension along one set of helical lines, and the existence of compression along the other set of helices, and he can infer from a previous discussion of shear that tangential stresses act across all horizontal lines and across all vertical lines.

STRESS-STRAIN RELATIONS.

Considerations of symmetry show that axes of stress and axes of strain coincide in isotropic substances. This is the first fact to be brought out in the discussion of stress-strain relations. Further development should, it seems, be based upon the experimental study of longitudinal stress and the strain produced thereby. Let P be the longitudinal pull, and let p', q' and r' be the corresponding stretches.

Experiment shows that when p' is small, it is proportional to P or equal to aP.

Also experiment shows that q' and r' are equal to each other and negative (*i. e.*, contractions), and that they are proportional to P or equal to bP. Therefore

$$\begin{aligned} p' &= aP \\ q' &= -bP \\ r' &= -bP \end{aligned} \qquad (1)$$

Similarly, a pull Q (parallel to the stretch q' above) produces the stretches:

$$\begin{aligned} p'' &= -bQ \\ q'' &= aQ \\ r'' &= -bQ \end{aligned} \tag{2}$$

And a pull R (parallel to the stretches r' and r'' above) produces the stretches:

$$\begin{aligned} p''' &= -bR \\ q''' &= -bR \\ r''' &= aR \end{aligned} \tag{3}$$

Experiment shows that the stretches (small) produced by a number of pulls acting together are equal to the sums of the respective stretches produced by the pulls acting separately so that the stretches P, Q and R acting together produce the strecthes:

$$\begin{aligned} p &= aP - bQ - bR \\ q &= -bP + aQ - bR \\ r &= -bP - bQ + aR \end{aligned} \tag{4}$$

The equations express the stress-strain relation for isotropic substances. They are based upon a semi-axiomatic fact and three experimental facts. The first experimental fact may be readily demonstrated experimentally, and the second may be illustrated by a rubber band, but the third is scarcely within reach of the hasty experimental methods of the lecture room.

South Bethlehem, March 13, 1900.

SPARK-LENGTH AS MODIFIED BY SOLID DIELECTRICS.

By W. J. Humphreys.

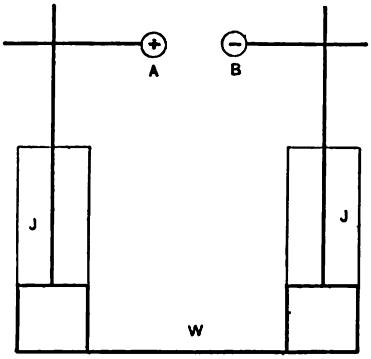
WHILE trying some experiments with an electrical influence machine I noticed that the spark-length was greatly increased by the presence of solid dielectrics, such as glass or ebonite, at certain points near the positive pole. The phenomenon was entirely new to me, and as I was unable to find it described, or even mentioned, in either those books or journals to which one would go for such information I decided to publish an account of my observations.

After my experiments were completed my attention was kindly directed to the fact that Messrs. Queen & Co., of Philadelphia, Pa., furnished an electrical influence machine with radically different poles—one being a metallic sphere while the other is a metal plate with an ebonite disk at its center. I have also learned that Professor Anthony once used a somewhat similar machine that was imported from Berlin about 1870. It seems, therefore, that the action of solid dielectrics on the spark-length has been known for a long while, at least to certain instrument makers, and I venture to publish the following account of my own experiments along this line because I have been unable, after considerable search and numerous inquiries, to find any reference in scientific literature to this interesting phenomenon.

During the course of the experiments several machines were tried and with equal success, but the one commonly used was a two-plate Winshurst machine, the plates being twenty inches in diameter, of ebonite and furnished with metallic sectors.

The accompanying sketch shows an arrangement that gives excellent results, though, as will appear from the experiments, it may be greatly modified and the phenomenon still obtained.

J, *J* are two small leyden jars, *W* a wire joining their outer coatings, *A* and *B* discharging balls connected one to the positive the other to the negative side of an influence machine, or other source of high potential, and also to the inner coating of its respective jar.



Let the inner coatings of the jars be oppositely charged, as indicated, and the distance between *A* and *B* gradually increased till sparks cease to pass. If now the potential difference be maintained and a glass rod say be placed quite near that side of the positive pole that faces the negative, the air will in general break down and allow a full noisy spark to pass. Nothing of the kind happens when the rod is placed next the negative pole, or in fact in any part of the field other than that very near the anode.

Various solid dielectrics have been used, all giving the same general result. Among them various kinds of glass, such as lead, soda and different varieties of enamel glass, some in the form of solid rods, others as tubes, even freshly drawn glass fibers of not over three-tenths of a millimeter in diameter, sticks of sulphur, rubber tubes, rods of ebonite, different varieties of wood and finally, as an extreme case, an ordinary silk thread stretched taut and approximately at right angles to the lines of force.

The increase in spark-length was usually very pronounced, often more than doubling the maximum length obtainable under ordinary conditions.

In most of my experiments the machine was kept constantly running at an approximately uniform speed, but this was simply for the purpose of holding the potential difference as nearly constant as possible, and not at all essential to the general phenomenon. The sparks of extra length could easily be obtained after the machine had come to rest, provided they were taken before the potential had fallen too low.

The following are among the conditions tested.

1. Polished poles, jars belonging to machine and their outer coatings connected, inner coatings oppositely charged. Results excellent, best obtained.

2. Polished poles, jars belonging to machine but insulated from each other, inner coatings oppositely charged. Increase in spark-length decided, but volume of spark small.

3. Polished poles of machine oppositely charged, no jars. Increase in spark-length still evident, but volume of spark of course very small.

4. Polished poles, one connected to inner the other to outer coating of a single jar. Results good.

5. Polished poles, one or more jars of large capacity. Results pronounced, but spark-length never so long or so greatly increased as with the jars belonging to the machine.

6. State of atmosphere such as to give only very short sparks. Increase in spark-length decided, but not very great.

7. State of atmosphere such as to give unusually long sparks. Increase in spark-length not great, but still evident.

8. Poles separated very widely, beyond the length even of the forced spark. In this case a small spark left the positive pole when the glass or other dielectric rod was brought in proper position and disappeared along numerous little branches.

9. Negative pole polished, positive pole very rough, in fact the broken end of a metal rod. Under ordinary conditions this gave only pinkish brush discharges, except when the poles were very close together, when numerous small, white and somewhat noisy sparks passed. On placing a glass rod next the positive pole, the white and noisy sparks were more than doubled in size and length.

Various other more or less important combinations were made, but these are sufficient to indicate the general nature of the phenomenon, the interpretation of which is probably the same no matter what the conditions under which it is obtained.

Just what the correct and complete interpretation really is, I do not pretend to say, but the increase of the spark-length when the discharge is between two oppositely charged jars, insulated from each other seems to preclude the assumption that it can be due to oscillations of any kind.

Possibly though an explanation is found in the fact that the solid dielectrics used in these experiments have specific inductive capacities greater than that of air, and consequently produced corresponding local concentrations of the lines of force, which concentration extended to a greater or less extent to the surface of the electrode when the dielectric was in its immediate neighborhood. It may be that the increase in the spark-length was due to this local increase of the electromotive intensity, but in framing an hypothesis to explain it, the fact that this increase must be next the anode has to be kept in mind.

For the sake of binding these and other experiments more or less closely together, I suggest the following hypothesis:

The dielectric between two poles is most sensitive to electrical disturbances at points next the anode.

The following are some of the facts that seem to support this idea.

(*a*) The-spark length is increased by the presence of a glass rod say next the anode, but not affected when the rod is in any other part of the field.

(*b*) The greater the increase of the spark-length, the nearer the glass rod must be brought to the positive pole.

(*c*) The greatest increase of spark-length is obtained with a glass rod of a certain size, other things being equal. That is the rod must not be either to large or too small for best results.

(*d*) When glass fibers are used they must be brought very near the pole, usually nearly or quite in contact with it.

(*e*) The same phenomenon follows when insulated conductors are used instead of glass or other dielectric.

With the view of testing this idea I tried, among other things, the effects of dielectrics of widely different inductive capacities, but otherwise approximately equal. The substances used were:

(*a*) A small thin-walled test tube, empty.

(*b*) Similar tube part filled with water.

(*c*) Similar tube part filled with metal.

The results were not conclusive, the increase in the spark-length remaining nearly the same, with if anything, the advantage in favor of the tube part filled with water, but this was not sufficiently pronounced to warrant deductions. Again the fact that insulated metal rods act no better than glass rods does not seem to be fully in accordance with the above assumption.

These, however, are the only experimental objections I am aware of, and it may be that they are by no means fatal.

In further support of this hypothesis, we have the increase of spark-length caused by a small spark drawn from the negative pole of two oppositely charged leyden jars when their outer coatings are connected, and also Faraday's experiments on spark-lengths between spherical electrodes of different sizes.

In speaking of these experiments of Faraday, J. J. Thomson says: "We may express this result by saying that when the electric field is not uniform the gas does not break down so easily when the greatest electromotive intensity is at the kathode as it does when it is at the anode."

It will be noticed that the hypothesis used to explain these experiments is only an adaptation of this sentence of Thomson's. Whatever the real action of solid dielectrics in increasing spark-length may be, it is a phenomenon of the anode, and to that extent indicates a greater sensitiveness of the air about this pole to electrical disturbances.

It would be interesting, but probably of little use at present, to speculate on the cause of this localized sensitiveness in the dielectric. Possibly it is due to the formation of Grothus' chains and the clustering of different types of ions about the two poles.

ROUSS PHYSICAL LABORATORY, UNIVERSITY OF VA., April, 1900.

ELECTRICAL RESISTANCE OF THIN FILMS DEPOSITED BY KATHODE DISCHARGE. II.[1]

By A. C. Longden.

The Electrical Measurements.

THE apparatus used in making the tests to which these resistances have been subjected was such as to afford results of the highest attainable accuracy; and in all cases where accuracy was considered important no painstaking effort has been spared in securing it.

The Wheatstone bridge and rheostat used in the Ryerson Laboratory was one of Queen's, of the kind known as "Set No. 1," No. 80. The rheostat consists of five sets of coils aggregating 11,111 ohms, and guaranteed by the manufacturers accurate to $\frac{1}{60}$ per cent. The bridge is provided with five pairs of reversible bridge coils ranging from a pair of units to a pair of 10,000 ohm coils. These are guaranteed accurate to $\frac{1}{100}$ per cent. The temperature of the coils was determined to within one hundredth of a degree by means of an electrical thermometer placed inside of the box.

Resistances beyond the range of this bridge were measured by the direct deflection method. In applying this method a new form of water battery[2] devised and constructed for this work was found useful, convenient, and comparatively inexpensive. In the construction of this battery, instead of using a copper plate and a zinc plate, two copper plates are used, one of which is amalgamated with mercury containing a little zinc. This plate presents a zinc surface to the water and is therefore equivalent to a zinc plate in the cell, but it does not dissolve in the cell, nor become brittle and break off at the surface of the water as an all zinc plate does. The resistance of this cell may be varied by varying its size, or, within certain

[1] Continued from page 55.

[2] Electrical World, Vol. 31, p. 681.

limits, by the introduction into the water of a *small* quantity ($\frac{1}{10}$ of 1 per cent.) of sulphuric acid. If the cell is properly constructed and sealed it requires no attention whatever for many months at a time, and its electro-motive force is practically constant as long as it is not overworked. Of course it will polarize quite rapidly if used on anything but very high resistance circuits. It is not intended to furnish much current.

In the work done at Columbia University, for measuring resistances of more than a few thousand ohms it was found convenient to use a bridge of the slide-wire type in which a potentiometer of 100,000 ohms forms the two variable resistances. The third resistance was the one under test, while the fourth was a standard of 10,000, 100,000 or a 1,000,000 ohms. The rest of the set consisted of a sensitive Thomson galvanometer and a battery of from one to one hundred dry cells.

The potentiometer used was designed at Columbia University by the late Mr. Holbrook Cushman, and was constructed for the university by a mechanician employed for the purpose. The coils were standardized and adjusted at the University by Mr. H. C. Parker and his assistants. A fuller description of the instrument is given in Mr. Parker's admirable work on Electrical Measurements.

The 10,000 and 100,000 ohm standards were made by Elliott Bros. of London. The 1,000,000 ohm standard was one of the platinum film resistances under discussion, but its resistance was frequently measured and it was found to be perfectly trustworthy.

With this apparatus a resistance as high as 20 megohms may be measured to within one part in 10,000.

In making the measurements for determining temperature coefficients, the standards of comparison were kept in the electrical testing room in the Fayerweather Laboratory. The resistances under test were first measured in that room at the same temperature as the standards, and again after being removed to an underground constant temperature room, located just outside of the main wall of the building, and entered from the sub-basement. One arm of the bridge in the testing room was connected with a mercury commutator in the underground constant temperature room by a well insulated, heavy copper wire circuit of small but known resistance.

For all high resistance work this circuit is negligible. If the resistance to be measured is small enough to make it at all important, the resistance of the line is deducted. The temperature changes in the line itself are very slight, but of course the method is not used for measuring resistances so small that temperature changes in the line itself would produce appreciable errors in the results.

The temperature of the testing room is ordinarily about 20° C., and does not usually vary more than a degree during the course of a day. The temperature of the underground room is generally about 10° or 11° and changes very slowly. These temperatures were measured by means of well seasoned thermometers which have been carefully standardized by comparison with a thermometer accompanied by a Reichsanstalt certificate dated May 7, 1898.

This method of measuring resistances at different temperatures proved to be at once convenient and, what is much more important, reliable. Convenient, because no special precautions were necessary in order to maintain constant temperatures, and reliable because there could be no doubt about the temperature of a film or coil which had remained undisturbed in a constant temperature room for two or three days.

In measuring the temperature coefficients of film resistances, it was soon noted that all of the very thin films had negative temperature coefficients, while all of the thick ones had positive temperature coefficients, and that for a certain range of thicknesses the temperature coefficients were close approximations to zero. It was also noted that, unfortunately, some other factor besides thickness enters into the condition which produces zero temperature coefficients. This other factor may be, and probably is, density; for this is obviously the most variable property of the films. When it becomes possible to hold all the conditions under which the films are deposited in perfect control, it will, doubtless, be possible to produce zero temperature coefficients or temperature coefficients of any desired value by simply producing films of certain thicknesses. Even under present conditions the range of thicknesses for negligible temperature coefficients is not small.

Before proceeding further with this part of the subject it may be well to state that for convenience a standard length and breadth were adopted for all films, so that, as the thickness was the only varying dimension, its relation to temperature coefficients might the more easily be discovered. Therefore, wherever in this article films are in any way compared, it may be assumed that they are films of equal length and breadth. The adopted total length of the glass plate is 7 cm., but as a centimeter at each end is covered with a *thick* film of negligible resistance for making the electrical connections, the length of the *thin* resistance film is 5 cm. The breadth is in all cases 15 mm.

A platinum film of the above dimensions thick enough to have a resistance of only about 400 ohms, has a positive temperature coefficient equal to 0.0003. The largest negative temperature coefficient measured was — 0.005. This was for a film so thin as to have a resistance of about 18 megohms. If the curve connecting temperature coefficients with thickness, were a straight line, zero temperature coefficients would occur at about one megohm, for films of these dimensions, as seen by the position of the dotted line in Figure 9. As a matter of fact, however, zero or negligible temperature coefficients frequently occur in resistances of less than 5000 ohms, and they occur all along from that point to somewhere in the neighborhood of 100,000 ohms. It must also be remembered that the very large negative coefficient mentioned above is an extreme case. The large number of results indicated by the points between the axis of ordinates and the two-megohm line in Figure 9 indicates that for films of a certain density the real position of the curve is probably not far from that represented in the figure. Fortunately the range of low temperature coefficients, from a few thousand ohms up to about a megohm, is an exceedingly useful

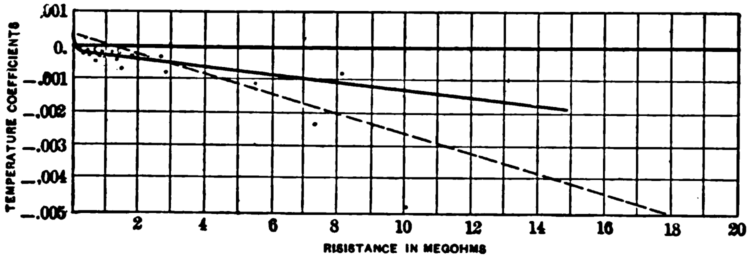


Fig. 9.

one. It could hardly have happened to occur in a more desirable place.

ADJUSTING AND STANDARDIZING FILM RESISTANCES.

It is important to consider in connection with the question of temperature coefficients, a method of adjusting these resistances to a particular value. The method referred to is substantially the same as the one described by Mr. Fawcett. It was also used in this investigation as early as June 2, 1898. It consists of simply cutting the film into sections, as in Fig. 10, so as to increase the length and diminish the breadth of the conductor. In this way the resistance of a film may easily be increased ten fold or even much more than that, if necessary.

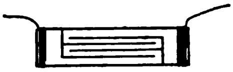


Fig. 10.

Of course a resistance may in this way be adjusted to any desired value, provided the final value is to be *higher* than the original value; but the importance of the method in connection with the question of temperature coefficients is in the fact that a film *thin* enough to have a resistance of several megohms is quite likely to have a rather large negative temperature coefficient (see Fig. 9), while a film of something less than 100,000 ohms resistance with a zero or negligible temperature coefficient may, by means of the method here described, have its resistance increased to the higher value without altering its temperature coefficient.

For making very high resistances with zero temperature coefficients it is advisable to use wider plates, so that the current may be made to traverse the length of the plate a greater number of times without making the conductor dangerously narrow.

The same result may be accomplished by bending a small glass tube or rod into the form shown in Fig. 11 and depositing upon it a film of such thickness as to have a zero or neglible temperature coefficient. In this case, however, the final adjustment to a required value is not quite so easy.

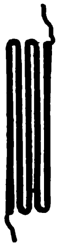


Fig. 11.

AGING AND PROTECTING THE FILMS.

The change in resistance which a film undergoes, quite rapidly at first, and less rapidly later, makes the "artificial aging," or "seasoning" process one of great importance. There are, however, no difficulties connected with it. It is the same process for films as for wire resistances. The films may be heated and cooled in the air, or they may be boiled in oil or in melted paraffin. In any case the process is rather a tedious one and should not be considered at an end until sometime after the point has been reached where no further change is noticeable. Otherwise the resistance will continue to undergo slight changes for a considerable length of time. The following table exhibits some of the results of perfect and imperfect artificial aging:

		Resistance in Ohms.			
Class A.	1	2265.8	2265.7	2265.8	2265.8
		37586.	37585.	37588.	37586.
Class B.	3	384.90	383.72	383.16	382.92
	4	9999.5	9992.5	9990.3	9989.4
	5	15722.	15697.	15685.	15681.
	6	23335.	23680.	23765.	23774.
	7	165200.	167680.	167880.	167930.
Class C.	8	84167.	83710.	83431.	83178.
	9	656030.	653600.	651620.	650110.
	10	4690000.	5008000.	5070000.	5080000.

The values given in the successive columns are for measurements about a month apart, and either made at the same temperature or reduced to the same temperature; the temperature coefficients of all the films having been carefully determined.

The films in class (A) were thoroughly seasoned; while those in class (B) were less perfectly seasoned and those in class (C) were very poorly seasoned. It will be observed that in class (C) the variations amount to as much as two or three tenths of 1 per cent. even during the third month. In class (B) the corresponding variations are only a few hundredths of 1 per cent.; while in class (A) no change greater than could be attributed to errors of observation was detected during the entire duration of the test.

In subjecting a number of films to the process of artificial aging, a rather important relation between this process and temperature coefficients was observed. By keeping the films connected with the Wheatstone bridge during the aging process, it was seen that the resistances did not continue to change in the *same direction* during the entire process. It was observed, first, that if the rapid change of resistance which occurs when the film is first placed in a hot bath is an *increase*, it will soon reach a maximum and then gradually *decrease* for a number of hours; second, that if the first rapid change is a *decrease*, the later and more gradual change will be an *increase;* and third, that if the resistance does not change at first it will not change at all. Now a film of the first kind always has a positive temperature coefficient, while one of the second kind always has a negative temperature coefficient. Of course the temperature coefficient of the third kind is zero.

The sign, and roughly the value of the temperature coefficient of a film resistance may be determined by holding a glowing incandescent lamp within about an inch of the film, while the latter forms one arm of a balanced Wheatstone bridge. If the resistance does not change at all the temperature coefficient is zero and the film does not need much artificial aging. Briefly stated, the changes in resistance which occur during the process of artificial aging, and the length of time required to bring the resistance to a permanent value, are proportional to the magnitude of the temperature coefficient. Therefore a low temperature coefficient is important, not only for its own sake, but also as a means to an end. Films having low temperature coefficients are very easily seasoned and extremely reliable after seasoning.

It is probably true that in films there are not as many causes at work to create a necessity for artificial aging as in the case of wires.

It has recently been found by C. de Szily[1] that if a wire be subjected to *torsion* its electrical resistance increases, and that if the torsion is maintained the resistance very slowly diminishes with time. Also that a wire, which, after dextral torsion, has been relieved and then twisted by sinistral torsion to the zero position, diminishes in resistance more quickly that when the twist is maintained. Also that when a wire has been twisted beyond its elastic limit, and has been allowed to untwist itself, subsequent changes of

[1] Comptes Rendus, Vol. 128, pp. 927–930.

resistance for torsion are less than those which occur during the first twist. It is probable that stretching, bending and even winding upon a spool all produce a similar condition in wires.

After the artificial aging process is at an end it is important that the films be protected from the air in order that no new conditions may arise to produce changes in resistance. They may be enclosed in glass tubes from which the air is exhausted, they may be simply imbedded in paraffin, or they may be coated with varnish prepared by dissolving India-rubber in carbon bisulphide.

THICKNESS OF FILMS.

In consideration of the relation which evidently exists between thickness of films and temperature coefficients, it seems desirable to be able to measure the thickness of the films.

It is to be regretted that no method of making an exact determination of the thickness of the *very thin* films has been discovered. Miss Stone calculated the thickness of her films from their weight and area, assuming that the density was the same as that of silver in its ordinary condition. However correct or incorrect this method may have been for silver films deposited from aqueous solutions, it certainly would not be very useful in determining the thickness of films of such varying compactness as those deposited by kathode discharge. Furthermore, many of the films considered in this research are entirely too thin to produce any impression whatever upon the most delicate balance; so that both density and weight are unknown quantities. Even if an exact determination of weight could be made, it would be of doubtful value, because of the varying and always indefinite amount of air and other gases absorbed by the film and weighed with it. The property of absorbing and occluding gases does not belong to platinum alone. It is possessed in some degree by a large number of metals.

For this and other reasons it was deemed desirable to attempt to determine the weight of a few films by the methods used in quantitative chemical analysis. Silver was the metal chosen for these experiments, because of the very delicate and exact existing method for its quantitative determination. After the films were deposited, they were converted into silver nitrate and then subjected to the volumetric process known as Gay-Lussac's method. The quantity of silver in these films was so small that although the method of analysis used was the one which Fresenius describes as "The most exact of all known volumetric processes," yet it was not possible in any case to make an exact determination of the amount of silver in a *transparent* film, and if the films were really *thin* it was not even possible to *detect* any silver in the solution. These facts are mentioned as forcible illustrations of the exceeding thinness of the films.

The assumption upon which Mr. Fawcett bases his method of calculating thickness is certainly unwarranted; for in very thin films, the ratio of the measured resistance to the calculated resistance is unquestionably very much higher than it is in thicker films.

Professor Michelson's interferometer furnishes a direct method of measuring the thickness of comparatively *thick* films, but that is not the important part of the problem; for even a thick film, in the sense in which the terms thick and thin are here used, is only one or two tenths of a wave length thick. However, a few comparative experiments were made upon thick films, determining the thickness by means of the interferometer and also from weight and area. In some cases the results were fairly concordant, indicating that in these cases the density was about normal.

A silver film which was just transparent enough to reveal the outline of a luminous gas jet was used in making one of these comparisons. A portion of the film was removed from the glass and the optical difference of path between the surface of the glass and the front surface of the film, was measured by means of the interferometer. The displacement of the interference fringes due to this difference of path was about .3 of a fringe. (Sodium light.) The thickness of the film is therefore about 0.00009 mm. The thickness of the same film, calculated from its weight, 0.0008 g., and area, 7.38 square centimeters, assuming a density of 10.55, was 0.00010 mm. A platinum film somewhat more transparent was found by the interferometer method to have a thickness of 0.00006 mm., while its thickness, calculated from its weight and area was 0.000063 mm. This, however, does not prove that the density of thin films is the same as that of thick ones or even that the density is the same for all films of the same thickness.

It must be remembered that both of the films just considered were *thick* films. The utter uselessness of attempting to weigh very *thin* films is illustrated by the fact that a platinum film 15 mm. wide and 5 cm. long, weighing only 0.00025 g. has a resistance of only about 2000 ohms. What would be the weight of a film thin enough to have a resistance of several megohms?

Photometric methods are of some service, for they at least give an indication of the amount of metal in a film which is too thin to be weighed, but the results obtained by this method do not agree to within 10 per cent.

There is yet one method of estimating thickness to be considered.

It seems reasonable to suppose that the rate of deposition must be uniform under uniform conditions, and that therefore the thicknesses of films ought to be directly proportional to the times required for their deposition; provided only that none of the conditions are allowed to change during the process. The electro-motive force, current and distance are under perfect control. The frequency of the Wehnelt interrupter is not very difficult to control if the temperature is kept constant by means of a cold water circulation. Even the vacuum may be kept within reasonable limits if very great care is exercised during the first part of the process, while the occluded gases are being expelled. It is therefore believed that the time element in the deposition of a film may be made the most reliable basis for estimating thickness; and it is this element that has been trusted more than any other in the estimates of thickness which have been made during the progress of this research.

CHOICE OF MATERIALS.

In the choice of a metal for the production of film resistances, permanence has been the chief consideration, and platinum, all things considered, is believed to be the most suitable metal. Gold is doubtless just as permanent as platinum as far as its ability to resist chemical action is concerned; but its molecular condition does not settle down to a permanent value as that of platinum does. H. L. Callendar in his work "On a Practical Thermometric Standard"[1] condemns gold as a material on the ground that there is a slow but

[1] Phil. Mag., S. 5, Vol. 48, pp. 519–547.

constant change of zero in a gold thermometer. This of course, means a slow but constant change of electrical resistance at a certain temperature. It is for this reason that gold is not a suitable metal for resistance standards.

A number of other metals have been experimented upon, but none of them seemed to possess any advantage over platinum.

SUMMARY OF RESULTS.

The conclusions drawn from this investigation, briefly stated, are as follows :

(1) The process of depositing metals by kathode discharge as illustrated by figures 3, 4 and 5, is probably simple *distillation*, in which the vaporization of the kathode is largely due to its electrification.

(2) The temperature coefficients of the very thin films are negative, and for films within a certain range of thicknesses, the temperature coefficients are approximately zero.

(3) The necessity of artificial aging is proportional to the magnitude of the temperature coefficient.

It may be said in general that the method herein described is capable of producing standard electrical resistances of a very high degree of precision and of any desired value, from a few ohms up to several megohms ; that these resistances may be made of pure metals—simple elementary substances—and that they are therefore free from the possibility of decomposition changes ; that the metals may be those least likely to enter into chemical combination with other elements ; and that in addition to these valued qualities as pure metals, they possess the only desirable qualities of alloys, namely high specific resistance and low temperature coefficients.

In closing this dissertation I wish to express my sincere thanks to Professors Rood and Hallock, of Columbia University, for their kindly interest in my work and for the many helpful suggestions which they have given me. Also to Professors Michelson and Stratton, of the University of Chicago, for valuable assistance during the early part of the investigation.

FAYERWEATHER PHYSICAL LABORATORY,
COLUMBIA UNIVERSITY, March 31, 1900.

EFFECT OF MAGNETIZATION UPON THE MODULUS OF ELASTICITY.

By James S. Stevens.

III. Elasticity of Traction.

In previous papers the relation between magnetization and the moduli of flexional and torsional elasticity has been discussed. The present experiment deals with elasticity of traction. My assistant, Mr. Stanley Sidensparker, undertook the work and carried it on with occasional assistance.

The form of the apparatus may be understood from the figure.

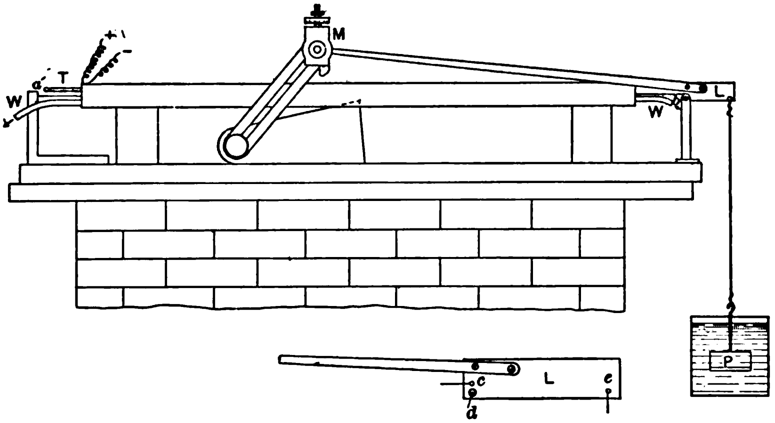


Fig. 1.

A coil of wire was wound on a brass tube through which passed a stream of running water, in which was carried the wire to be tested. One end of the wire was fastened to a rigid iron support and the other was attached to a bent lever with a multiplying power of about 11. At the other end of the lever were attached weights which gave a tension on the wire of 5.39×10^6, 10.78×10^6 and 19.6×10^6 dynes respectively. The changes in length were communicated to another arm of the same lever, which multiplied the effect 110 times. The free end of this lever moved under the field of a micrometer microscope. A thermometer whose bulb was inserted in the stream of water completed the arrangement.

DATA.

Length of coil	131.1	cm.
No. of turns of wire	1612	
Diameter of wire	0.13	cm.
Length of wire tested	175	cm.
Diameter of wire A	0.73	mm.
Diameter of wire B	0.98	mm.
Tension weight A	5.5	kgm.
Tension weight B	11.0	kgm.
Tension weight C	20.0	kgm.
Multiplying power of first lever	11	
Multiplying power of second lever	110	
Microscope constant 28.33 turns per min.		
Current	2.2–7.6	Amp.

The procedure consisted in setting the cross wires of the microscope on some point for zero, then turning on the current and noting the deflection of the end of the lever. The tables which follow give the current, zero readings, the difference, and the change in the length of the wire reduced to centimeters.

TABLE I.

Wire B. Tension B.

Current.	Initial reading.	Final reading.	Difference.	Change in length in cms.
2.12-2.18 amps.	42	88	46	15×10^{-6}
	75	18	43	14 "
	79	10	31	10 "
	68	16	48	15 "
	82	26	44	14 "
	79	15	36	11 "
	59	92	33	11 "
	41	80	39	12 "
	4	36	32	10 "
	81	14	33	11 "
	56	5	49	15 "
	51	87	36	11 "
	41	74	33	11 "

The above table shows complete readings in order to set forth the degree of constancy which the experiment afforded. This was about an average set of readings. The tables which follow this give the mean only of a number of readings, usually about 12.

TABLE II.

Wire B. Tension B.

Current amps.	Change in length cms.	Current amps.	Change in length cms.
2.62	13×10^{-6}	5.30	26×10^{-6}
3.95	16 "	6.50	48 "
4.50	20 "		

TABLE III.

Wire B. Tension A.

Current amps.	Change in length cms.	Current amps.	Change in length cms.
2.6	6.0×10^{-6}	4.2	12.8×10^{-6}
3.5	9.0 "	4.7	16.0 "

TABLE IV.

Wire A. Tension A.

Current amps.	Change in length cms.
4.0	12.8×10^{-6}
7.7	25.4 "

TABLE V.

Wire B. Tension C.

Current amps.	Change in length cms.	Current amps.	Change in length cms.
6.9	15.0×10^{-6}	7.6	19.2×10^{-6}
7.1	16.1 "		

TABLE VI.

Wire B. Tension B.

Current amps.	Change in length cms.
6.8	10.2×10^{-6}
8.0	11.2 "

From the tables we may draw the following conclusions :

1. The modules of elasticity of traction *increases* when the body is magnetized.

2. The increase is fairly proportional to the magnetizing force.

3. With the same wire the change in the modulus varied with the stretching weight.

4. With wires of different cross sections, stretched by the same load, the change in modulus was greater with the smaller wire.

It was considered that errors due to change in temperature were entirely avoided or reduced to negligible quantities. The stream of water in which the wires were placed kept the temperature so constant that changes greater than 0.1 degree did not occur in fifteen minutes. Moreover the change in the length of the wire due to magnetization was noted in about fifteen seconds. It may therefore be assumed that temperature changes during the time in which the observations were made were about of the order of $\frac{1}{600}$ of a degree F. This was about $\frac{1}{3}$ the value of the smallest observed change. The principal test for absence of temperature effects, however, was the fact that as in former experiments already noted, the changes in length due to magnetization are sudden and easily distinguished from the temperature effects which are creeping. Whatever changes may be due to temperature must of course be added to the results above given since they take place in the opposite direction. The same remarks will apply to changes in the length of the wires due to magnetization alone.

IV. ELASTICITY OF COMPRESSION.

In this experiment the apparatus was constructed, and the measurements taken by Mr. H. G. Dorsey of the department of physics at this University.

The figure shows the arrangement of apparatus. The bar was fastened firmly at one end and moved a lever arm when changing in length. The other arm of the lever moved under the field of a micrometer microscope. The compressing force was measured by a spring balance. The whole apparatus was bolted rigidly to the bed of a planer, so that there was little chance for slipping or friction. The knife edges and bearings were of hardened steel and variation of pressure was applied by moving the bed plate of the planer.

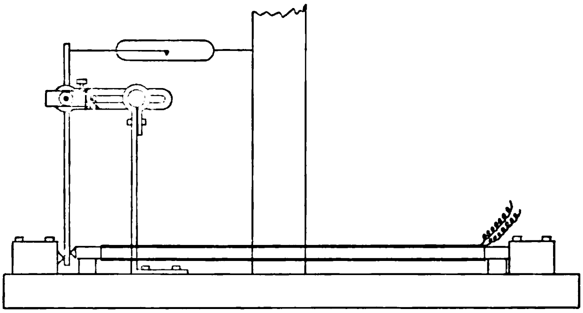


Fig. 2.

DATA.

Length of rod	134.5 cm.
Diameter of rod	2.54 cm.
Resistance of coil	3.3 ohms.

Wound with No. 16 double cotton covered magnet wire 1830 turns.

Ratio of power arm	1 to 120.
Ratio of index	1 to 106.7

Constant of micrometer microscope, 28.33 turns to 1 mm.

The tables which follow are self explanatory.

	270 kilos.		540 kilos.		1080 kilos.
C.	*change.*	*C.*	*change.*	*C.*	*change.*
.5	.00038+	.5	.00014+	1.	.00030+
1.	.00073+	1.	.00031+	2.	.00034+
1.5	.00068+	2.	.00083+	4.	.00023+
2.	.00051+	3.	.00028+	6.	.00000
2.5	.00021+	3.5	.00025+	8.	.00021−
3.	.00019+	4.	.00012+	10.	.00029−
3.5	.00009+	5.	.00011+	15.	.00052−
4.	.00000	6.	.00000	23.	.00180−
4.5	.00002−	8.	.00018−		
5.	.00017−	10.	.00034−		1350 kilos.
6.	.00018−	17.	.00130−	*C.*	*change.*
7.	.00032−	23.	.00139−	1.	.00033+
8.	.00039−			2.	.00019+
17.	.00112−			4.	.00015+
20.	.00145−			6.	.00000
23.	.00154−			10.	.00012−
				15.	.00043−
				22.5	.00112−

Current in Amperes. Changes in mm. + Increase in length of bar.
— Decrease in length of bar.

When these results came to be plotted it was found that very little difference could be detected for changes in compressing force varying from 270 to 1350 kilos. It is noticeable also that the curves resembled those of other experimenters who have tested the changes in length of a bar when magnetized. They are particularly similar to those obtained by Dr. L. T. More. (See PHYSICAL REVIEW, November–December, 1895.)

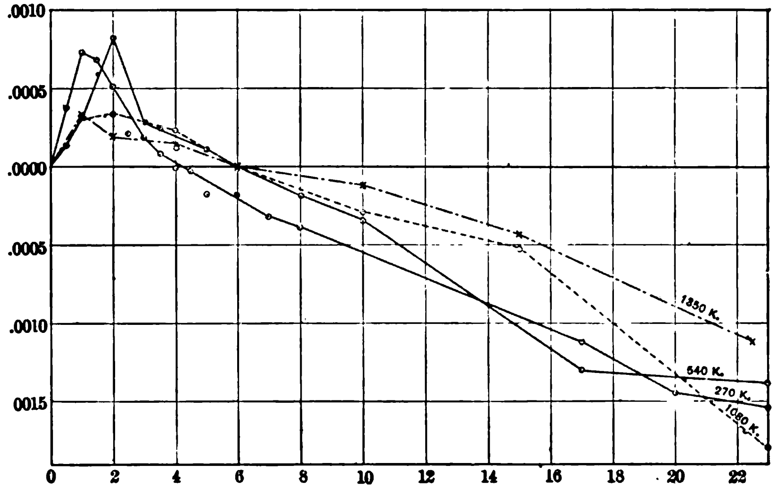


Fig. 3.

It may be said therefore that within the limits of this experiment magnetization does not change the elastic modulus for compression. If it had been possible to apply a pressure nearer to that of the elastic limit of the metal, the results might have been different.

Of the four experiments described in this JOURNAL, three have shown clearly that magnetization increases the elastic modulus in case of torsion, traction and flexion. We began these investigations with no theory as to the results. An experiment along a different line previously performed, seemed to indicate that magnetization did not change the modulus; another (*Phil. Mag.*, June, 1899), suggested that the modulus by flexion was decreased.

The behavior of magnetized bodies offers an attractive field for investigation and cannot fail to throw light on the general theory of magnetization, and indirectly on the constitution of bodies.

THE UNIVERSITY OF MAINE.

SPECTRA OF MIXTURES.

By Carl J. Rollefson.

AMONG the results obtained from a photographic study of arc spectra, the following may be of some interest. The aim of the experimenter was to note the change, if any, in the spectrum of a metal when a salt of this metal was introduced into the arc together with the salt of some other metal.

The apparatus used were about the same as those used by Foley,[1] and described by him in the Physical Review. A Rowland concave grating of 6½ feet radius and 14,438 lines to the inch, was placed upon a Brashear mounting. The light from the arc lamp used was reflected upon the slit by a concave mirror of ten-inch aperture and twelve feet radius. In Fig. 1, *L* represents the arc lamp, *M* the mirror, *S* the slit, *G* the grating, *C* the camera box. The slit, *S*, had the usual adjustments for width, length and direction. The width used varied from .015 mm. to .02 mm. The room was not darkened, but the space from *S* to *G* and from *G* to *C* was inclosed so as to exclude all light except what entered through the slit *S*. The tube from *S* to *G* consisted of a number of metal rings placed crosswise at intervals along the track, and about these rings black cotton flannel was wrapped. In order to

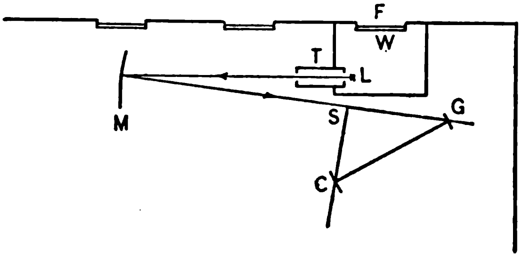


Fig. 1.

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[1] Arc Spectra, Arthur L. Foley, Physical Review, Vol. 5, p. 129, 1897.

prevent the ultra-violet rays of the first spectrum, and also the rays from spectra of higher orders from falling upon the sides of the tube leading from *G* to *C*, from which they would be reflected upon the plate, a tapering box, shown in Fig. 2, was used. Along the inside walls of the box were fastened strips of tin, and the whole interior was painted black. The camera box was constructed for 2 x 8 inch plates, curved to 6½ feet radius. In order to conform to the grating the plates should have been bent to 3¼ feet radius, but the plates would break when bent to this curvature. That the spectrum remains a normal one when the plates are bent to a radius of 6½ feet, is shown by Foley in the article already referred to.

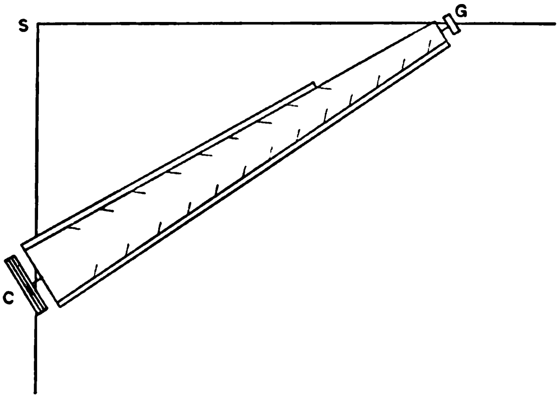

Fig. 2.

The metallic salts experimented with were at first introduced into both carbons, but later only one carbon was filled, and this was usually made the positive electrode. A pair of carbons was used for each salt or mixture.

All of the exposures were made in the primary spectrum in the region where the actinic effect is greatest, from $\lambda = 3200$ to $\lambda = 5000$.

I.

On a plate containing the spectrum of strontium and lithium were noticed some lines that did not seem to coincide with any known lines. As these same lines were obtained when a mixture of the chlorides of strontium and potassium was used instead of strontium oxide and lithium carbonate, and did not appear under any circumstances when strontium was not introduced, the conclusion was drawn that these lines were due to strontium. The wave-lengths were found to be 3780.68 and 3807.51. Afterwards it was found that Rydberg, by the aid of his formula for the strontium series, had predicted a group of three lines in the cyanogen band at $\lambda = 3800$, and that Kayser and Runge[1] had then studied the spectrum of strontium again, introducing carbon dioxide into the arc to weaken the cyanogen band, and had discovered the triplet at λ 3780.58, 3807.51 and 3865.59.

It will be noticed that the two lines on my plate correspond to two of the lines found by Kayser and Runge, and the third line $\lambda = 3865.59$ was found immediately after reading the account of their experiments.

Besides these three lines there are six more that have not been recorded before, that seem to be due to strontium. The wave-lengths are 4139.89, 4313.64, 4701.29, 4704.42, 4707.65, 4714.42. The reasons for believing these to be due to strontium are that they have not been found except where strontium has been introduced into the arc, and they have the same general appearance as the known lines of strontium.

Besides these lines there is also a band extending from $\lambda = 4400$ to $\lambda = 4410$, which seems to correspond to the magnesium oxide bands, but is very weak in comparison with them. It is probably due to strontium, or strontium oxide, as it has been found only on plates containing the spectrum of mixtures of which strontium oxide, or chloride, formed a part.

II.

In her study of arc spectra Miss C. W. Baldwin[2] found that there is a decided difference in the spectrum of the several sheaths of the arc, and that "the difference is largely due to a fading out of the carbon bands and of all lines from the positive carbon, and an increase of intensity in the outer sheaths of those lines which are most brilliant near the negative carbon." This was later investigated by Foley,[1] whose results do not agree with those of Miss Baldwin. He found that many of the metallic lines are relatively much stronger in the outer region of the arc, than other lines due to the same element, but did not find any that have a maximum of intensity in the outer sheaths.

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[1] Beiträge zur Kenntniss der Linienspectren. II. Kayser u. C. Runge, Wied. Ann 52, 1894.

[2] Physical Review, Vol. III., Nos. 17 and 18, 1896.

Some of the plates obtained in the present investigation are of some interest in this connection. Among the metals experimented with were bismuth and lead. In studying the spectra of these two metals the horizontal arc was used. The center of the arc was focused on the middle of the slit so that the ends of the slit extended out through the outer sheaths of the image of the arc. The object of the experiment being to see if the lines of either bismuth or lead were influenced in any way when a mixture of the salts of these two metals was introduced into the arc, three sets of carbons were prepared. Two of them contained the salts separately, and the third contained the mixture. The salts used were nitrate of bismuth and peroxide of lead. When the plates had been exposed and developed it was found that strontium had been present as an impurity either in the carbons or in the salts used, and that the strontium line at $\lambda = 4607.52$ was not continuous. It was not visible in the middle, while both ends were strong and extended beyond the carbon lines. This result was afterwards verified.

The effect of introducing strontium with bismuth and lead was then tried. When only a small quantity of strontium is introduced the line is stronger at the ends than in the middle, but the plates seem to indicate that as the quantity of strontium introduced is increased, the middle of the line increases in intensity relatively more than the ends.

The barium line $\lambda = 4554.31$ is also stronger in the outer sheath under the same conditions. It does not, however, on the plates obtained, extend out in both directions as the strontium line does, but only at the end produced by the rays coming from the flame above the arc.

It is doubtful if any of the calcium lines mentioned by Miss Baldwin are affected by bismuth or lead.

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[1] PHYSICAL REVIEW, Vol. V., p. 129, 1897.

DESCRIPTION OF A NEW ATWOOD'S MACHINE AND STOP CLOCK.

By Will. C. Baker.

THE Atwood's machine and stop-clock recently built in the mechanical laboratories of this university for use in the physics department, presents some novel features that are of interest.

The clock consists of a uniformly rotating shaft carrying a loose sleeve to which are rigidly attached a wheel of one hundred teeth and a pointer or hand that indicates the angular position of the sleeve on a fixed dial graduated with one hundred divisions. (See Figures 1, 2 and 3.)

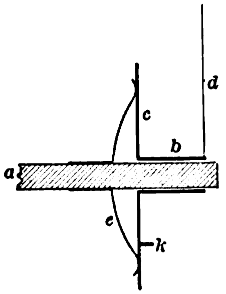

Fig. 1.

Section of Shaft and Sleeve showing frictional connection.

a, uniformly rotating shaft.
b, loose sleeve.
c, wheel of 100 teeth.
d, pointer or hand.
e, light spring forming friction clutch.

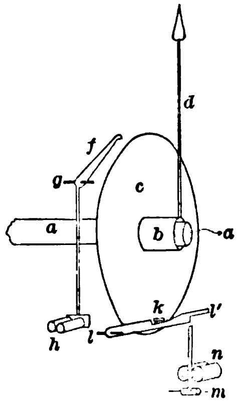

Fig. 2.

Perspective view of release lever, stop pawl, and magnets.

f, stop pawl.
i, release lever.
k, peg.
h and *n*, electromagnets.
g, *l* and *m*, pivots.

Rigidly attached to the shaft and pressing lightly against the back of the toothed wheel is a spring that communicates the motion of the shaft to the sleeve and hand except when these are held by catches, in which case the spring slips over the surface of the wheel. Immediately over the toothed wheel is a pawl, *f*, (Fig. 2) pivotted at *g*, which, by means of an electromagnet, *h*, may be caused to fall into one of the notches between the teeth of *c*, and so arrest the motion of the hand. The wheel *c* carries also a small peg situated at *k*, diametrically opposite to the pointer *d*. A second catch pivotted at *l*, and notched as shown, is supported by a vertical piece hinged at *m*, and arranged so that when it is in the position shown, *k* catches in the upper notch of the release lever, stopping the hand at the zero of the dial. Now, when the electromagnet, *n*, is energized, the support is drawn back, the release lever falls free of *k* and the hand and sleeve, being very light, take on the motion of the shaft practically instantaneously.

In the Atwood's machine itself the cylinder that is to descend with its rider is supported by a small horizontal platform, *p*, as shown in Fig. 3, and in detail in Fig. 4. The energizing of the electromagnet *q*, as may be easily seen, permits the platform to drop and the cylinder to fall. This electromagnet is put in series with a key, a battery cell, and the electromagnet *n* of the clock. The closing of the key releases the clock hand and the following cylinder at the same instant.

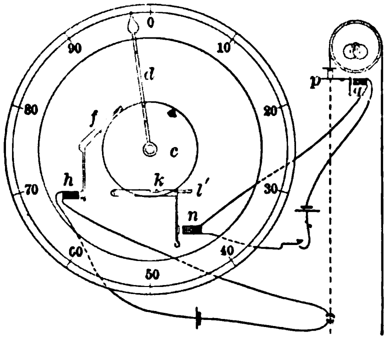

Fig. 3.

Diagrammatic view of connections. *p*, dropping platform.

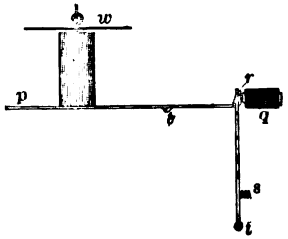

Fig. 4.

Detail of dropping platform.
q, electromagnet.
r, armature.
s, spring.
t, *t*, pivots.
w, cylinder and metal rider.

The ring at the base of the machine consists of two half circles of lead held by an insulating support. These are connected to the opposite poles of a battery cell, by wires, one of which has in current the electromagnet *n* of the clock. When the metallic rider falls across the ring it completes the circuit, throwing the pawl, *f*, into a notch of the wheel, *c*, and the hand is stopped at once. Fig. 3 shows these connections diagrammatically.

The regulation of the clock-shaft *a* (driven by a weight through a train of wheels) is affected as follows: A toothed wheel rigidly attached to the back end of *a* works into an endless screw on a vertical shaft, that bears a heavy cross bar supporting fans that may be set at any angle to their path. (See photograph, figure 5.) As this regulator has a high moment of inertia the change of the angular velocity of the main shaft, *a* is very slow[1] and the stopping of the motion of the sleeve, even for a few seconds produces but a very slight effect. The following method of reading is therefore adopted: The shaft is allowed to attain a state of uniform rotary motion, the sleeve moving with the shaft. The cylinder and rider of the Atwood's machine are raised into position. Just after the stop *k* has passed the notch, the release lever is raised[2] and the vertical piece springs forward and supports it. As soon as the hand, at the beginning of the next revolution, is checked by *k* striking the notch, the observer closes the key and immediately the hand starts on again from zero and the cylinder with its rider begins its fall. When in its descent the rider falls across the ring, the second circuit is closed and the hand stops. The reading is now taken and the rider removed at once. Sometimes, especially for rapid falls, the rider bounces from the ring, merely checking the hand for an instant, but even in this case no difficulty has been found in reading.

The cylinder is raised to place again and all is ready for a second observation.

The machine has not been used to determine the velocity produced by a certain fall[3] but to show that the acceleration produced in a given mass varies directly with the force acting, and that the acceleration produced by a given force varies inversely with the mass acted on.[1]

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[1] This may be measured by a hand fixed to *a*.

[2] This is done (from the outside of she case) by a small lever (seen in figure 5).

[3] This would require a mere change of connections and a second adjustable platform.

Forty-one consecutive readings of a certain fall were made to test the constancy of the apparatus. The time value of one division (S) was 0.016 seconds, and the values obtained were: 327 S, 14 times; 328 S, 17 times; 329 S, 6 times; 330 S, twice; and 331 S, twice. A set of thirteen readings (all 197 S or 198 S) was made, as in the

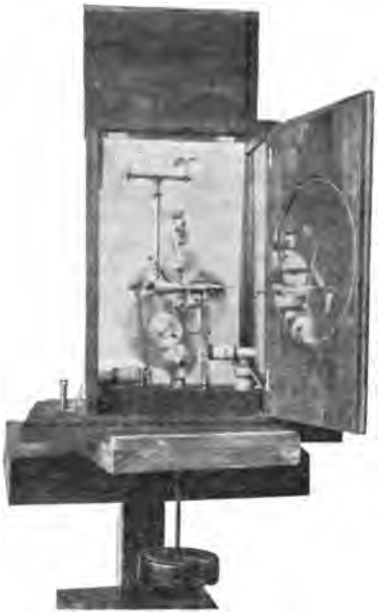

Fig. 5.

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[1] By using two riders A and B and four cylinders M, N, x and $\bar{y}$ having the mass relations $A > B$, $M = N$, $x = y$ and $M > x$. Now if $A + M$ falls, raising $B + N$ the force acting is $(A - B)g$. If $A + B + M$ raise N the force is $(A + B)g$ and the mass moved is the same as before. The force ratio $\frac{A - B}{A + B}$ should be equal to the acceleration ratio $\frac{a_1}{a_2}$ which is given by $\frac{t_2^2}{t_1^2}$, t_1 and t_2 being the lines of falling the same distance in the first and second cases.

Again if $A + x$ be made to raise $B + y$ the forces in the first and third cases are the same but the mass ratio is $\frac{M_1}{M_3} = \frac{A + B + M + N + W}{A + B + x + y + W}$, W being the correction for the wheel of the Attwood's machine. The mass ratio should be the inverse of the acceleration ratio. This method is given by Ames and Bliss.

second case of the preceding footnote, with the result $\frac{F_1}{F_2} = 0.364$ and $\frac{a_1}{a_2} = 0.362$, a discrepancy of about the half of one per cent.

The apparatus was designed by Professor N. F. Dupuis, Dean of the Practical Science Faculty, and was constructed under his supervision by students of that department.

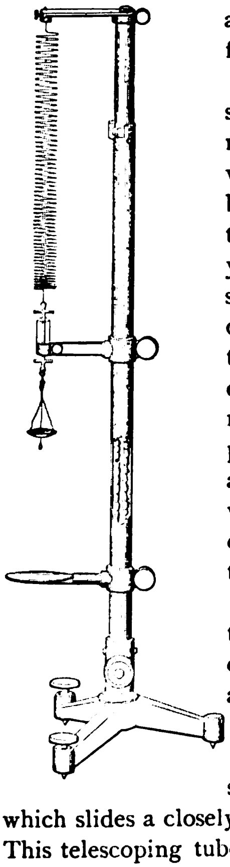
The photograph, Figure 5, shows the clock and driving weight, the dial and shaft hand being removed. The peg K and release lever in its raised position may be seen rather better as reflected in the clock face than directly. The screw regulating the stop pawls, may also be seen. To the spectator's sight is visible part of the small lever for raising the release lever from the outside of the case.

Physical Laboratory, Queen's University,
Kingston, Ont., May 7, 1900.

A NEW FORM OF THE SPIRAL SPRING BALANCE.

C. E. Linebarger.

THE determination of weight by means of a measurement of the elongation of a spiral spring has become very common and "spring scales" of all sorts and sizes abound. The reason for this lies in the circumstance that such instruments are self-contained, have no easily separable parts and are very simple to read, a change of length alone being the measure of the force.



In scientific laboratories the form of the spiral spring balance devised by Jolly is in general use, mostly for the finding of specific gravities and the verification of Hooke's Law. Although the Jolly balance is so popular and has been in use for over thirty years in essentially the same original form, yet it must be conceded that it has some rather undesirable features. Thus, when it is moved, the spring often flies about in a most disagreeable manner; there is no arrangement for steadying and dampening the spring's vibrations; the platform can be moved only up and down; the reading of the position of the head on the mirror-scale is trying and hard on the eyes; it is almost impossible to use with it any accessory apparatus because the latter cannot easily be adjusted to follow the movement of the bead.

The present modifications in the construction of the spiral spring balance avoid to a considerable extent the undesirable features of the "Jolly" and give a wider range of usefulness to this style of balance.

Upon an iron tripod base provided with levelling screws is fixed a nickel-plated brass tube, within which slides a closely-fitting tube bearing a millimeter scale (500 mm.). This telescoping tube may be moved up and down by means of an endless chain running around two cogged shafts fastened near the ends of the outer tube, the lower one being attached to a milled head. The upper end of the outer tube is cut out a little and bears a vernier. A third tube (10 cm. long), sliding in the telescoping tube and provided with a clamp, is for the purpose of lengthening the apparatus in case a spiral spring of extra length is used. It bears a horixontal arm for the suspension of the spring.

At the lower end of the spring is hung a piece of aluminium foil, cut in the shape shown in the figure. It plays in a glass tube held by an adjustable clamp. By this arrangement only a slight up and down or sideways movement of the spring is possible. One-half of the foil is blackened and a fine line is etched around the glass tube. The instrument is adjusted for reading by bringing the foil so that the line of division between its blackened and uncolored parts may coincide with the line around the glass tube. This reading device has proven to be very convenient and accurate. The platform is capable of sideways or vertical motion and moves very smoothly.

This instrument has been found applicable to a variety of experiments. Thus, besides being adapted to the verification of Hooke's Law and to the finding of specific gravities, it may be employed together with some simple accessory apparatus to illustrate the cohesion of liquids (a glass or metallic disk being used), to measure surface tension (a film of liquid being drawn up by means of a wire or foil frame) and to ascertain the relative intensity of magnetization in various parts of a bar-magnet by the "test nail method." With suitably wound coils and an iron core, ammeters and voltmeters of the Kohlrausch type may be readily set up, and by suspending from the spring a piece of copper foil immersed in a solution of copper sulphate, a voltameter may be arranged with which current strengths may be ascertained without removing the electrodes from the electrolyte or breaking the circuit. Also, with a steam calorimeter (serviceable ones may be made from lamp chimneys, glass tubings and corks) latent heats of vaporization and fusion and specific heats may be found.

APPARATUS FOR DEMONSTRATING BOYLE'S LAW.

By F. J. Rogers.

WHILE making use of an inclined tube of mercury for measuring pressure, it occurred to me that an apparatus based on this principle might be used with advantage for verifying Boyle's Law.

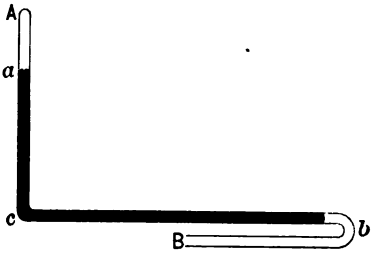


Fig. 1.

$Ac = 50$ cm.
$bc = 70$ cm.
$bB = 30$ cm.
$Aa = 13$ cm.

A glass tube 3 mm. internal diameter and 160 cm. long was bent into the form illustrated by Fig. 1. The tube was partially filled with mercury, and while lying in a horizontal plane the end A was sealed, the end B being left open. The apparatus in its simplest form was now ready for use.

If the bore of the tube is uniform the volume of confined air is proportional to the length Aa. And while the tube is lying in a horizontal plane the pressure to which the confined air is subjected is equal to that of the atmosphere or to H cm. of mercury. If the tube be now placed in a vertical plane as in Fig. 2, the volume of confined air is reduced while the pressure is increased by the column of mercury bc, so that it becomes $H + h$ cm. of mercury. If the tube be now turned 180° about

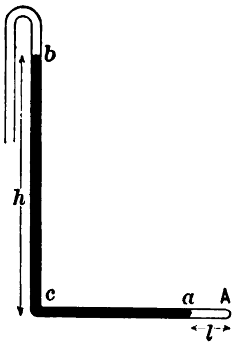


Fig. 2.

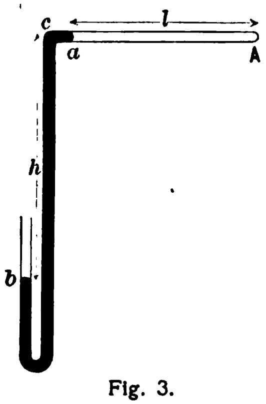


Fig. 3.

a horizontal axis it will be in the position illustrated by Fig. 3. In this case the volume of confined air has increased, while the pressure is less than the atmosphere by an amount represented by the column of mercury of height h (see Fig. 3). In any intermediate position, the part of the tube bc being inclined to the vertical, the pressure to which the confined air is subjected is $H \pm h$ in which h is the *vertical* distance of the free surface of the mercury column above or below the horizontal tube Ac.

The following table gives the first set of data taken with the first apparatus constructed. The dimensions of this apparatus were approximately as given in Fig. 1.

Volume of Air = Aa = V	**Vertical height of $bc = h$**	**Pressure = $H + h$ = p.**	**Pressure × volume = $p \times v$.**
42.0 cm.	−41.7 cm.	32.0 cm.	1344
31.5	−31.0	42.7	1345
27.1	−24.3	49.4	1339
21.6	−12.0	61.7	1333
18.1	00.0	73.7	1334
13.7	+24.4	98.1	1344
10.3	+57.7	131.4	1353

Atmospheric pressure from barometer, $H = 73.7$ cm. of mercury.

In the last column of the table the numbers represent the product of pressure and volume. The average deviation from the mean value is less than ½ per cent. which is very satisfactory in the introductory laboratory work for which this experiment is suitable.

The advantages of this apparatus are: (1) the same tube and mercury may be used many times in succession without getting dirty; (2) the apparatus consists of a single piece of glass without rubber tubing and joints to leak air; (3) a large range of pressures, both greater and less than an atmosphere, is obtained combined with a very small cost for apparatus and mercury.

To secure the best results certain precautions are necessary: (1) if the internal diameter of the tube is much less than 3 mm. the parts of the mercury column will not be easily united when accidentally separated; if much greater than 3 mm. a good meniscus will not be formed in the horizontal part of the tube; (2) the part of the tube containing the confined air must have a uniform section; (3) the parts of the tube and the amount of confined air must be properly proportioned; (4) proper correction must be made for the irregularity due to sealing the end of the tube. This correction may be made in the following manner: let the length of the air-column measured from some fixed point near the closed end of the tube be l; then $l + x$ is the equivalent length in which x is the desired correction. Now take some other value of the length when the pressure is different, and we shall have for the equivalent length $l' + x$. If the corresponding pressures are $H + h$ and $H - h'$, we have, assuming Boyle's law to be true,

$$(l + x)(H + h) = (l' + x)(H - h').$$

If this equation is solved for x, using known values of l and h we shall know how far the true zero-point for measuring volumes differs from the assumed zero-point. Taking the least and greatest values for l and the corresponding values of h from the preceding table and solving for x we find that $x = -.09$ cm. This means that the assumed zero-point for measuring volumes is .09 cm. on the negative side of the true zero-point.

For regular laboratory work the tube should be furnished with a suitable mounting. For example, an L-shaped frame may be used as illustrated by Fig. 4. The closed branch of the tube may be graduated, or a scale may be attached to the frame with its zero-point properly adjusted by trial as above described. The end of the frame at the closed end of the tube should be heavily weighted for the sake of stability.

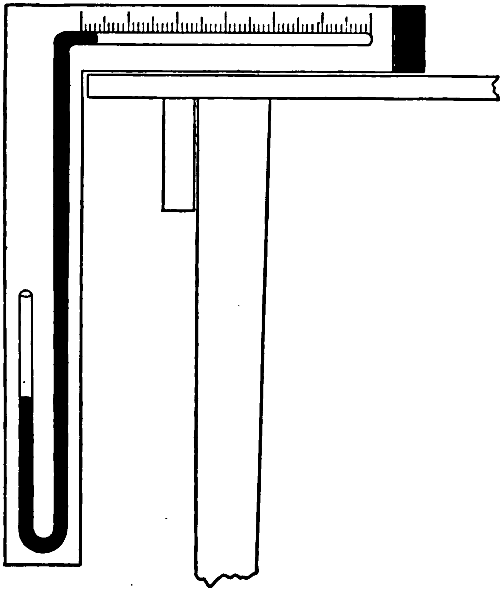


Fig. 4.

THE M. K. S. ABSOLUTE SYSTEM OF UNITS.

By F. J. Rogers.

THE C. G. S. units of force and work are extremely small, so small that any ordinary problem in mechanics will require for its solution the use of enormously large numbers. For example, the force of gravity acting on 50 Kg. is 49,000,000 dynes; the work required to lift 100 kg. a height of 100 m. is 980,000,000,000 ergs; one horse-power-hour is equal to 26,800,000,000,000 ergs. Of course, these large numbers may readily be expressed by using powers of 10, as for example, one-horse-power-hour $= 268 \times 10^{11}$ ergs. But there are objections to this. Students who are just beginning to use the C. G. S. units generally have little facility in the use of powers of ten in numerical calculations. A more serious objection is the fact that the units used are almost imperceptibly small while the numbers which express the actual quantities dealt with are almost inconceivably large.

These difficulties have led me to make use of a system of mechanical units based on the meter, the kilogram and the second, or briefly the M. K. S. System. In this system the unit of force is that force which, acting on one kilogram for one second, will give it a velocity of one meter per second. This unit of force I have called a *large Dyne* or merely *Dyne*, spelled with a capital letter to distinguish from the C. G. S. dyne. The relation between these two units of force may be expressed as follows:

$$1 \text{ Dyne} = 1 \text{ Kg.} \frac{\text{M}}{\text{sec}^2} = 1000 \times 100 \text{ g} \frac{\text{cm}}{\text{sec}^2} = 100{,}000 \text{ dynes.}$$

In the M. K. S. system the unit of work or energy is the work done when a force of one Dyne moves its point of application one meter. This unit is equal to ten million ergs and is, therefore, exactly equal to one Joule. It follows that the M. K. S. unit of power is equal to one Watt.

The use of this system requires the introduction of only one new unit, namely, the large Dyne. Even this unit may advantageously replace the megadyne which is occasionally used.

In addition to the greater convenience in solving problems in mechanics, the M. K. S. system has the advantage of bringing the Joule and Watt into a harmonious mechanical system involving the meter and the kilogram.

I have used this system of units in a first year university class in physics during the past year with little danger of confusion with the C. G. S. system which was also used.

Of course, no system will be really understood unless a large number of problems are solved, and, moreover, problems of some degree of complexity.

A MODIFICATION OF MANCE'S METHOD OF DETERMINING BATTERY RESISTANCE.

By Louis W. Austin.

ONE of the most elegant methods of determining the resistance of a battery cell is undoubtedly the Wheatstone's bridge method of Mance. As ordinarily used, however, it labors under certain disadvantages. Leaving out of account the annoyance of adjusting the galvanometer zero by means of the control magnet, the main difficulty in the use of the method lies in the fact that at each observation the battery circuit must be kept closed long enough for the galvanometer to come to rest, which in the case of open circuit batteries gives ample time for the errors due to polarization to appear unless large resistances are inserted in the bridge, thus reducing the sensibility of the method and rendering impossible the use of the convenient slide wire bridge for such cells. The placing of a condenser in series with the galvanometer as suggested by Professor Lodge does away with the first and smaller difficulty, but does not help the second. The substitution of a low resistance telephone in place of the galvanometer very largely obviates both. It possesses the advantages that it responds only to the changes of the current, and is always ready for an observation; hence no delay is necessary after the battery circuit is closed.

To carry out this modified method the slide wire bridge is connected as in the ordinary Mance's method, the telephone being connected to the ends of the bridge, and an empty wire across from the point between the two resistances to the bridge wire. No sliding contact piece is used, but contact is made by touching the slide wire with the empty wire with a stroking touch—a fine wire gives the best sound. A key is placed in circuit with the cell which is closed only at the instant that the connection on the bridge wire is closed. The sharp click of the closing of the key differs so much from the rustling noise of the other contact that there is no trouble in distinguishing the one from the other. This closing of the battery circuit in a series of almost instantaneous contacts gives little chance for polarization, and the resistances of even the most rapidly polarizing cells in common use may be found with a considerable degree of accuracy. For approximate determinations the method is very rapid, the resistance being obtained with an accuracy within a few per cent. almost instantly, and with a little care the determinations can be made practically as exact and satisfactory as with the more complicated Kohlrausch alternating current method.

The University of Wisconsin, April, 1900.

NEW BOOKS.

Physics: Experimental and Theoretical. Vol. I., *Mechanics, Hydrostatics, Pneumatics, Heat, Acoustics.* By R. H. Jude, D.Sc., M.A., Lecturer in Physics, Rutherford College, New Castle on Tyne. London, Chapman & Hall, 1899. Pp. xxiv + 926.

It is difficult to specify the character of this volume in any brief terms. In size, it almost equals the great compendious text-books so common in France and Germany, but as yet, for some reason, almost without existence in English. If the second volume is to be as large as Volume I., the work will exceed in bulk any modern general text-book of physics in our language.

Upon opening Dr. Jude's book, one is struck at first sight by the unusual allotment of space to the various topics. One is not so much surprised at the lack of space given to sound, for it is quite in vogue among modern writers to curtail this part of physics, as at the relatively large amount of space accorded to heat, as compared with mechanics. Looking more in detail at the selection of subjects for discussion, many other disparities meet the eye. The entire subject of capillarity, for example, is given within the limits of a single page, while one may easily select from neighboring portions of the book paragraph after paragraph given to unimportant and comparatively irrelevant matters, such as life belts, safety whistles, ball cocks, and the like. These might well have been sacrificed to the fuller treatment of the more vital and interesting phenomena which arise from the properties of the surface film of liquids. One might point out many other instances of the same kind. It is indeed a peculiarity of the work that a vast number of isolated topics receive brief and inadequate consideration. Some of these are of considerable interest in that they serve to illustrate some principle of physics, but the application is not always clearly brought out.

Among the faults of omission are the lack of an index, the almost complete absence of references to original sources, omissions which impair the usefulness of the volume considered as a book of reference (and it is too bulky to be conveniently used in any other way), and failure to indicate fully the sources from which the numerous tables of physics constants have been compiled. The value of these tables would have been greatly increased had the origin of the data been in all cases carefully given. The volume contains a very large number of miscellaneous examples and exercises, many of which are very good, while some are of doubtful utility. In these the metric system and the British systems of weights and measures are hopelessly mixed. For this, considering the deplorable example set by boards of science examiners throughout Great Britain, we ought perhaps not to hold the author wholly responsible, but we must hold him responsible for the introduction of new and questionable names for various derived units. One can but protest against such names as *velo* for a unit of velocity, *celo* for the unit of acceleration, *pul* for the unit of momentum, and above all against the use of *torque* for the unit of moment. Torque is already in universal use among physicists and engineers for the moment of a force tending to produce rotation.

The author attempts to justify himself in this matter in his preface, on the ground that it is "tolerably certain that much of the vagueness attaching in student's minds to sundry physical conceptions, arises from their units wandering about in an unbaptized state * * * ." As to the advantage of increasing the list of names assigned to science units, even when the nomenclature has been placed in the hands of committees of men best qualified for such work, we should differ fundamentally from Dr. Jude. When it comes to the free giving of names to such units on the part of individual authors, we desire to record vigorous disapproval.

In a book containing so many details, errors of all sorts are pretty sure to creep in. The vigilance necessary to exclude them all is something well-nigh superhuman. A British author could perhaps scarcely be expected to know that the Falls of Niagara which are, as he states, about 160 feet high, and may be assumed to deliver a certain amount of water per minute, do not strike Lake Ontario at all, since the base of the falls is distant several miles from the head of the lake; but when it comes to the scale of the hydrometer, as given on p. 129, one might expect the author to know that such a scale is not one of equal parts, and to demand of him that it should be drawn in accordance with the laws of the instrument.

It would be a thankless task to go through the volume, and point out the petty errors which meet the eye of the reader. Most of these will doubtless be corrected in later editions. I refer to such matters as the substitution of Döppler instead of Doppler, a slip which seems to be well-nigh universal among English writers, the statement that lampblack is a "perfect absorber," and the like; but there are certain deeper-lying faults in this volume which it is difficult to characterize and the existence of which makes the reading of the book, in the main, unsatisfactory. These we fear are not likely to be eliminated. They are those which arise, in all probability, from a failure on the part of the author to thoroughly assimilate and master all the minutiæ of the vast amount of information which he has attempted to get inside the covers of his work.

Such power over the details of a broad science like physics is rare indeed, and few, therefore, are the men whom one would like to see in charge of the task of writing a compendious treatise.

The style of Dr. Jude's book is vigorous, but not always dignified. One catches now and again a strident jarring overtone which reminds one too strongly of the recitation room; the insistent tone with which too often an overwrought teacher strives to impress the principles of his subject upon a listless class. It may perhaps be said of the treatise, with justice, that it is too obviously didactic in style; that it savors too strongly of the class-room and not sufficiently of the laboratory. It ought, in justice likewise to be said that there is a great deal of important material in the book, not always, alas, to be quickly found when wanted, and that some of the chapters on heat in particular are of very great interest and value.

E. L. N.

Physical Chemistry for Beginners. By Dr. Ch. van Deventer. With an Introduction by Prof. J. H. van't Hoff. Authorized American Edition from the German Edition. Translated by Bertram B. Boltwood, Ph.D. New York, John Wiley & Sons; London, Chapman & Hall, Limited., 1899.

The Arithmetic of Chemistry. Being a Simple Treatment of the Subject of Chemical Calculations. By John Waddell, B.Sc., Ph.D., D.Sc. New York, The Macmillan Co., 1899.

The Spirit of Organic Chemistry. An Introduction to the Current Literature of the Subject. By Prof. Arthur Lachman. With an Introduction by Prof. Paul C. Freer. New York, The Macmillan Co., 1899.

In 1893 there appeared in Amsterdam a little book on *Algemeene Scheikunde* (General Chemistry), for the use of students beginning the study of chemistry. The book was written in van't Hoff's laboratory; and van't Hoff used it in his classes as a text for that side of his course which dealt with general chemical principles and the physical aspect of chemical changes. It forms a very satisfactory, brief, introductory treatise on physical chemistry. Shortly after its appearance in Dutch it was reproduced in a German edition; and now, very properly, we have an English version.

The book considers, first, the conservation of mass, the laws of constant, multiple, and combining proportions, and Gay-Lussac's volume-combination law; then the properties of gases, and the ideas of molecular weight and of valence; and, next, the simpler facts of thermochemistry and the current conception of dissociation phenomena and mass-action. All this is followed by a brief outline of the theory of solutions, a few pages on chemical optics, and, finally, a chapter on the periodic classification of the chemical elements. The author's treatment is clear and good; and he presents only matters which form an essential part of any broadly conceived introductory course on general chemistry.

Our next book, Waddell's "Arithmetic of Chemistry," is a collection of chemical problems, with careful explanations of the methods used in solving them. The principles applied are the stoichiometric laws and the theorems of Boyle and of Gay-Lussac (Charles). The author says: "The form that this book has taken is due to a very considerable experience of the difficulties encountered by students when attempting to make chemical calculations. The effort has been made to smooth away these difficulties, and to give an accurate, as well as a simple and systematic, treatment of the subject." Like the foregoing book, this one is intended as an aid in elementary chemical instruction. An appendix presents some miscellaneous matter, such as an account of the metric system, a comparison of thermometric scales, a table of combining weights, and a table of logarithms.

The final book under review, Lachman's "Spirit of Organic Chemistry," is a gem in its way. It is an attempt to give to the advanced student of chemistry a vivid picture of the historical development of a number (nine) of important lines of work in organic chemistry. It is thus designed to serve as a supplement to text-books on organic chemistry. That such a book must be a great help to the earnest student is obvious to anyone; the need for it is made clear by Professor Freer, in his brilliant Introduction, in the following words:

"The magnificent science of modern Organic Chemistry, with its countless compounds, * * * fascinates with its possibilities, while it appalls with its vastness. The student stands discouraged at the very threshold. How can he ever hope to master the general classification, let alone the minor details, which must become a part of his very being, if he too wishes to do his share, however small, toward completing and rounding out the still unfinished structure? The answer is plain: he can do this only by comprehending the *spirit of the science*, by learning its great theories, not as mere mnemonic efforts, but as the result of a development for which many of the most earnest and acute minds known to the history of science have fought and toiled."

J. E. TREVOR.

Outlines of Industrial Chemistry. By F. H. THORP. 8vo, pp. xx + 541. New York, The Macmillan Company, 1899.

In the preface of this book the author states that it is intended "to furnish an elementary course in industrial chemistry" and that it is for the use of students who are at least "familiar with the elements of general chemistry, both inorganic and organic, and with the elements of physics." That a second edition was called for in a year after the first issue is sufficient evidence that a book of this kind is needed, and that it has been useful to students and those engaged in industrial chemical work.

The book opens with a very useful description of the more common manipulations in technical chemical work. An account follows of fuels and their preparation, especially coke and gaseous products; sulphur and its oxides and especially sulphuric acid are fully treated; no mention is made, however, under fuming acid of the use of platinized asbestos for making the anhydride with sulphur dioxide and air under pressure, the product being then absorbed by strongest oil of vitriol. Lunge gives a full description of this process and states that it is used in several factories in Europe.

Following the soda industries, concerning which the author states that the Leblanc process which was first used a century ago "still produces about half of the world's supply," the chlorine industry is amply treated; after describing the two most common methods in practice with and without the use of manganese binoxide, he describes some of the electrolytic processes for producing chlorine and caustic soda, and concludes with the opinion that they are not yet so perfected as to lead manufacturers to invest in them.

For getting the ammonia from the gas liquors Feldman's apparatus is the one most used in this country; a part of the heat for the distillation of the ammonia is produced by the combination of the base with sulphuric acid to produce the sulphate; some of the ammonia used in this country is made by the dry distillation of waste animal matter from slaughter houses and tanneries.

Under fertilizers the statement that the majority of the artificial fertilizers contain only one or two of the three plant foods that go to make a complete mixture is hardly correct; an examination of the lists of analyses made at the various experiment stations shows that by far the larger number of these mixtures contain all three of the plant foods, nitrogen, potash and phosphoric acid. In most of the experiment stations the same value is allowed for the reverted as for the water-soluble phosphate; therefore the other statements that the phosphate fertilizer is usually valued according to its percentage of water-soluble phosphate, and that reversion of the soluble to the reverted condition is a serious matter for the manufacturer and the buyer are not quite right.

The preparation of lime, cements, ceramics, pigments and a number of chemicals in use on a large scale occupies a large part of the book, as does also the treatment of the textile industries. Under illuminating gas the so-called water gas, consisting largely of carbon monoxid and the hydrocarbons necessary for the luminosity, is mentioned as having replaced coal gas very extensively, over which it has many advantages, such as greater illuminating power, a smaller plant, and less labor; the one serious objection due to its high percentage of carbon monoxide, and consequent exceedingly poisonous character, cannot be avoided.

Under fermentations Hansen's eight typical species of yeasts are briefly described, and the advantages gained in using only pure yeasts, especially in the manufacture of beer, are particularly emphasized. The process of pneumatic malting is claimed to insure a good product in any season; a constant stream of moist air, kept at a temperature of 14° C., passes through the grain in a wire gauze drum.

Under textile industries the several kinds of fiber used, and the processes of bleaching and mordanting are described; a long list of coloring matters is given, and the methods of dyeing and textile printing are very fully explained.

An interesting account of the manufacture of wood pulp paper is given under the paper industries. Explosives, and especially those with nitroglycerine as a basis, receive special treatment.

It would take too much space to enumerate all the industries described. All the important manufactures in which chemistry is concerned are included in the list. The author confines himself to methods and processes that are in actual use or in which there is some prospect of usefulness, and practically no space is wasted in the discussion of unsatisfactory processes. The print is clear, the illustrations though plain are well defined, and the descriptions are as complete as could be expected in consideration of the number of topics treated.

G. C. CALDWELL.

Central Station Electricity Supply. GAY AND YEAMAN. London, Whittaker & Co., New York.

This book is primarily written for those connected with the design and operation of central stations, and it furnishes an excellent basis for a general knowledge of these subjects. The machines and other apparatus described are mainly of English manufacture, but the sections which discuss engineering practice show this to be similar to that of the United States in many particulars. The work should be appreciated by all engineers who have any connection with the supplying of electrical energy.

The authors of the work are practical men in charge of electric lighting interests, and the text gives the essentials of this particular branch of engineering. The title of the work would have been more complete had it included the statement that the electricity supply was for electric lighting purposes. The hints and suggestions as to the best modes of operating the several parts of a station equipment for best efficiency are very helpful. The authors have wisely adopted the practice of summarizing the data in regard to particular pieces of apparatus so that their various features may be seen at a glance. In their desire to make the work complete they have occasionally included descriptions of commercially unsuccessful apparatus in such a manner that an inexperienced reader might have difficulty in distinguishing it from that in common use. As a rule, however, they discriminate between the really useful devices and those which merely look well upon paper.

The plan of the work is systematic and progressive. After a brief summary of the legal and business history of electric lighting and a description of the experiments leading up to its commercial application, the location and general arrangement of the building are discussed. The steam plant and the electrical equipment are given detailed treatment in a manner which shows intimate knowledge of the requirements of the problem. The book brings out clearly the extent to which the accumulator or storage battery forms a part of the modern central station and thus shows that this useful device for increasing station economy is well appreciated in England. In connection with the steam and electrical plants all of the modern labor-saving appliances are described, and while no distinct recommendations as to the choice of particular pieces of apparatus are made, the advantages and disadvantages of each are clearly set forth. With these data the engineer can make an intelligent selection for a particular case. Some attention is also given to the distribution of the electrical energy after its generation and a number of appendices give practical matter which does not properly belong to the body of the work. The book, which forms one of the well-known and excellent "Specialist's Series," may be read with profit by both engineer and student.

HENRY H. NORRIS.

Text-book on Physics. Sound. By POYNTING AND THOMSON. London, Chas. Griffith & Co., 1899. Pp. x + 163.

This, which was announced as the second of a series of four volumes constituting the text-book of Professors Poynting and J. J. Thomson, is the first to appear. The first volume of the series, on *the properties of matter*, is promised shortly. The remaining volumes are to treat of *heat*, *magnetism*, *electricity* and *light*.

There is greater need, perhaps, at the present moment of a concise treatise on the subject of *sound*, not too difficult for the beginner and at the same time sufficiently rigorous in its treatment, than in any of the other parts of physics. This the authors have succeeded in giving us. The volume contains just enough of the physiological and the musical elements of the subject to relieve it from the dryness and severity almost unavoidable in treatises on wave motion, and at the same time it deals at sufficient length with the analysis of vibrations to give the student a definite idea of the manner in which curves are built up by the superposition of harmonics and of the application of Fourier's theorem to the determination of their components.

That the work in spite of its brevity (163 pages), is not without a certain completeness will appear from the following enumeration of the points covered in discussing the experimental determinations of the velocity of sound. We find in the few pages devoted to this topic a statement of the corrections for moisture and temperature, a brief but entirely definite and intelligible account of Derham's experiments on the effect of wind, a discussion of the method of reciprocal observations, in which it is shown that the velocity is given with a sufficient degree of approximation by dividing the distance between the stations by the mean of the observed intervals. Then follows a statement of the results of the French Academy's observations in 1738, of the experiments of 1822, of the long-distance determinations made by Moll and Van Beck in 1823 near Utrecht, of the Tyrolese and Swiss experiments at different levels, of the measurements of the velocity of sound in the Arctic regions, of Stone's determinations at Capetown in 1871, and of Regnault's work in which sound waves were made to record their arrival electrically. The section closes with an account of Jacques's experiment on the influence of intensity on the velocity of sound, a description of the extraordinary observations made upon the sound waves from the Krakatoa eruption, and a brief statement concerning the velocity of sound in water and in isotropic solids.

All of this material is given in the space of eight pages. The reader who desires simply the final data will find what he needs, while he who wishes to read the full accounts of these various experiments, will obtain all the necessary references in the footnotes. We cite this as an example of the manner in which Professors Poynting and Thomson have worked out the physics of their subject. It is to be hoped that the remaining volumes of the series will exhibit the same remarkable combination of clearness, completeness and consciseness, the same good judgment in the selection of material and the same accuracy of statement.

We note the substitution, usual in English text-books, of *Döppler* for *Doppler*, and the not uncommon omission of the final s in *Lissajous*. Where the name of the latter observer is used in the possessive case, only the most strenuous effort on the part of author and proofreader will prevent printers from placing the apostrophe before the finals. E. L. N.

Magnetic Induction in Iron and Other Metals. By J. A. Ewing. Third edition revised. Pp. xviii + 393. New York, D. Van Nostrand Co., 1900.

This is the third edition of Prof. Ewing's well-known treatise. It contains in addition to the subject matter of former editions a new chapter on practical magnetic testing, in which the author describes the permeability bridges of Eickemeyer, Kennelly, and Holden, the yoke apparatus of Koepel and Kath, Brüger's apparatus for testing the induction in the air gap by means of the bismuth coil, and du Bois' magnetic balance. He also gives a brief account of some of his own more recent apparatus; such as the Ewing magnetic balance, and the Ewing hysteresis tester. With the exception of the very brief reference to the use of bismuth in magnetic measurements just alluded to, no account is given of the very important and interesting researches of Lenard and others upon the properties of this metal in the magnetic field. No mention is made of the very beautiful method recently developed by Braun in which the deflection of the kathode rays is utilized for the studying of alternating magnetic fields; nor of the attempts made by Maurain and by Wien to separate losses of energy due to true hysteresis from those resulting from the action of Foucault currents. Indeed, with the exception of the description of the practical apparatus mentioned above, the present edition is essentially a reprint of those which have gone before, with a few additional references in the footnotes to investigations which have appeared since the date of the previous edition, and a few new articles such as those describing the effects of temperature as determined by Dr. Morris, p. 190, and Partridge's study on the aging of iron by prolonged exposure to a moderate temperature, p. 193.

E. L. N.

Oswald's Klassiker der Exakten Wissenschaften, No. 109. Pp. 121. Leipzig, Wilhelm Englemann, 1899.

This number of Ostwald's series contains a translation of Felici's three memoirs on the mathematical theory of electro-dynamic induction by Dr. Dessau of Bologna. These papers, which appeared in the original in the annals of the Tuscan University, Pisa, 1854 and 1855, and are in that form inaccessible to many readers, are of no little importance in the development of our knowledge of induction. Felici was the first to test experimentally the laws of voltaic induction, and throughout his work experiment went hand in hand with theory. He made it his object not only to determine as far as possible the causes of induction, but likewise to give an exact mathematical formulation of the laws thus experimentally established.

In translating these important memoirs into German, and thus bringing them conveniently to the hand of physicists in general, Dr. Dessau has placed that portion of the scientific world which interests itself in mathematical theory under great obligations.

At the end of the volume, there is a brief but appreciative sketch of Felicic's life, and a series of useful notes by the editor. E. L. N.

The Theory of Electrolytic Dissociation and Some of its Applications. By HARRY C. JONES. Pp. vi + 289. New York, The Macmillan Co., 1900.

After an introductory chapter on "The Earlier Physical Chemistry" (pp. 1–70), the remaining three chapters of this volume are devoted to "The Origin of the Theory of Electrolytic Dissociation" (pp. 71–103), "Evidence Bearing upon the Theory of Electrolytic Dissociation" (pp. 104–170), and "Some of the Applications of the Theory of Electrolytic Dissociation." The contents of the book are fairly indicated by these headings.

The first chapter is quite in place, though, in the reviewer's opinion, it might with advantage have been somewhat abridged as regards numerical data. The second and third chapters give an elementary account of the subjects treated. They do not show any depth of research and might have been computed almost entirely by the mere aid of Ostwald's "Lehrbuch." Nevertheless, they are remarkably easy reading, and serve their purpose, which is more or less qualitative, very well.

The last chapter is less satisfactory. Even in an elementary book, the author should be careful about logical accuracy and be sure that, however much or little he may *say*, he *knows* everything of importance that has been said on the subject. The reviewer's impression from this chapter is that the author has not worked sufficiently on the original memoirs, and has not so absorbed the subject as to be able to present it in a clear and complete form which is his own and not Ostwald's or Nernst's. There is sometimes a lack of definiteness in the statement of problems, and insufficient stress on the more difficult points which are sure to occur to the keen student.

A more unpleasant defect, to the reviewer's taste at least, lies in the fact that the English of the book, though perfectly clear, is often faulty from a grammatical point of view. It may be useless to expect a proper use of shall and will, or of should and would, and it may be carping to object to split infinitives; but we wish that the author had paid more attention to his moods and tenses in conditional sentences.

But after the disagreeable task of finding fault, it is pleasanter to return to the good points. The student will not be injured seriously by a few slips; for if the subject is of any importance to him, he will certainly go on, and then upon working it through for himself he will easily correct what few false impressions he has received. Dr. Jones's book will hardly be indispensable to anybody, but it forms an easy and agreeable introduction to the study of Electrolytic Dissociation, and we think it will prove useful to many chemists who are beginning the subject.

EDGAR BUCKINGHAM.

THOMAS PRESTON

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THE

PHYSICAL REVIEW.

THE SURFACE TENSION OF WATER ABOVE 100° C.

By Chas. T. Knipp.

THAT the surface tension diminishes more or less regularly with increase of temperature and ultimately vanishes when the critical temperature is reached has been fairly well established by direct experiment in the case of a large number of liquids. The means most available when the pressure is above atmospheric is the familiar capillary tube method. In fact the experimental part of the problem, by this method, presents no difficulties other than those attending high temperature and pressure work. Investigations on water at high temperatures, however, are limited, and this is due, no doubt, to the difficulty of observing the phenomena at these temperatures because of the water vapor attacking the glass.

Frankenheim[1] was the first to study the effect of temperature upon the surface film. He found that the capillary ascent diminishes more rapidly with rise of temperature than the density diminishes. For water he found that if the coefficient of capillarity at 0 degrees C. is 1, the variation with temperature between 0 and 100° C. may be expressed with fair accuracy by the equation,

$$l = 1 - .00191\, t$$

or

$$\gamma = \gamma_0 (1 - .00191\, t)$$

where γ_0 represents the surface tension at 0° C. He was also the first to notice that at certain high temperatures the surface tension ceased, and that this temperature was different for different liquids. Brunner[1] in 1847 and Wolf[2] in 1857 found a proportionality between capillary ascent and temperature, but their work was not very accurate. They observed, though, that the proportionality did not always hold. Schiff[3] in 1884 was the next to take up the subject, but his work was limited as the observed heights at only two temperatures, and these were below the boiling point of the liquid. Two years later Eötvos[4] boldly attacked the problem experimentally. He tried 106 different substances; some organic and some inorganic, and also a few liquefied gases. He was not, however, able to go above 210° C. in the case of water because of the water vapor attacking the glass. In 1893 Ramsay and Shields[5] made a very exhaustive study of a number of liquids up to their respctive critical temperatures. For the reason before stated they were not able to make observations with water. The same year E. C. de Vries[6] worked with ether. Its low critical temperature (193°.5 C.) made it possible to use wide barometer tubing for his containing tube. The tube was 8.3 mm. in diameter. The capillary tube was placed in the center and the whole connected with the pressure apparatus and exhausted of air. As a means of heating he used the chemist's method, *i. e.*, heated various liquids whose boiling points differ. His range of temperature was from — 102 to 160° C. The variations through small ranges of temperature have been investigated by P. Volkmann,[7] H. Sentis,[8] and V. Monti.[9] J. Verschafflet[10] worked mostly with carbonic acid. The capillary tube method was used. The containing vessel was connected to the pressure apparatus by steel capillary tubes, and the heating was effected by flowing water. Pellat[11] announced that the surface tension is a linear function of the absolute temperature.

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[1] Jour. Prakt. Chem., vol. 23, p. 401.

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[1] Pogg. Ann., vol. 70, p. 514.

[2] Ann. Chim. Phys., 3, vol. 49, p. 230.

[3] Annalen, vol. 223, p. 47.

[4] Annalen, vol. 263, p. 448.

[5] Phil. Trans. Roy. Soc., vol. 184 A., pp. 647–673, 1893.

[6] Arch. Neerland, No. 28, 1894–'95.

[7] Annalen, vol. 56, p. 457.

[8] Jour. de Phys., vol. 6, pp. 183–187, 1897.

[9] Nuvo. Cim., 4, 5, pp. 5–27, 186–203, 1897.

[10] Comm. Leiden, No. 28.

[11] C. R., vol. 118, p. 1193, 1894.

From the foregoing it appears that but little has been done in the case of water. Eötvos was able to reach 210°C., which falls short about 150 degrees of the critical temperature. To extend the investigation over this region is the object of the present research.

APPARATUS.

As before stated, water vapor attacks glass at high temperatures and pressures, and renders it opaque and unsafe. The exclusion of glass from the apparatus at once multiplies the difficulties, for complete cognizance of what is going on within must be conveyed to the observer by other than direct vision methods. In the present investigation an attempt was made to measure the surface tension indirectly by lifting a vane partly submerged in the liquid. The drag that is experienced when a vane is thus lifted is, for the most part, due to the surface film. That due to viscosity is but a small part, and in the case of water is negligible, as will be shown later. To lift the vane the sucking action of a coil carrying a current was employed. A preliminary test was made before the apparatus proper was constructed.

It is evident that the success of an investigation like this, where it is proposed to determine the surface tension by lifting a vane in an enclosed vessel reached only by electrical circuits and both vessel and circuits being subjected to a wide range of temperature and pressure, depends in a large measure, if not entirely, on the delicacy yet strength of the apparatus. For this reason considerable time will be given to a description of the same.

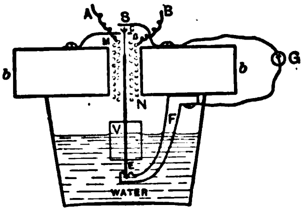

Fig. 1.

A coil *mn* (Fig. 1) of No. 16 copper wire, consisting of two layers of about ten turns each, was supported in a wooden block *bb*. The inside diameter of the coil was 5 mm. The core was made of iron and copper wire, the part *de* of No. 21 iron wire, and *dc* of No. 24 copper wire. This core was supported at the center of the coil by the conical cup at *e* and the guide *g* at the upper end. To avoid friction as much as possible, the hole through *g* was made about double the diameter of the copper wire, and a conical washer stamped out of copper foil was soldered to cd. The height of the guide g was adjusted so that the core ce occupied the axial position of the coil. Thus when the core was lifted it would always drop back to its former central position. A stop s was placed above the top of the core. A mica vane v, 1 x 1.5 cm., was fastened to the core just below the coil. The apparatus was placed over a beaker, the water in which came about half way up the vane, as shown in the figure.

The apparatus thus set up, a current was passed through the coil mn, which, by adjusting the resistance in the circuit, was increased until the vane just lifted. An ammeter in the circuit gave the value of the current. Before taking each reading the vane was wetted by lowering slightly then raising. The following values of the current required were obtained:

To lift vane in water at 19° C.,	5.8 amp.
	5.9 "
Average,	5.85 amp.
To lift vane in air,	4.35 amp.
	4.4 "
Average,	4.37 amp.

The range was 1.48 amperes.

These results were considered quite satisfactory, the range was amply sufficient to arrange for accurate readings, and these readings were, to within the errors of observation, consistent among themselves. However, the above alone hardly guaranteed that the method would be a success. Working in the open air with the apparatus in full view was no assurance that it would work enclosed in a steel vessel and under varying temperature and pressure conditions.

To this end the preliminary apparatus (Fig. 1) was rebuilt, made smaller and more compact so as to work in a space whose walls could be made strong enough to withstand from 200 to 300 atmospheres pressure without making the vessel unduly heavy and cumbersome. The coil was made of three layers of eleven turns each of the same wire as before. The inside diameter was 4 mm. and length 1.3 cm. The length of the core was 3.5 cm. Instead of observing by the eye when the vane was lifted, a galvanometer was used as a signal; the circuit was *GfecsG*. The current required to lift the vane in water and air was 4.2 and 2.3 amperes, respectively, giving a range of 1.9 amperes.

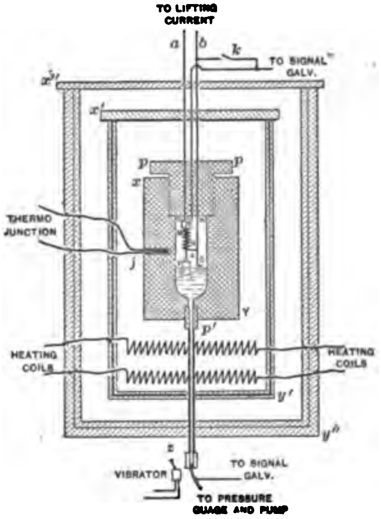

Fig. 2.

The above preliminary tests being sufficiently satisfactory, the apparatus was constructed in its final form. It consisted of a containing vessel in which the water under investigation was placed, and which contained the coil and vane for making surface tension measurements. This vessel, *xy* (Fig. 2), was made of a piece of car axle. The cavity was 7 cm. deep and 3.2 cm. in diameter. To the lower end a small plug, *p'*, made connection with a steel bicycle tube, which in turn connected by a union to the copper capillary tube leading to the pressure gauge and pump. The upper end of the containing vessel was closed by a large plug, *pp*, which screwed home on an annealed copper washer placed on the shoulder *ww*.

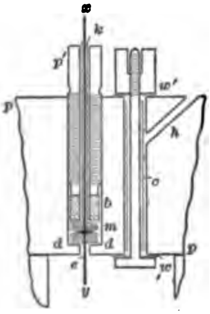

Fig. 3.

Three insulated circuits were run through this plug. This was a difficult task. Two forms of insulated circuits were used. There seems but little choice between them. One may be styled the bolt, and the other the mica disc method. Prof. G. S. Moler, of the department, suggested the former. As will be seen from figure 3, it consisted of a bolt extending through the plug *pp*. A mica collar *c* insulated the bolt from the iron plug. Mica washers, *w* and *w'*, were placed under the head and nut. It was found later that platinum washers placed on either side of the mica washer *w* aided greatly in preventing vapor leaks. The platinum is soft and readily conforms to any irregularity of the surface as the nut is tightened. A diagonal hole *h* was drilled to act as a vent. This prevented any accumulation of moisture and for this reason this form of circuit remained at all times well insulated, unless moisture condensed on the inner surface of the containing vessel.

The second form, the mica disc method, was more difficult to construct. A half-inch hole was drilled nearly through the plug. The lower end *dd* was reamed out square, and a small hole *e* was drilled on through. Mica discs were stamped out to fit the half-inch hole. A flat head or disc was hammered on a No. 16 copper wire. Two heads thus prepared were soldered together so as to form a straight wire *xy*, the heads forming the disc *m* shown in the figure. A hole that tightly fitted the wire was drilled through the pile of mica discs. Half of these discs were then slipped on one side of the disc *m* and half on the other side. Care was taken to reject all discs that contained radial cracks or seams. The upper end of the half inch hole was threaded out to receive a long finely threaded plug p'. A hole through the axis of this plug and lined with mica *k* received the wire *xy*. Between the plug p' and the mica discs was placed a thick brass washer *b*, the expansion of which aided in compressing the discs while the vessel was being heated. After each heating the plug p' could readily be turned; this, however, was tightened before again heating. This form of circuit has the disadvantage of the pressure within the vessel tending to reduce the pressure on the shoulder *dd* on which the mica discs rest, while in the former the pressure within lessens the chances for a leak. However, the pressure in the opposite direction, due to the plug, was so great in comparison as to cause no serious trouble, yet the circuit at times leaked. One time the leak was traced to one of the discs having a radial crack, which, of course, no amount of pressure would close. On the other hand the latter form of insulated circuit has a decided advantage in that it can be tightened up from time to time and not interfere with the alignment of the apparatus fastened to the wire *y* on the inside of the vessel, while the bolt invariably turned when the nut was tightened which was fatal to the mechanism fastened to it within, and which also ground out the surface of the mica washer, hence whenever the bolt circuit leaked (vapor) it caused considerable trouble, but fortunately it leaked less often than the mica disc circuit.

With these precautions regarding the insulated circuits and with the annealed copper washers under the large plug *pp* and small plug *p'*, the vessel (Fig. 2) was enabled to withstand 180 to 200 atmospheres pressure without serious leaks. Later, in several runs, the pressure was pushed up to 240 atms. before the apparatus showed any signs of leaking.

A diagrammatical sketch of the containing vessel *xy*, of the oven, and of the various connections and electrical circuits is given in figure 2. The oven consisted of a sheet iron box, *x'y'*, 8 x 8 x 12 inches. It was lined both inside and outside with one-fourth inch of asbestos. To still further guard against radiation the oven was surrounded with an asbestos lined wooden jacket *x''y''* leaving an air space of about two inches between the two. Windows of heavy glass were placed in the opposite walls of the oven and jacket. The internal apparatus for determining the surface tension is also shown. The lifting coil *mn* was made of the same wire (No. 16) as in the preliminary tests and consisted of two layers of eleven and twelve turns each. The wire was stripped of its insulation and wrapped, in making, on a form so that in the completed coil there was at least .5 mm. of space between the consecutive turns and between the two layers. The ends of the coil connected rigidly to two of the insulated circuits (*a* and *b*) that passed through the large plug *pp*. This formed the lifting circuit.

The lower end of the remaining insulated circuit through the plug terminated in the stop *s* with which the core *ce* made contact when lifted. By closing a key at *k* the water level was adjusted by means of the terminal *o*. These two circuits led to the same galvanometer and were styled the signal circuits, for they acted merely as signals. Obviously, then, a small electrical leak caused but little trouble. Neither was there any appreciable error introduced in case the insulation of the circuit conducting the lifting current was not perfect, for the resistance of the coil was negligible in comparison to that of the insulation.

In the final form the core *ce*, the vane *v*, and the guide *f* were considerably modified (see Fig. 1, then Figs. 4a and 4b). In the preliminary tests it was found that it would not do to have the core extend below the vane. The core became more or less contaminated, due to oxidization of the iron, because of the difficulty of getting an absolutely perfect platinum plate, and thus the drag due to the surface film was altered. Instead of extending the core some distance below the coil with the vane fastened to the side, it was allowed to extend only a few millimeters below and the vane was suspended from its lower end as shown in figure 4b. A small symmetrical globule was formed on the end of a piece of No. 24 copper wire by heating in a blast flame. This globule was turned down in a jeweler's lathe to the shape *h* with a small point *a* projecting (Fig. 4a). To the point *a* was fused a small platinum wire tip *c* which made contact with the stop *s*. In the preliminary tests the copper and iron wires forming the core were joined by soldering; this, however, would not do since ordinary solder melts at 190 or 200 degrees centigrade. The process was as follows: The iron wire was carefully copper plated. The two were then joined temporarily with solder. The joint was polished and plated with copper for a few minutes, the remainder of the core being protected by beeswax. It was then taken from the bath and heated beyond the melting point of solder which by expansion was forced out through the plate in several places and collected in small globules on the surface. These were brushed away, the surface of the copper polished, and the joint again plated with a heavy coat. The core was now taken from the bath, polished over its entire length and plated with platinum. The vane *v* (Fig. 4a) was of platinum foil rolled down until it was as thin as could well be handled. It was fastened to the lower end of the core by a thin platinum wire as shown at *c*. The average density of the core and vane was 12.24, and its weight was 145 mg. The approximate proportionate amount of copper, iron and platinum in the core and vane was, Cu 38 mg., Fe 57 mg., and Pt 50 mg. The density of the core from these by calculating was 12.2.

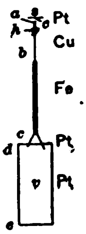

Fig. 4a.

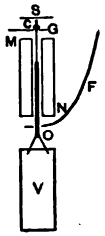

Fig. 4b.

Figure 4b shows the guide *g*, the stop *s*, and the form and position of the lower guide *f*. In the final arrangement the guide *f* was placed above the vane as before stated. It consisted of platinum wire, the upper end was fastened to the lower face of the large plug, and the lower end terminated in a loop through which the core passed. In taking readings care was taken to have the coil *mn* in a vertical position so that the core hung free of *f* from its upper support *g* (Fig. 4b). It is understood from the drawings and from what has been said that the coil, core, in fact all of the internal mechanism was mounted on the lower face of the large plug. This, of course, was necessary in order to be able to remove the plug from its seat in the containing vessel, and, if desired, to make observations at atmospheric pressure and ordinary temperatures.

The interior of the containing vessel and the apparatus within was plated with platinum on copper. This included the annealed copper washers under the two plugs. A platinum surface resists best the action of the water vapor and does not oxidize. The plating was done according to approved formulæ.[1] To place a continuous deposit on the interior of the containing vessel and the steel tube immediately below required the greatest care and patience. The tube plated was 22 cm. long by 3 mm. internal diameter. Several attempts were made before a successful deposit was obtained. The method shown in figure 5 was quite successful. After thoroughly polishing the interior of the tube it was mounted in a vertical position. The upper end connected to the vessel *mn* made of a large flat cork fitted into an argand lamp chimney. A glass tube drawn out and thrust through a cork was fitted into the lower end of the steel tube *b*. This tube was connected to a funnel *x* whose elevation was a foot or more above the vessel *mn*. The electrical connections for plating are shown in the figure. A copper wire *ab* (in the case of copper plating) served as the anode and was stretched taut through the center of the tube. A small

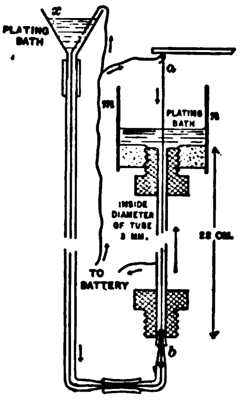

Fig. 5.

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[1] Langbein's Treatise on Electro-Plating, 1898.

globule of copper at *b* held the lower end, care being taken not to obstruct the passage. The plating bath was placed in the funnel. The bath thus flowed through the steel tube from the bottom up and collected in the vessel *mn*. The speed of the flowing liquid could be varied at will by raising or lowering the funnel. By this method the gases that collect on the anode and object are rapidly carried to the surface of the liquid as also are any precipitates that may be formed (after the bath has been used for some time) by the action of the current. In fact when the bath was allowed to flow *down* through the tube the interior surface at the lower end of the tube was covered with dark blotches indicating that the circuit was short-circuited by the precipitates collecting. The method just described worked equally well in depositing the platinum plate.

In conducting the experiments four electrical circuits were employed, viz., the lifting, the signal, the temperature, and the heating circuits. The current for lifting the vane was supplied by 15 chloride accumulators. In series with the cells and the lifting coil (Fig. 6) was a 3-ampere fuse, an ammeter *A* reading to 15 amperes, a standard .03-ohm coil of manganin wire, some tin resistance, and a sliding German silver resistance for accurate current adjustment. For the determination of the current flowing in the circuit the fall of potential through the .03-ohm coil was measured by looping around it a Direct Reading Laboratory Standard Weston voltmeter. This instrument read to 150 millivolts. The ammeter served to indicate roughly the amount of current flowing, thus acting as a protection to the millivolt meter. Under these conditions the maximum current was about 5 amperes. To still further guard against accident a fuse was included in the circuit.

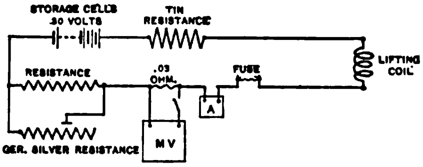

Fig. 6.

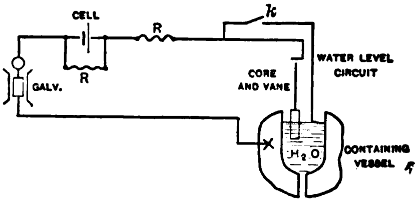

Fig. 7.

The signal circuit consisted of two parts—one to adjust the water level and the other to know when the vane was lifted (Fig. 7). The circuit comprised a cell properly shunted, a high resistance, and a galvanometer. The water level was adjusted by closing the key *k*.

A copper-platinum thermo-junction placed in a small hole drilled into the side of the containing vessel (Fig. 2, j) was used in determining the temperature. The depth of the hole was 2.6 cm.; the distance of the junction from the interior was 7 mm. Figure 8 shows the electrical connections. The galvanometer used was of the D'Arsonval type. In order to guard against any change in the sensitiveness of the galvanometer its deflections were compared from time to time with certain standard deflections obtained by a Clark cell. This check circuit is shown in the figure.

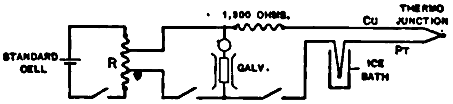

Fig. 8.

Figure 9 shows the electrical connections for heating the oven. The heating coils consisted of iron wire, the degree of heating being readily adjusted by tin resistance frames. Shunted around a portion of the tin resistance was a circuit that contained a vibrator that was an important factor in making observations. Indeed the accuracy of all the readings was due to its use. The vibrator was simply an ordinary door-bell with the bell removed. The hammer was allowed to play on the steel tube that projected beneath the oven. It is shown at *z* in figure 2. A tremor was sent through the containing vessel and all tendency of the core sticking was removed. In fact it was impossible to get consistent readings by means of the signal circuit without this vibrator.

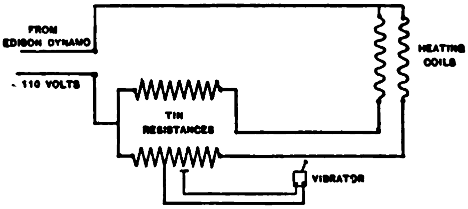

Fig. 9.

Calibrations.

The thermo-junction was calibrated by comparison with a standard 600° C. mercurial thermometer whose zero, steam, and sulphur points were accurately known. The calibration curve that was first constructed was found to be considerably in error because no correction was made for the stem of the thermometer, or what amounts to the same thing, the stem was not kept at the same temperature as the bulb. A new curve was constructed in which every precaution was taken to avoid the difficulty. Readings for the upper end were made first. The Cu-Pt junction was tied to the bulb of the thermometer and both placed on the interior of the large mass of iron that composed the containing vessel (Fig. 10). A heavy asbestos collar *mn* was placed around that part of the stem that was outside of the oven yet inside of the outer jacket. This collar did not fit tightly on the stem, and allowed the heat from the interior of the oven to circulate up through it. The upper end, however, was tightly packed with asbestos. By this arrangement the lowest reading on the thermometer stem was the 320 degree mark. The heating was conducted very slowly. The temperature and galvanometer deflections were read simultaneously. The galvanometer did not change its sensitiveness, which simplified matters. Readings were also taken while cooling. Care was taken to adjust the rate of cooling to the same as that of heating. The points obtained locate the upper end of the calibration curve (Fig. 11). The lag, as shown by the curve, was negligible.

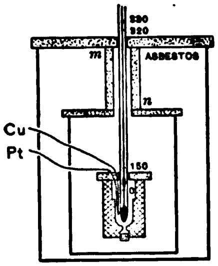

Fig. 10.

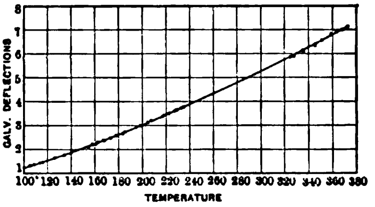

Fig. 11.

The lower end of the curve was next established. To do this the asbestos collar was removed. The containing vessel in the oven and the oven itself were raised so as to allow as much of the stem as possible to protrude. The lowest readable mark on the stem was 150 degrees. A similar set of readings was taken as before. It was thought best not to go higher than 225 degrees in order to avoid the error due to the stem. For temperatures below 150 degrees another thermometer was used whose boiling point was carefully tested. The calibration curve was drawn through these three sets of readings.

The Ducretet pressure gauge was calibrated through the range to be used, *i. e.*, to 225 atmospheres. It was rather suspected that after years of use it would give values too large due to weakening. A manometer tube such as described by Cailletet and Colardeau[1] was made and filled with a known volume of dry air. The volume of the cistern was 42.89 cu. cm., while that of the manometer tube was 668.7 cu. mm. The latter was made of a heavy-walled glass tube of about 1 mm. internal diameter. The upper end of this tube was drawn out into a very fine capillary tube. This was closed after the tube was carefully calibrated and filled with dry air. The error due to disturbing the calibration by sealing the capillary tube was insignificant. The calibration of the gauge was made upon the assumption that Boyle's law is approximately true for air. The piezometer containing the manometer cistern was connected to a high pressure pump. The temperature of the room was 20° C. To guard against a possible error due to the heat generated by the more or less sudden compression of the gas the pressure gauge was read at 2 minute intervals for each elevation until it became constant. The correction curve is shown in figure 12. The lowest reading obtainable on the manometer was that corresponding to a pressure of 70 atmospheres.

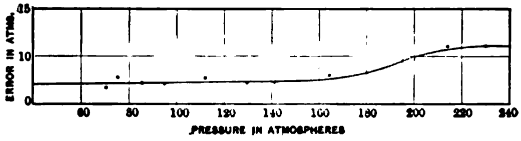

Fig. 12.

I. Pressure-temperature curve of saturated water vapor, and the critical constants.

Before beginning the measurements on surface tension, the apparatus, in a slightly different form, was used in making observations on the vapor tension of water. The object was to reestablish the critical temperature and pressure.

Nadejdine's method enabled him to determine the critical temperature and density, but not the critical pressure. His method was based on the supposition that at the critical temperature the densities of the vapor and liquid are the same. He balanced the tube which was to contain the liquid on a knife edge. Then it was partly filled with the liquid. The tube was no longer in equilibrium. Heat was applied. The balance would be restored under three conditions; when the liquid is evaporated before the critical temperature is reached, when the liquid expands and fills the vessel, and when the critical temperature is reached before either the liquid expands or is all evaporated. The critical temperature was taken as the highest temperature at which equilibrium was restored, and this was obtained by placing different initial quantities of the liquid in the vessel. The initial volume of the liquid and the volume of the vessel being known the critical density was deduced. Battelli placed the water in a steel tube connected to an open air manometer of reduced height. He observed both the temperature and pressure. Cailletet and Colardeau made, perhaps, the most careful investigation of the constants for water. They also used a steel tube connected to a carefully calibrated hydrogen manometer. Strauss in 1882 determined values for the constants by calculation.

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[1] Cailletet and Colardeau C. R., tom. cxii., p. 563, 1891.

The following table contains the results of different investigators:

TABLE I.

Critical constants.

T	p	δ	Investigator.
358°.1	——	.429	Nadejdine.[1]
364.3	194.6	——	Battelli.[2]
365.	200.5	——	Cailletet and Colardeau.[3]
370.	195.5	.207	Strauss.[4]
365–368	——	——	F. W. D. Dery.[5]

Instead of the large plug *pp* (Fig. 2) another was used having only one insulated circuit running through it. The original intention was to use this circuit in making measurements on the conductivity of different liquids at high temperatures. However, observations along this line have not as yet been attempted as it was desirable

[1] Biebl. 9., p. 721, 1885.
[2] Mem. d. R. Acc. d. Sc., Torino (2) 41, 1890.
[3] C. R., tom. cxii., p. 563, 1891.
[4] J. d. russ. phys.-chem. Ges., 14, 1882.
[5] Bull. Akad. roy. Belg. 29, p. 277, 1895.

not to introduce other liquids into the vessel until the observations on water were completed. The distilled water used was forced through the pump. At first thought this seems undesirable since the water might be more or less contaminated. If it had been possible to construct the containing vessel with its insulated circuit perfectly tight so as not to leak then the necessity of the water going through the pump could have been easily obviated, by some sort of a displacement vessel, but since the leaks were considerable such a vessel constructed within the limits of safety would not be adequate. Hence the only precaution against impurities was by thoroughly cleansing the pump.

The large plug was screwed home. The containing vessel was inverted and filled with distilled water through the lower opening. The steel tube was then firmly screwed in place and it in turn filled with water. In connecting the pump after the containing vessel was put in place care was taken to have the capillary tubes also filled with water. The thermo-junction was then inserted in its proper place, and after all openings in the oven and jackets were closed with asbestos, the heating current was turned on. The rate of heating was such that it required from 7 to 8 hours to heat the vessel from room temperature to 370° C. At this slow rate of heating the temperature lag was not a disturbing factor. The temperatures and pressures were read simultaneously at intervals of 10 to 20 degrees. When the temperature rose to about 110° C., the cock connecting to the pump was slightly opened and about half of the water in the vessel was allowed to be forced out. The first few runs were interrupted by the vessel springing a leak when the temperature rose to about 300° C., the corresponding pressure being 90 atmospheres. However, on several occasions the temperature was carried to 370°, and even to 400° C., without serious leaks. The pressures read from the gauge were corrected, so also were the temperatures whenever rendered necessary by the galvanometer changing its sensitiveness.

Figure 13 shows two tension curves superimposed. The data was taken on different days. They are nearly unique up to the critical point, but beyond that they both take slightly different directions depending on the quantity of water in the vessel before the critical point was passed. The curve below the critical temperature is independent of the quantity of water in the vessel. The branch *ab* indicates that the vessel was nearly full when the critical point was passed. The quantity of water was less in the case of branch *ac*. Where these two branches intersect is the critical point, and this, as indicated by the curve, is, for the critical temperature 359 degrees centigrade, for the corresponding pressure 205 atmospheres. The data for the curve in figure 13 is given in the following table.

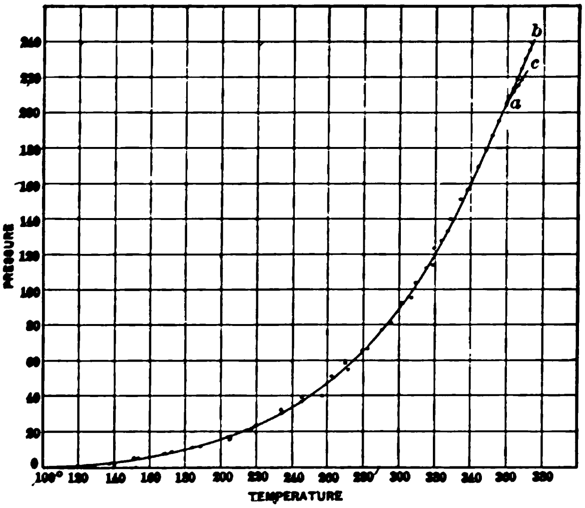


Fig. 13.

TABLE II.

Temperature.	Pressure.	Temperature.	Pressure.	Temperature.	Pressure.
180°	9.6 atms.	260	47.1 atms.	330°	138.9 atms.
190	12.4 "	270	56.0 "	340	159.8 "
200	15.5 "	280	65.9 "	350	182.2 "
210	19.0 "	290	76.8 "	355	194.0 "
220	23.0 "	300	89.0 "	359	205.0 "
230	27.8 "	310	103.2 "		
240	33.6 "	320	120.0 "		
250	39.9 "				

In another run the resulting curve was found to branch at a lower point, viz: 358°.5 C., and 204 atmospheres.

These results do not exactly agree with those that have been obtained by others, though, no doubt, they represent the critical point with a fair degree of accuracy.

II. Surface Tension.

In order to express the surface tension in absolute units with any degree of accuracy, the relation between the lifting current and the weight lifted should be linear. If the relation is not a linear one the instrument must be calibrated by liquids whose surface tensions at given temperatures are definitely known. Before attempting such a calibration the relation above mentioned was investigated experimentally.

The plug, *pp* (Fig. 2), was mounted on a suitable support, the lifting and signal circuits and also the vibrator being properly connected. The value of the current that was required to lift the system in air was observed. Known weights were added and the corresponding current noted. Table III. gives the data obtained. The resulting curve (Fig. 14) is a straight line. That the relation was linear was fortunate, as it simplified matters. This, no doubt, was due to the fact that the core scarcely moved from its axial position in the coil before the signal was given, and also that the core was saturated. The current flowing through the coil was about 5 amperes.

Table III.

Current in Mv.	Weight of system.	Weight added.	Total weight lifted.
96.	145 mgs.	0 mgs.	145 mgs.
100.5	" "	14 "	159 "
104.2	" "	28 "	173 "
113.5	" "	57 "	202 "
117.2	" "	66 "	211 "
123.4	" "	90.5 "	235.5 "
137.	" "	129. "	274. "
146.	" "	160. "	305. "

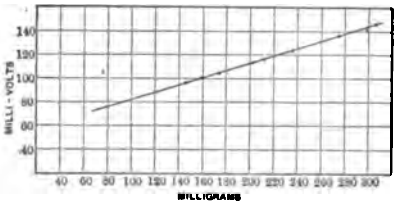
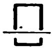
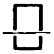
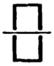
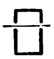
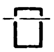


Fig. 14.

The possible effect of the depth of the vane in the liquid upon the current required to lift it was also investigated experimentally. The vane was submerged to five different depths in the water. The electrical connections were the same as before. The lifting of the vane was observed directly by the eye, and then by the signal galvanometer. In using the signal the resistance in the lifting circuit (Fig. 6) was adjusted to make the current larger than necessary. The current was then decreased and its value (by means of the millivoltmeter) was noted the moment that the core broke contact at *s* (Fig. 2), which was indicated by the deflection of the signal galvanometer falling to its zero. This method of using the signal galvanometer in determining the value of the lifting current was used in all the surface-tension readings. The observations tabulated in Table IV. show that the differences between the two methods of observing were no greater than the errors of observation, and consequently that the signal was reliable. It also shows that the difference in buoyancy, when the vane was submerged to different depths, was not noticeable; the water displacement due to the volume of the vane was exceedingly small. In the light of the above it would hardly be reasonable to expect an effect due to viscosity.

TABLE IV.

Depth of Vane.	Observed directly by eye.	Observed by signal Galvanometer.		Average.
	mv.	*mv.*	*mv.*	*mv.*
	146.	146.1	145.2	145.7
	145.4	146.2	145.8	145.8
	145.1	145.6	145.3	145.3
	145.8	145.5	146.2	145.5
	145.3	145.6	145.6	145.5
Average	145.5	145.8	145.6	145.6

It is natural to expect a more or less marked temperature effect as the iron core is heated. To test for this the apparatus was placed in the oven and the heating current turned on. The data obtained was as follows :

TABLE V.

Temperature.	Mv.	Temperature.	Mv.
116°	90.6	231	91.6
156	91.6	283.5	90.8
192.5	90.4	322.5	got no response.

Platting these values (Fig. 15) a straight horizontal line is obtained. Unfortunately the signal refused to respond at the end of last interval, *i. e.*, 322°.5, and further observations could not be taken. A poor contact was found afterwards when the apparatus was taken apart. The data, however, shows no temperature effect. It might be well to state that in the above three investigations different sized platinum vanes were used, hence the results cannot be compared. The data given in Table III. was taken with the vane as last constructed.

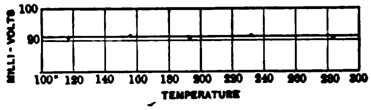


Fig. 15.

Being satisfied that no appreciable error would result from the change in the temperature of the core, from the depth in the water to which the vane was submerged, and that the relation between the current and the weight lifted was linear, the observations on surface tension were begun. The large plug *pp* (Fig. 2) was screwed down tight on its seat in the containing vessel. The cavity was filled with distilled water and connected to the pressure pump as previously described. The different electrical circuits were connected to their proper terminals. The heat was applied very slowly; to heat through the entire range of 360° requiring seven or eight hours.

When the temperature was raised sufficiently part of the water was allowed to flow out by slightly opening for a moment the stopcock connecting to the pump. It was impossible to determine the water level at *o* (Fig. 2) immediately because the walls of the vessel and the objects within were covered with a film of water. This was known to be the case by the insulation of the signal circuits in part breaking down. The lifting coil acted like a sponge and held a large mass of water which extended to the core and guides *g* and *f*.

It was found that the water would not evaporate sufficiently to make the signal circuits of any use short of 200° C. However this difficulty was overcome by allowing the water to be forced out of the containing vessel but not out of the steel tube immediately below. This caused all of the moisture in the vessel to evaporate in a very few minutes, and the two signal circuits became perfectly insulated. The water was then slowly pumped into the vessel. The circuits remained insulated for some time, when gradually the zero of the galvanometer began to shift, showing that there was a slight accumulation of moisture, however the water level was readily determined. The water level being adjusted, the key *k* (Fig. 2) was opened and the lifting current turned on. Before each reading the vane was wetted by raising then lowering the water. Temperature, pressure, and the current required to lift the same were read simultaneously.

To get consistent readings for the surface tension below 220° C. was quite difficult even when the circuits were almost perfectly insulated. The values were generally small. However, when moisture was condensed on the coil, core, etc., as indicated by the signal circuits, the readings were generally large. When the behavior of the circuits became too erratic it was taken as an indication that the coil, core, and the guides *g* and *f* were enveloped in moisture. This was always overcome by evaporating the moisture as described above, when the readings would return to their former more or less irregular small values. But as the heating was pushed farther a steady condition set in that was remarkable. Not only did the current, required to lift the vane, steadily decrease, but the two signal circuits became better insulated. Above 240° C. there was no trouble whatever of moisture condensing; however the chances for leaks were greater. The vessel leaking proved a great source of trouble. When they were slight the observations were continued by pumping in water, but more frequently the work was interrupted permanently. The high temperature of the apparatus prevented further work that day. Before attempting another run all plugs were tightened and circuits examined. By the vessel leaking a ready opportunity was afforded for taking readings of the current required to lift the system *in* the *vapor*. These values, obviously, give an indication of the density of the vapor and may be styled base line readings.

Seven runs were made for the purpose of getting surface-tension readings, though each contained a number of values taken in the water vapor. One run was made solely to obtain values of the lifting current with the system in the vapor, *i. e.*, to establish the base line. To be sure that the vapor was saturated the water was let out until the pressure began to drop. It was then pumped in and the pressure rose to a certain value and remained constant on further pumping. In order for the vane to swing free of the water the pumping was stopped soon after the pressure became constant. Since the pressure is determined by the temperature the amount of water in the vessel was immaterial. Table VI. contains data relative to the eight runs. The order is that in which they were made. The 17 readings in the fourth run were taken while the vessel was cooling down from 323° C. These readings agree very closely (Fig. 16) with the curve obtained on heating. This shows again that the lag was inappreciable. The figure shows a portion of the upper end of the curve where the greatest lag would be expected.

TABLE VI.

Number of readings.

Order of run.	In liquid.	In vapor.	Vessel began leaking at	Permanently interrupted at
1	15	3	347°	357°.8
2	14	—	324	337.
3	15	—	—	305.
4	17	—	cooled from	323.
5	9	2	334	340.5
6	23 (?)	6 (?)	—	363.8
7	7	4	—	342.
8	0	13	—	368.

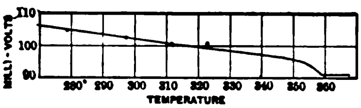


Fig. 16.

The data suggested by Table VI. was divided into groups, each group forming a new point. The points thus formed are shown in curve *a*, figure 17. The data for the readings in the vapor is represented by the curve *b*. This curve or base line is not horizontal, but drops as the density of the vapor increases. The form of the curve indicates that the density of the vapor increases very rapidly as the critical temperature is reached, and becomes constant when that point is passed.

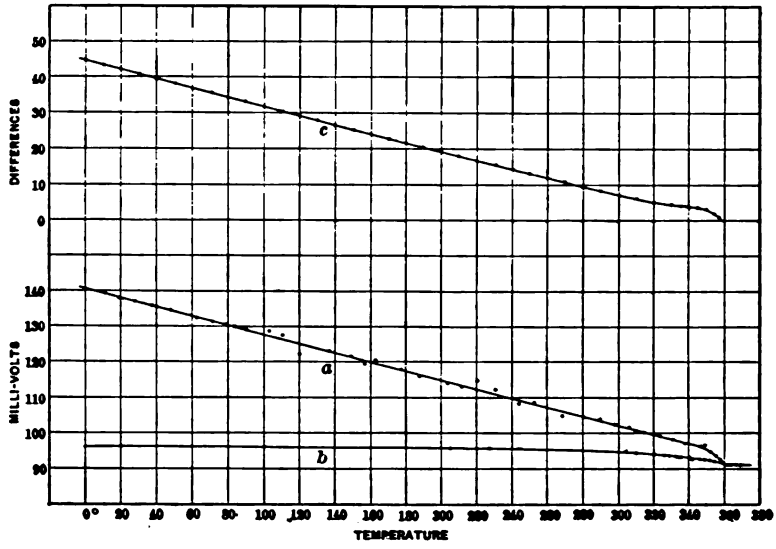


Fig. 17.

To aid in the further discussion of the surface tension curve it was extended back to 0° C. The arrangement of the apparatus for obtaining data for less than 100° C. is shown in figure 18. The water to be investigated was placed in the beaker *mn* surrounded with a large water bath that was constantly agitated. The plug *pp* with all of its electrical connections, including the vibrator, was suitably mounted above the beaker as indicated in the figure. The vane was wetted before each reading by compressing the bulb *b* and forcing water (at the temperature of the bath) from *A* into the beaker. When the

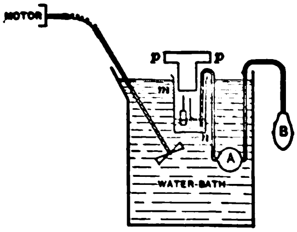


Fig. 18.

bulb was released the original water level was restored. A thermometer was placed in the bath. The data obtained is represented in curve *a* (Fig. 17), and is a straight line as other observers[1] have shown it to be. It will be noticed that the curve continues a straight line on through the boiling point and for some distance above. It was pleasing to have the two curves gotten under such different conditions join so perfectly. This seems to indicate conclusively that the apparatus was capable of giving accurate results, and that the curve above 100 degrees undoubtedly represents the true values of the surface tension.

Table VII.

Temperature.	Numbers proportional to surface tension.	Temperature.	Numbers proportional to surface tension.
0	44.2	200	19.3
10	43.2	210	18.2
20	41.7	220	17.
30	40.5	230	15.7
40	39.1	240	14.5
50	37.8	250	13.2
60	36.6	260	12.1
70	35.4	270	10.9
80	34.1	280	9.7
90	32.9	290	8.5
100	31.6	300	7.2
110	30.4	310	6.2
120	29.1	320	5.5
130	27.9	330	4.7
140	26.7	335	4.4
150	25.5	340	4.1
160	24.1	345	3.9
170	23.	350	3.4
180	21.9	352.5	3.
190	20.6	355	2.1
		357.5	1.

The current required to lift the system in air at atmospheric pressure was 96.4 mv., at 200° C., the current required to lift the system in the vapor was 96 mv., at 300° C., 95 mv., and at 360° C., 91 mv. From these values, which are taken from the base line, it is evident that the curve *a* is displaced by an amount equal to the drop in the base line. In the curve *c* (Fig. 17) temperatures are platted as abscissas and differences (between the curves *a* and *b*) as ordinates. This gives the true form of the curve, the data for which is contained in table VII.

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[1] Landolt and Börnstein, Phys.-Chem. Tables, p. 44.

The surface tension can be expressed in any system of units when once its value is known for any given temperature, *i. e.*, if we accept 74.22 dynes as its value at 20° C., then the surface tension at, say, 200° C. is $19.3 \times 1.78 = 34.35$ dynes.

From the data obtained we infer that the surface tension does not decrease gradually to zero as the critical temperature is reached, but falls rather suddenly. This may seem strange since it has been shown by other investigators[1] that the surface tension curves of such liquids as acetic acid, ethel acetate, etc., approach the axis asymtotically. That the forms of curves *a* and *b* (Fig. 17) are correct there can be but little doubt when the conditions under which the data for them was obtained is considered. It will be remembered that above 240° C. the readings were obtained with great accuracy. The different runs extending over several weeks gave for this region almost exactly the same results. On two different instances the observations were extended beyond 355° C. As was pointed out on page 148 the reading below 240° C. (*i. e.*, from 100° –240° C.) were subjected to two disturbances. The one was due to the condensing of moisture on the different parts within, and which caused the values obtained for the surface tension to be large; the other effect was noticeable when apparently the circuits were well insulated and caused the values to be small and fluctuating. It is evident that the large values in the first instance were due to the increased weight of the system. However the small values in the second instance are not so easily accounted for. It is reasonable to suppose that they were due to impurities in the water and on the vane. These impurities, however, had *no effect* on the surface tension above 240° C. as was before remarked. Therefore they must have been rendered inert or were dissolved by the heat· By the method employed in locating the points (see page 149) in curve *a*, figure 17, it is seen that these two effects have practically no tendency to change the direction of the curve in that region.

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[1] Ramsay and Shields, Phil. Trans. Roy. Soc., vol. 184 A., p. 647, 1893.

The same consistency that characterized the upper end of the curve *a* was also true of the base line *b*. The readings for this curve were obtained on five different days, and but one point was located that was not on the curve. The points on the curve *b* are actual observations. The fact that the base line became horizontal is evidence in itself that the critical temperature was passed. It would seem that the water becomes quite compressible as the critical temperature is approached, and that the critical density is greater than is usually given. The density of the saturated vapor at any temperature can be computed by aid of the curves in figures 14 and 17, when the volume of the system (*i. e.*, core and vane) is known. However, further discussion of this method of determining vapor density will be postponed. It is the intention to slightly change the apparatus and make it more suited to this phase of the investigation. By increasing the volume of the displaced vapor the readings can be made more sensitive.

SUMMARY.

The principle employed was that of lifting a platinum vane partly submerged in the water. The lifting force was the sucking action of a coil carrying a current. The coil and core were mounted on the inner face of a large plug that screwed into a steel containing vessel made of a piece of car axle. Insulated electrical circuits were run through the plug which enabled observations on surface tension to be made. The containing vessel was in communication with a pump and a pressure gauge. The interior of the vessel and the apparatus within was platinum plated on copper. A copper-platinum thermo-junction was used in determining the temperature. Both the gauge and the thermo-junction were carefully calibrated. The heating was by the electric current. The position of the junction in the large mass of iron that composed the containing vessel made the temperature lag inappreciable. It was placed in a small hole drilled to within 7 mm. of the inner cavity.

The measurements consisted of two parts, viz., the determination of the critical constants for water and the determination of the surface tension from 100° C. to the critical temperature. In determining the critical constants another plug was used instead of the one containing the insulated circuits and surface tension apparatus. Temperatures and pressures were read simultaneously. The critical temperature obtained was 359° C. and the corresponding pressure was 205 atmospheres.

The observations on surface tension were repeated a number of times. Various difficulties were encountered, such as moisture condensing on the interior of the vessel, the joints leaking vapor and as a result the insulation breaking down. The moisture was evaporated by letting out some of the water and thus superheating. Leaks were finally stopped by repeated tightening of the plugs. But little trouble, however, was encountered from condensed moisture above 240° C. The apparatus gave remarkably consistent results above this temperature. The apparatus was also used in getting surface tension values from zero to 100° C. The final curve shows that the decrease in surface tension is gradual except when near the critical temperature where it drops suddenly to zero.

The form of the apparatus also enabled the density of saturated water vapor to be determined by noting indirectly the loss in weight of the system (core and vane) when lifted in the vapor.

This investigation was made under the direction of Professor E. L. Nichols to whom I express my thanks for suggestions made and for the facilities placed at my disposal.

PHYSICAL LABORATORY OF CORNELL UNIVERSITY, June, 1900.

BECQUEREL RAYS, A RESUME.

By Oscar M. Stewart.

THE importance of the subject of Becquerel rays has increased almost beyond expectation during the past year or two. Not content with attributing to Becquerel rays all the properties of X-rays, physicists and chemists have found that these rays also possess properties hitherto attributed to kathode rays alone. In its changed aspect it is believed that work along this line will throw light not only on the subjects of kathode rays and X-rays, but possibly on the constitution of matter itself. It has therefore been thought advisable to bring up to date the résumé given over two years ago in this Journal.[1]

In his earlier work Becquerel thought that he had proven beyond all doubt that the radiations from uranium could be reflected, refracted and polarized. It was upon these experiments that the ultraviolet light theory of Becquerel rays was based. No one, including Becquerel himself, has been able to repeat these decisive experiments, and Becquerel now admits that he was mistaken and probably deceived by the presence of a "secondary" radiation. With this exception all the early work on uranium radiations has been confirmed.

Uranium no longer stands as the only element known to emit of its own accord a persistent radiation, and the name Becquerel rays is now applied to the radiations from a number of substances. Some of these radio-active substances have been found to be elements which were hitherto unknown.

Mme. Sklodowska-Curie[2] in searching for other elements or substances which gave radiations similar to those emitted from uranium, found that the oxide of thorium appeared to emit a radiation even more intense than that from uranium. Electrified bodies were more quickly discharged when brought near thorium oxide than when near uranium. G. C. Schmidt[3] independently discovered the radiant activity of thorium and also found that both the element and its compounds imparted conductivity to gases and affected photographic plates. He failed to find any evidences of polarization but thought that he obtained refraction by glass. Mme. Curie had noticed[4] that pitch blende or uraninite and chalcolite (phosphate of copper and uranium) were more active than should be expected from the amount of uranium they contained. In fact they were even more radio-active than uranium itself. This led her to suspect that they contained some material more active than uranium. This seemed more probable when she found that the radiation from natural chalcolite was more intense than that from artificial chalcolite, although they contained the same amount of uranium. After chemical analysis M. and Mme. Curie found that the bismuth compounds obtained from pitch blende were very active. They concluded after examining a large number of elements and substances that they had found as an impurity in the bismuth salts a new element. This element, after Mme. Curie's native country, they called *polonium*. When tested by Demarçay no characteristic lines were observed in its spectrum. But this is also true of uranium, thorium and tantalum, which give innumerable fine lines difficult to recognize. At first only a trace of polonium was found in the bismuth salts. Even this gave a radiation several hundred times as intense as that from uranium. As polonium behaves analytically almost like bismuth, they did not succeed in entirely isolating it from bismuth. The partial separation obtained was by utilizing a difference in solubility and volatilization. For Mme. Curie's researches in magnetism and the discovery of polonium and the radiant activity of thorium the French Academy of Sciences awarded her the *Prix Gegner* of 4000 francs.

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[1] Physical Review, VI., p. 239, April, 1898.

[2] Comptes Rendus, 126, p. 1101.

[3] Wied. Ann. 65, p. 141, 1898.

[4] C. R., 126, p. 1101.

Continuing this work on the pitch blende derivatives, M. and Mme. Curie and G. Bémont[1] found that the barium salts obtained from pitch blende contained in small traces another element analytically behaving as barium, but extremely active in its radiant power. Chlorides of this new element[2] were obtained by fractional precipitation from the barium chloride. This barium chloride was extracted from a large quantity, 100 kg., of pitch blend (the residue after the removal of the uranium) obtained from the Austrian government. Although obtained only in very small quantities, a few milligrams, the chloride of this new element, called *radium*, gave about 900 times the intensity of uranium when measured by the conductivity imparted to air. Later by using still larger quantities of pitch blende, salts of radium were obtained which were still more active. So closely related is the chemical behavior of barium and radium that as yet radium has not been obtained pure, but always in some salt of barium. The proportion of radium in a barium salt may be increased by utilizing the difference in solubility of the two salts. Barium salts are more easily dissolved in pure water or alcohol.

E. Demarçay[3] found that the spectrum of the barium salts containing radium contained quite a number of bright lines hitherto unknown, and that the brightness of these lines increased as salts that were more active were tested. One line ($\lambda = 4627.4$) was the center of a peculiar nebulous band; the other lines were all sharp. The wave-lengths of the brightest of these lines were 3814.7, 4683, 4340.6, 3649.6 and 4826.3.

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[1] C. R., 127, p. 1215.

[2] This element *radium* was independently discovered by Giesel, Physikalische Zeitschrift, 1, p. 16.

[3] C. R., 127, p. 1218 and 129, p. 716.

Using the silver-nitrate reaction Mme. Curie[1] determined the atomic weight of the barium-radium. The radio-active salts gave values ranging from 140 to 145.8 while the inactive barium salts gave 137.7 by the same method. This indicates that the atomic weight of radium is greater than that of barium.

A. Debierne[2] has isolated another substance from pitchblende which is comparable in its radiant power to radium and polonium. Its radiation resembles that from radium, although it is not self-luminous like some radium salts. He has found in *actinium*, as he calls this element, all the properties possessed by radium rays. It resembles thorium in its chemical behavior so closely that he is lead to suspect that the activity of thorium may be due to the presence of actinium in thorium salts.

In most of the work which has been done only very small amounts (a few mg.) of the radium or polonium-containing salts were available. Within the last few months, however, more has been available and masses of a gram or more have been used.

The radiation from the earlier radium salts was about nine hundred times as great as that from uranium, while the polonium salts gave about one-half as much as the radium salts. The salts obtained later gave an intensity 5000 to 50,000 times that from uranium. They would affect a photographic plate in half a minute where uranium required hours. The discharge[3] of electricity by this radiation is comparable to the discharge by a flame and could be detected even when the rays penetrated 4 mm. of brass. The radiation, like X-rays, excites a fluorescence on a barium platinocyanide screen even through thin sheets of aluminum.

The intensity of the radiation from these salts increases[4] for some days or weeks after solidification, the final activity being five or six times the initial. Afterwards the intensity remains practically constant as long as kept free from moisture. The activity of polonium, however, slowly falls off so that in the course of a year a considerable decrease occurs.[5]

Small temperature changes produce little effect on the radiation. Heating the salt will destroy its radiant power, but after a day or so it recovers a large part of it.[1] Behrendsen[2] found by reducing the temperature of radium to that of liquid air, that the intensity was decreased. But on restoring the salt to ordinary temperatures, the radiation was more intense than before.

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[1] C. R., 129, p. 760.

[2] C. R., 129, p. 593 and 130, p. 906, April 2, 1900.

[3] Elster and Geitel, Wied. Ann., 69, p. 83.

[4] Mme. Curie, C. R., 129, p. 760. Giesel, Physik. Ztschr., 1, p. 16, 1899.

[5] Curie, Science Abstracts, p. 465, 1900. Behrendsen, Ann. der Physik., 2, p. 335, 1900.

Usually radium salts give greater intensity of radiation than polonium salts. Giesel obtained, however, a polonium preparation which exceeded the best radium salt which had been prepared up to that time. However, as will be seen later, the radiations from the various sources cannot be accurately compared.

FLUORESCENCE AND PHOSPHORESCENCE.

In general all substances which fluoresce or phosphoresce under the action of X-rays or ultraviolet light are excited by radium and polonium, while those which are excited by visible light are not affected by Becquerel rays. Barry,[3] in testing a large number of substances, particularly the alkaline metals and earths, found that all those that were excited by X-rays were also excited by radium rays and those that were not affected by X-rays were also not affected by radium rays. Becquerel[4] noticed some exceptions to this. A diamond that was not excited by X-rays became quite luminous under the action of radium rays. Calcium sulphide is scarcely affected by X-rays, yet it is by radium rays. Some specimens of fluorite which fluoresced strongly by ultraviolet light became nearly as luminous under the action of radium rays. This fluorite, after an exposure to radium rays, showed a visible phosphorescence for more than twenty-four hours.

Some of the radium salts are self-luminous. Giesel[5] prepared some that without exposure to any source of light, gave a bluish light so bright that one could read by it. This luminosity is probably due to the excitation of a fluorescence of the barium salt in the radium preparation by the rays from the radium. Salts which are not self-luminous give out more intense Becquerel rays than the more luminous salts. This is to be expected, as the rays which produce the fluorescence must be absorbed by the barium salt. Salts freshly prepared gradually change their color. Some crystals prepared by Giesel[6] were at first green, then yellow and at last brown. With the change in color the fluorescence of the salt became weaker and the Becquerel rays stronger.[7]

[1] Elster, Verhandl. Phys. Ges., Leipzig, Jan. 5, 1900.

[2] Ann. der Physik., 2, p. 335, 1900.

[3] C. R., 130, p. 776.

[4] C. R., 129, p. 912.

[5] Physik. Ztschr., 1, p. 16.

[6] Ibid.

[7] Also M. and Mme. Curie, C. R., 129, p. 823.

Giesel[1] found that, like X-rays, radium rays produced a faint sensation of light when brought near the eyes,—even when they were closed. This sensation is probably due to a fluorescence excited within the eye itself.

Some of the Curie polonium preparations do not excite fluorescence although more active in discharging electrified bodies than some of the de Hāen preparations that give relatively strong fluorescent action.[2] This indicates a difference in the radiations from different salts which is confirmed in other ways.

CONDUCTIVITY OF GASES.

The radiations from the salts of uranium, thorium, polonium, radium, and actinium impart a temporary conductivity to gases. This conductivity has been studied in detail in the cases of radiations from uranium and thorium by Rutherford[3] and Owens.[4] The conductivity imparted to gases by Becquerel rays does not differ essentially from the conductivity imparted by other methods, as by X-rays, kathode rays, or flames. The results in all cases so far obtained agree with the ionization theory first suggested by Giese[5] and Schuster[6] and later more fully developed by Professor J. J. Thomson. This theory assumes that under the influence of X-rays or Becquerel rays some of the molecules of the gas are broken up into ions, half of which are charged positively and the other negatively. These ions may in some cases become nuclei for clusters of molecules, as in the case of the gases from flames where the ions seem to have greater mass than usual. Under the influence of electrical forces the ions move with a velocity approximately proportional to the intensity of the field. A charged body placed in the gas will therefore attract one kind and repel the other. Upon coming in contact with a metallic conductor these ions will give up their charges and cease to exist as ions. The ratio of the number of ions existing in any gas to the total number possible is very small, so that the gas when ionized by any radiation behaves like an extremely dilute electrolyte. From this it follows that the electrical current between two electrodes immersed in the gas is not always proportional to the difference of potential, that is, it does not obey Ohm's law. For small potential differences, the current will be approximately proportional to the potential difference ; but for much larger potential gradients, all the ions may be used in carrying charges as fast as generated by the rays. Therefore as the current increases it will approach a limit where for further increase in potential no increase of current will result.

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[1] Physik. Ztschr., 1, p. 43, 1899.

[2] Meyer and von Schweidler, Physik. Ztschr., p. 209, February 10, 1900.

[3] Phil Mag., 47, p. 109.

[4] Phil. Mag., 48, p. 360.

[5] Wied Ann., 17, p. 538, 1882 and 38, p. 403, 1889.

[6] Proc. Roy. Soc., 37, p. 317, 1884.

The velocities of the positive and negative ions in various gases have been determined by several different methods. It is almost universally true that the negative has a greater velocity than the positive ion in the same electrical field. In dry air the velocity of the negative ion is about 37 per cent. greater than the velocity of the positive.[1] Rutherford determined the velocity of the ions generated by the radiations of uranium and found that they were, within experimental errors, the same as those generated by X-rays. While the method was not at all accurate for absolute determinations, it afforded a good method for comparison. As in the case of air ionized by X-rays, the negative ion moves faster than the positive ion in the same electrical field. This explains the results obtained by Beattie, who observed that the gas near uranium in an electrical field was generally positively charged. As the positive ions move through the electrical field more slowly than the negative, an excess of positively charged ions will be found in the field.

The ionization produced in a gas is proportional to the absorption of the rays by the gas. It is therefore those rays that are most strongly absorbed that will produce the greatest conductivity in a gas. The absorption of polonium rays by air is greater than the absorption of radium rays, so that for the same intensity polonium rays will produce greater ionization. The discharging action of Becquerel rays from different sources is therefore not a true measure of their intensities. The radiation from these various substances is also known to be non-homogeneous in character. For example, Rutherford has found that the radiation from uranium may be classified in two groups, α and β rays. The α rays produce relatively a greater amount of ionization than the β rays. It is thus seen that any measurements of the relative intensities of Becquerel rays from different sources by the conductivity they impart to the air is liable to considerable error. It may be likened to the measurements of the intensity of the different sources of visible light by a method which really measured the intensity of one color only, as, for example, green. The same objection can be made to photographic or fluorescent methods.

The effect of Becquerel rays on a spark gap has been tested by Elster and Geitel.[2] When a spark gap, 1 cm. long, consisting of a positive knob and a negative disc, was exposed to these rays, the spark or brush discharge was changed into a glow discharge. The spark gap was found to be very sensitive to the presence of the rays. The electrodes did not need to be polished as in the case of ultraviolet light.

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[1] Zeleny, Phil. Mag. 46, p. 120; Proc. Roy. Soc. 66, p. 238, March 1, 1900.

[2] Wied. Ann., 69, p. 673.

ABSORPTION.

By measuring the absorption of uranium rays by different numbers of thin plates, Rutherford concluded that uranium rays were not homogeneous, but could be divided into two classes, α rays and β rays. The β rays were much more penetrating than the α rays. Relatively greater ionization is produced by the α rays, as they are more strongly absorbed by gases. The α rays therefore appear to be the more intense, as measured by the conductivity imparted to gases. As a result of this composite character, the absorption of Becquerel rays from uranium does not obey the ordinary law of absorption. A thin sheet of metal or a sheet of paper reduces the intensity considerably, but as the number of layers is increased their relative effect decreases. The first layers absorb the α rays, which apparently are the most intense, and additional layers affect only the β rays, which are more penetrating. Aluminium foil, 0.002 cm. thick, reduced the intensity to about $\frac{1}{20}$ of its original value. But an additional thickness of 0.05 cm. only reduced the intensity to about one-half of the intensity transmitted through 0.002 cm. Rutherford's results indicate that the β rays are homogeneous. Their penetration is comparable to that of X-rays from an ordinary Crookes' tube. X-rays from the modern "hard" tube are however much more penetrating. The α rays have about the same penetrative power that "secondary" X-rays have. Since the α rays are the most active in producing ionization and the β rays have the greatest photographic action, determinations by these two methods of the absorption of uranium rays by different substances give results which do not agree. The α rays appear to come from the surface, while the β rays seem to come from the interior. Thus the radiation from a thin layer consists chiefly of α rays, while the proportion of β rays is increased if the radiation is from a thick layer.

The radiation from thorium[1] is in general more penetrating than uranium rays when measured by the conductivity imparted to gases. Schmidt[2] found that the absorption of thorium rays by a number of layers of metal was less than the sum of the absorption by the layers taken separately. This shows that these rays too are not homogeneous. However Mme. Curie[3] noticed a difference between the radiation from thin and thick layers of the thorium compound. Owens[4] found that the radiation from thin layers was homogeneous, and that the radiation from layers a millimeter or more in thickness was not homogeneous.

Owens showed that the absorption of thorium rays by gases followed the order of the gas densities.

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[1] Mme. Curie, C. R., 126, p. 1101 and Rutherford, Phil. Mag. 47, p. 123.

[2] Wied. Ann., 65, p. 141.

[3] C. R., 126, p. 1101.

[4] Phil. Mag., 48, p. 360.

Becquerel[1] observed, while obtaining fluorescence of different substances by radium rays through absorbing screens, that the screens affected the fluorescence in some salts more than it did in others. The simplest explanation of this is that the radiation from radium is also not homogeneous in character. This is confirmed by the researches of Meyer and von Schweidler[2] who found, as in the case of uranium and thorium, that the absorption of radium rays by quite a number of substances did not obey the ordinary absorption law. Using the rate of discharge of an electrified body as a measure of the intensity they found that a thin layer of metal diminished the intensity greatly, but additional layers produced but little effect. Through a thin sheet of platinum, 0.0025 mm. thick, 49 % of the original intensity was transmitted, while two such sheets let through 36 % and a layer, 0.12 mm. thick, gave 15 %. 28 % was transmitted through one sheet of aluminium, 0.16 mm. thick, while 16 % was transmitted through six sheets. A sheet of tin, 0.011 mm. thick, transmitted 44 %, two sheets 31 % and fifteen sheets 15 %. 38 % penetrated one sheet of paper, 0.06 mm. thick, and 20 % twenty-two sheets. Glass of 0.16 mm. thickness, transmitted 26 % and ten plates 16 %. Similar effects were obtained through quite a number of other metals and substances. These results were obtained with the radiation from a radium barium carbonate prepared by M. and Mme. Curie. The rays from a bromide and a chloride of radium prepared by Giesel behaved similarly, but were less penetrating than the rays from the salt obtained from the Curies.

As has been shown, the radiations from uranium, thorium and radium consist of at least two kinds of rays, one which is greatly absorbed and another which is far more penetrating. R. J. Strutt[3] has found that the absorption of the more penetrating component of radium rays depends only on the density and thickness of the absorbing substance. The absorption by various substances, ranging in density from that of platinum, lead and silver, to that of aluminium, paper and sulphur dioxide, follows the same order as their densities. He further showed that within a reasonable approximation, the quotient of the coefficient of absorption divided by the density is a constant. This is the same law observed by Lenard[4] in the case of kathode rays. The absorption however was about five hundred times greater in the case of kathode rays.

Polonium rays seem to correspond to the less penetrating rays of radium, being readily absorbed by all substances. While rays have been obtained[5] from radium which have penetrated 1 to 2 cm. of metal, polonium rays are greatly absorbed by paper and card board. Giesel said that polonium rays gave better radiographs of the hand as they showed more contrast. However these radiations will probably never be found very useful in surgical work owing to the strong absorption of them by air and light substances. A few centimeters of air will totally absorb the rays from the polonium prepared by M. and Mme. Curie, while rays from radium sufficiently intense to excite a fluorescent screen have been obtained through 40 or 50 cm. of air.

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[1] C. R., 129, p. 912.

[2] Physik. Ztschr., 1, p. 209.

[3] Nature, 61, p. 539, 1900.

[4] Wied. Ann., 56, p. 255.

[5] Giesel, Physik. Ztschr., 1, p. 16.

Mme. Curie[1] noticed that the absorption of the rays from their polonium increased as the distance from the source is increased. They will penetrate a very thin aluminium plate, but their penetrative power is decreased thereby. Or as it may be stated, the coefficient of absorption increases with increase of the thickness of the absorbing medium. These rays seem to behave like small projectiles which rapidly lose their velocity and therefore their penetrative power.

Evidence of a feeble but more penetrating radiation from radium closely resembling X-rays has been recently found by Villard.[2]

CHEMICAL EFFECTS.

M. and Mme. Curie[3] found that the glass of bottles in which radium salts had been kept for some time turned violet and then black. The effect seems to gradually penetrate through the glass. Villard[4] noticed the same effects with X-rays. He said that a radiograph can thus be obtained on glass by a sufficiently long exposure to X-rays. This coloring of the glass has also been observed with kathode rays. Giesel[5] found that rock salt or potassium bromide when exposed to radium rays for a day took the same coloring that kathode rays give them. M. and Mme. Curie noticed that a barium platinocyanide screen was fatigued by the impact of radium rays in the same way that it is fatigued by kathode rays. Radium rays act in many ways like light, reducing silver salts, peroxide of iron and bichromate of potash in the presence of organic substances. Besides coloring glass they will also color porcelain and white paper.[6]

The radiation from radium salts seems to convert oxygen into ozone. This was first noticed by Demarçay, to whom M. and Mme. Curie had sent some radium salt in a tightly stoppered bottle. On opening the bottle, the characteristic odor of ozone was distinctly noticeable. A coloring of a mixture of starch and potassium iodide,[7] a common test for ozone, was obtained. Giesel, however, failed to detect the formation of ozone by any of the salts he had.

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[1] C. R., 130, p. 76.

[2] C. R., 130, pp. 1010 and 1178.

[3] C. R., 129, p. 823.

[4] C. R., Nov. 27, 1899.

[5] Verhandl. Phys. Ges., Leipzig, Jan. 5, 1900.

[6] Science Abstracts, p. 465, 1900.

[7] M. and Mme. Curie, C. R., 129, p. 823.

In the course of time the radio-active salts undergo changes of structure. This is quite noticeable in some of the radium barium salts which change color for several days after they are made. A greenish yellow barium platinocyanide screen prepared from barium salts containing radium was observed by Giesel to be at first yellow then orange and finally a brownish red. A single one of these crystals will appear a clear yellow but on crossing two of them only brown red light is transmitted. The same effect is noticed when one of these crystals is tested with a Nicol. The usual barium platinocyanide salts as well as the freshly prepared active salts show no such polarization.

SECONDARY OR INDUCED ACTIVITY.

Sagnac[2] discovered that X-rays had the power of giving to bodies on which the rays impinge the ability to radiate rays which are called "secondary" rays. The action of these secondary rays ceases the instant the X-rays are cut off from the excited substance. As Becquerel rays seemed to be similar in action to X-rays, it was therefore natural to expect such rays in this case. M. and Mme. Curie[3] found that the rays from radium were apparently able to induce in other substances such as zinc, platinum, bismuth and even paper the temporary power of emitting Becquerel rays. These secondary rays were readily detected by the conductivity they imparted to gases even after the removal of the radium. Increasing the length of exposure to radium rays increased the acquired activity. On removal of the radium the induced radiation rapidly decreased, in two or three hours being reduced to about one-tenth. The secondary activity induced in the different substances varied from one to fifty times the intensity of the radiation from uranium. An explanation of this phenomenon is readily suggested. If the radium gives off a vapor or something similar to a fine dust, some of it may be deposited on neighboring substances. Each of these small particles may give off Becquerel rays, but owing to its small size it is unable to keep up this output of energy. However, it was found that the induced activity could not be destroyed by washing. And as the radium rays would produce this effect through thin sheets of aluminium, they felt confident that it could not be due to the presence of a vapor or to traces of the radium salt on the substance. It seemed to be more allied to a phosphorescence. However, if it were a real induced effect, differences should be detected in the activity excited in different bodies, as is the case with secondary

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[1] Verhandl. Phys. Ges., Jan. 5, 1900.

[2] C. R., 125, pp. 573 and 942, also 126, p. 521.

[3] C. R., 129, p. 714.

X-rays. But no such difference was observed, all substances apparently showing the effect equally.

Becquerel[1] observed that this activity was induced in pieces of the fluorite which showed such intense phosphorescence. After an exposure to radium rays it was able to discharge electrified bodies. However, the acquired activity could be destroyed by heating or by washing with water. This is contradictory to the results of M. and Mme. Curie. He further found that although this activity, which was acquired by many different substances, could discharge electrified bodies, it could not affect a photographic plate.

Elster and Geitel[2] while freeing some radium salts from water in a vacuum tube found that a cold glass body brought into the tube became coated with a scarcely visible deposit. This glass body was then found to emit rather intense rays. At first they thought that they had a case of the induced activity similar to that announced by M. and Mme. Curie, but they found that this activity could be destroyed by washing. These results of Becquerel and Elster and Geitel, together with the work of Rutherford on thorium, to be referred to later, make it quite probable that some of these salts, in addition to emitting Becquerel rays, send out very small particles, perhaps of molecular dimensions, which retain for a time the ability to give out these rays. Rutherford[3] has in the case of thorium found very strong reasons in support of this view. He has found that these small particles can apparently penetrate screens of paper and thin sheets of metals.

It now seems probable that Becquerel rays are similar in character to kathode rays, that is, are small particles of matter carrying negative charges and moving at a high velocity. By some writers it has been thought necessary to explain the apparent penetration of thin screens of metal by assuming that when a particle strikes one side another particle on the opposite side is in some way driven off. Hence if kathode rays impinge on one side, new rays are emitted on the opposite side, giving the metal screen the appearance of transparency. Following this line, Villard[4] has attempted to show that in the case of radium rays the apparent penetration is due to secondary rays which are emitted by the opposite face. These secondary rays are of an entirely different nature from the induced rays of Curie or Rutherford. They cease the instant the incident radiation is suppressed. Villard thought that he had proven that the chief radiation beyond a screen started from the screen itself going in a path perpendicular to its surface. While under some circumstances the secondary emission from screens may be the predominate one Becquerel[1] denies that it is always so. He found evidences that some of the rays passed directly through, accompanied, however, by a diffuse radiation from the screen. With thick screens, or thin ones placed obliquely, the secondary rays are relatively more intense, but for thinner screens they become less important.

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[1] C. R., 129, p. 912.

[2] Verhandl. Phys. Gesell., Leipzig, Jan. 5, 1900.

[3] Phil. Mag., 49, pp. 1 and 161.

[4] C. R., 130, p. 1010.

The secondary rays may differ from the incident beam, but to what extent is not known. Different observers disagree. Dorn[2] says that the rays from the salt consist chiefly of rays which are not deflected by a magnet but are transformed by a screen into deflectable rays. Villard[3] says that secondary rays are produced by the deflectable rays and always leave the screen perpendicularly. Becquerel thought that part of the secondary rays could not be deflected while the rest could.

THORIUM "EMANATIONS."

Mme. Curie[4] noticed that the radiation of thorium from thick layers was apparently more penetrating than than from thin layers. This was confirmed by Owens[5] who also observed that the radiation from thick layers of thorium was greatly affected by slight air currents. Rutherford[6] has taken up the subject and has found that thick layers, one millimeter or more, of thorium oxide and other thorium compounds seem to have the property of giving off small particles which can radiate Becquerel rays for a time. This vapor or "emanation," as Rutherford calls it, can pass through very thin sheets of metals and through many layers of foolscap paper. It gradually diffuses itself through the air and may be carried away by light currents. When the "emanation" is drawn by a current of air into the space between two metallic plates, one of which is connected to a sensitive electrometer, the intensity of its radiation may be measured by the electrical current which will pass through the gas. It was found by this method that the electrical discharge would last about ten minutes after the air current is stopped. If the conductivity had been due to the ordinary gas ions brought over from the neighborhood of the thorium, it would have lasted for only a second or so. For considerable potential differences, the ions that carry the charges in the conduction in gases, are used and thus destroyed as rapidly as they are produced. Rutherford, however, found that the rather slow decay of the electrical current after the air motion had stopped, was independent of the potential difference employed. This too contradicts the supposition that ions alone were present, for in this case for large potential differences the ions would be used up immediately. It seems evident that there must be something in the air between the plates of the condenser which, by sending out Becquerel rays, made the air conducting. Air containing these emanations could be drawn through wool or bubbled through water, dilute or strong sulphuric acid, without destroying the emanations. Ions are destroyed by such processes. The emanation is also independent of the nature or pressure of the gas surrounding the thorium.

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[1] C. R , 130, pp. 979 and 1154.

[2] Beiblätter, p. 572, 1900.

[3] C. R., 130, p. 1010.

[4] C. R., 126, p. 1101.

[5] Phil. Mag., 48, p. 360.

[6] Phil. Mag., 49, pp. 1 and 161.

Rutherford found that under certain conditions this emanation has the property of making other bodies radio-active. Any substance, charged to a negative potential, will become able to emit Becquerel rays after it has been kept for a few hours near thorium or in contact with air containing the emanation. By keeping a wire exposed to air containing the emanation, negatively charged, the radio-activity may be concentrated on it instead of being distributed over the neighboring surfaces. Bodies made active in this manner will retain this property for several days. Rutherford showed that this induced activity could not be due to the direct radiation of the thorium rays or to any ordinary dust from the thorium.

It seems probable that the small radio-active particles which diffuse from thorium in some way take on a small positive charge. The particles will then be attracted by negatively charged bodies. In this way a body may become partially coated with this active material. It was found that the activity which ordinarily would last for several days could be removed by a long and vigorous polishing with sand paper. On placing a very active platinum wire in sulphuric acid, the radio-activity was largely destroyed. The residue of the acid obtained by boiling it down on a sand bath was then found to be strongly active.

The radiation from the different bodies that had been made active, differed from thorium radiation. It was in some cases more intense, as when the activity was concentrated on a negatively charged wire. It was more penetrating than the radiations from any of the uranium, thorium, polonium, or radium compounds which he had been able to test. If this activity is due to the presence of a radio-active substance, its power per unit mass of emitting Becquerel rays is large as compared to that of thorium.

Debierne[1] has recently found a very active element, actinium, which so closely resembles thorium in its chemical behavior that he thinks the activity of thorium is in part, at least, caused by the presence of this element. He points out that the experiments of Rutherford can be explained by the presence of this element as an impurity.

Rutherford thinks that the reason the vapor particles are attracted by a negative charge is due to the fact that they become positively charged by the diffusion of the ions in the gas. As negative ions are known to move more rapidly than the positive they tend to leave the gas with an excess of positive ions. M. and Mme. Curie have proven that some radio-active substances tend to become positively charged since negative charges are carried off by the rays. In this way the presence of a slight positive charge on the active particles which diffuse from thorium may be explained.

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[1] C. R., 129, p. 593, and 130, p. 906, April 2, 1900.

Attempts to obtain this emanation or something similar to it from uranium, polonium, and radium have failed. Rutherford's specimens of radium and polonium were, however, not pure. S. Meyer and E. R. Schweidler[1] observed that when a radium barium carbonate prepared by M. and Mme. Curie was removed from a metallic cylinder within which it had been kept, the air within the cylinder retained its conductivity for some time. But radium salts prepared by Giesel did not show this. These facts, together with the induced radio-activity observed by M. and Mme. Curie, suggest that possibly some of the radium salts give off a vapor similar to that observed from thorium.

MAGNETIC DEFLECTION.

Elster and Geitel[2] observed that the discharge by Becquerel rays of an electrified body in a vacuum tube was retarded by a magnetic field. Two explanations at once suggest themselves: the effect may be due to the deflection of the rays by the magnetic field, or, as in the case of discharge by ultra-violet light, part of the ions may be deflected back on the charged body. Attempts by them to observe a direct magnetic deflection of the radium rays were at first failures. This failure was probably due to the nature of the salt used as well as the method. Later a magnetic deflection was observed almost simultaneously by S. Meyer and E. R. v. Schweidler,[3] Giesel,[4] and Becquerel.[5] It has since been verified by Elster and Geitel[6], Warburg[7] and M. and Mme Curie[8] and several others. In all cases the deflection is in the same direction as for kathode rays, *i. e.*, in the direction corresponding to the deflection of a negative current.

The deflection is not obtained with the rays from all the salts of radium and polonium. Meyer and Schweidler failed to find it with the rays from

[1] Physik. Ztschr., p. 209, Feb. 10, 1900.

[2] Wied. Ann. 69, p. 83.

[3] Physik. Ztschr., Nov. 25 and Dec. 2, 1899.

[4] Wied. Ann., 69, p. 834, Dec., 1899.

[5] C. R., 129, p. 996, Dec. 11, 1898.

[6] Verhandl. Deutsch. Phys. Ges., Leipzig, Jan. 5, 1900.

[7] Ibid.

[8] C. R., 130, pp. 73 and 76.

a radium barium carbonate and a polonium nitrate. Both Becquerel and M. and Mme. Curie failed to find any deflection of the rays from any of the polonium salts prepared by M. and Mme. Curie. Rays from some polonium prepared by Giesel have been deflected.[1] This has led to a classification by M. and Mme. Curie[2] of the rays under two heads, the rays deflectable by a magnetic field, and those not deflectable. Many salts of radium give off both kinds. This classification corresponds to that already given under the head of absorption. The more penetrating rays of radium are the deflectable ones, while the less penetrating ones are not deflectable. The rays from the polonium prepared by M. and Mme. Curie are not deflectable, and they do not contain any of the more penetrating type. M. and Mme. Curie[3] have found that the non-deflectable rays of radium behave like the rays from their polonium. As measured by the conductivity imparted to air the penetrating or deflectable rays of radium are a small fraction of the total radiation. However, the less penetrating rays are almost totally absorbed by 7 cm. of air, so that at even small distances from the source the penetrating rays are in excess. Debierne[4] has shown that the rays from actinium belong to the deflectable class. Thorium rays have not as yet been deflected.

Becquerel[5] has recently obtained a magnetic deflection of uranium rays. It has been thought that part at least of the radiation from uranium is due to the presence as an impurity of actinium. Becquerel attempted to remove the actinium from the uranium salt. This decreased the activity of the uranium and the foreign substance extracted was found to be active. As the radio activity of uranium was not entirely destroyed, Becquerel thinks that uranium has a radiation of its own. The deflectable and the more penetrating rays, β rays, of uranium are probably due to actinium.

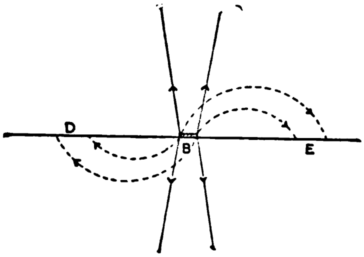

Fig. 1.

The usual method of observing the magnetic deflection is to place the radiant substance on a photographic plate or on a fluorescent screen. The radio-active salt is placed in a hole in a little block of lead so that the rays are emitted only in a direction perpendicular to the plate or screen. When the field is excited to about 2000 C. G. S. units the rays are deflected in a circular path and fall on the plate or screen on one side of the radio-active substance. As used by Meyer and v. Schweidler, the substance B (see figure) is placed below a fluorescent screen. With no magnetic field the rays travel rectilinearly away from B exciting luminosity only on a spot directly above B. When a uniform magnetic field, perpendicular to the plane of the figure and in direction away from the observer, is created, two luminous spots will appear, D and E. An opaque screen placed above B will cause the spot E to disappear. When placed below B it will affect D, or when placed below E no effect is observed. In this way the path of the rays is readily traced.

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[1] Elster and Geitel, Meyer and Schweidler, and Giesel.

[2] C. R., 130, pp. 73 and 76.

[3] Ibid.

[4] C. R., 130, p. 906.

[5] C. R., 130, p. 1585, June 11, 1900.

When the rays are perpendicular to a uniform field they move in arcs of circles, but when the direction is not perpendicular to the field, or the field is not uniform, the path is more complex. In all cases the action is similar to that observed with kathode rays. Becquerel found that when the direction of the rays was not perpendicular to the field the path became a spiral. If the field were not uniform the rays could be concentrated or focused by it.[1] For example, if the radium salt be placed on or near one pole face of an electromagnet of the horseshoe type and a screen placed near the opposite one, the diffuse illumination of the screen could be concentrated by the field to the part nearest the pole piece. This requires an electromagnet of rather small pole pieces.

These effects are all readily explained if we assume that the rays are, as is now generally conceded in the case of the kathode rays, small particles carrying negative charges.

Until the determination of the magnetic deflection of Becquerel rays, the known phenomena of these rays were almost identical with those of X-rays. The discovery of the magnetic deflection of Becquerel rays led Strutt[2] to make a further attempt to deflect X-rays. By carefully arranged apparatus he was able to assert that if there were a deflection, the radius of curvature of the path was greater than 19,800 cm., in a field of 3270 C.G.S. units. With this field the radius of curvature of Becquerel rays was about a centimeter.

If the deflectable Becquerel rays consist of rapidly-moving negatively charged particles, it may be shown that

$$m\frac{v^2}{\rho} = Hev$$

or

$$\frac{e}{m} = \frac{v}{H\rho},$$

where e/m is the ratio of the charge to the mass of the particle, v its velocity, H the strength of the magnetic field, and ρ the radius of curvature of the path of the rays. It has been observed by several that the rays are not equally deflected. Becquerel[1] found that in the radiation from a single sample of radium the value of $H\rho$ varied from about 350 to nearly 3,000. As is evident, a magnetic spectrum of Becquerel rays is always produced with a source that gives such a non-homogeneous radiation. The values of $H\rho$ are of the same order of magnitude as in the case of kathode rays.

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[1] Becquerel, C. R., 129, p. 912, also Meyer and v. Schweidler, Physik. Ztschr., 1, p. 113.

[2] Proc. Roy. Soc., 46, p. 75.

That different parts of this spectrum have different properties is readily shown by its penetrative power. Those rays which are deflected the most will penetrate only very light bodies or very thin sheets of paper and aluminium foil. For the portion of the spectrum corresponding to a small deflection the rays will penetrate thicker or denser metals. The penetrative power increases as the deflection decreases. Or for those rays for which $\frac{e}{mv}$ is smaller the penetration is greater. This is what would be expected if the difference in the rays is due to a difference of velocity of the charged particles, since the particle with the greater velocity is more penetrating and is also less deflected by the field. To observe this difference in absorption, Becquerel laid side by side on a photographic plate long narrow strips of the different metals. The direction of the strips was parallel to the direction of the dispersion. In this way it was easily shown that the thin strips of the lighter materials let through almost the entire spectrum, while the denser strips let through only the part deflected the least.

From Becquerel's observations may be deduced, at least approximately, that the amount of spectrum transmitted by different substances of the same thickness, follows the inverse order of their densities. This is in confirmation of the view that the difference in deflection is caused by a difference in velocity of the particles. Following this hypothesis of small particles, the non-deflectable or less penetrating rays are probably particles of much greater mass. Whether these particles are charged or not is not certainly known, as the deflection of larger particles may be so small as to have escaped detection.

Becquerel[2] found that an absorbing screen placed near the source permitted rays giving greater deflection to pass through than when the screen is placed farther from the source. Others[3] have noticed that the absorption of a screen increases with the distance from the source. This may be explained in two ways. The penetrating power of the rays seems to depend on their velocity. If the velocity is decreased by passage through the air then the rays have less penetrative power at greater distances from the source. Or, the apparent transmission may be largely due, as some writers think, to a secondary radiation emitted by the screen. This secondary radiation may travel at a different velocity and may be deflected more than the incident beam.

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[1] C. R., 130, p. 372.

[2] C. R., 130, pp. 372 and 979.

[3] See C. R., 130, p. 979.

Becquerel[1] tried in several ways to find out if the velocity was altered by transmission through air or other substances. Any change in velocity should be detected by a change in the magnetic deflection. Although unable to settle the question positively, as the experimental difficulties are rather great, he could detect no change in the velocity or rather in the ratio $\frac{e}{mv}$. Some experiments on this subject are misleading. Becquerel[2] obtained a magnetic deflection in a vacuum and then in air. Measurements showed them equal. But he protected his plates with black paper. As he has since pointed out, if the rays which penetrated the paper in a vacuum had their velocity diminished in air, their penetrative power would be decreased, and although deflected more the photographic plate would not show it as they could not then penetrate the paper. The great difficulty in determining any change in the velocity is due to the many different velocities present, and an inability to distinguish them. In this connection it may be stated that no change in the velocity of kathode rays has been observed when they have penetrated screens.

ELECTROSTATIC EFFECTS.

If the deflectable rays carry negative charges they should, as is the case with kathode rays, give negative charges to bodies which they strike, and should be deflected by an electrostatic field. At first[3] no such effects were obtained. As the rays make the surrounding air a conductor, any negative charge imparted to a body will be conducted off by the air. This conductivity of the air also makes a deflection of the rays by an electrical field difficult to observe, since the air being a conductor tends to serve as an electrostatic screen.

There seem but two ways open to detect the negative charge imparted to a body struck by the rays. If the body be enclosed by a very high vacuum it can then retain its negative charge. This may be varied by placing the radium salt in the vacuum, where it will accumulate a positive charge. The second method is to entirely enclose either the target or the radium itself in a solid dielectric like paraffin or ebonite. One side of this dielectric should be thin enough to permit the rays to traverse it. Using this latter method M. and Mme. Curie[4], with a sensitive electrometer, observed not only that the rays carried with them negative charges, but also that the rays by carrying away a negative charge left a positive charge on the radium. The rate at which a negative charge was carried off by the rays as measured in amperes was 4×10^{-12} per sq. cm. of the radium salt.

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[1] C. R., 130, p. 979.

[2] C. R., 130, p. 206.

[3] Becquerel, C. R., 129, pp. 1205 and 130, p. 206.

[4] C. R., 130, p. 647.

E. Dorn[1] was the first to obtain a deflection of Becquerel rays by an electrostatic field. The rays used were those from a bromide of barium and radium. Lenard[2] found that the excitation of a fluorescent screen by the impact of kathode rays could be either increased or decreased by an electric field parallel to the direction of the rays. When the field accelerated the charged particles the fluorescence was increased. Reversing the field produced a diminution. Dorn repeated this experiment, using radium rays instead of kathode rays, and obtained the same results as Lenard.

Becquerel[3] independently obtained an electrostatic deflection of the rays. Although he used an intense electrical field of about 10,000 volts per cm., the deflection observed on the photographic plate was only 4 mm. Hence it was necessary to use a very narrow beam of rays such as obtained through a narrow linear slit. In the path of this narrow beam was placed a thin screen, mica covered with tin, parallel to the linear source and edge on, so that its shadow on the plate was linear. By deflecting the beam at right angles to this shadow a broadening of it was obtained. As in the case of magnetic deflection it was observed that the rays were unequally deflected, again showing their composite character.

Knowing the field strength and the amount of the deflection the value of mv^2/e is obtained. Since mv/e is obtained from the magnetic deflection both the velocity and the ratio of charge to mass may be computed. The difficult thing is to identify in the two spectra the corresponding radiations. By observing the parts of the two spectra which seemed to show the same penetration he obtained the values for e/m and v,

$$\frac{e}{m} = 10^7, \quad v = 1.6 \times 10^{10} \text{ cm.}$$

The ratio of charge to mass and the velocity are therefore of the same order of magnitude as in the case of kathode rays. These values can be regarded only as approximations. The velocity according to the above figures is about one-half the velocity of light. The measurements of Dorn on the deflection by an electric field agree in magnitude with Becquerel's measurements. Using the value of the current obtained by M. and Mme. Curie, which is equal to Ne, where N is the number of particles given off in one second, and the observed value of mv^2/e, $\frac{1}{2} Nmv^2$ or the energy per second of the radiation is obtained.

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[1] Abh. d. Nat. Ges. zu Halle, March 11, 1900, also Physik. Ztschr., p. 337, May 5, 1900.

[2] Wied. Ann., 65, p. 509, 1898.

[3] C. R., 130, p. 809.

$Ne = 4.10^{-12}$ amperes or 4.10^{-13} C. G. S. and $mv^2/e = 2.57 \times 10^{13}$.

Then $\frac{1}{2} Nmv^2 = 5.1$ C. G. S. units. Or the energy emitted per second per square cm. is about five ten-millionths of a watt.

From the above ratio of e/m and the current the mass of the particles carried off in a second may be computed. This amounts to about a milligram in a thousand million years.

THEORIES.

Many theories have been suggested concerning the origin of the energy of the Becquerel radiation. Mme Curie[1] first suggested that the energy may be due to a very penetrating radiation hitherto unknown which could excite fluorescence in some substances. Sir William Crookes suggested in the Presidential address before the British Association in 1898 that in some way the energy might be obtained from the rapidly moving air particles. Elster and Geitel[2] took a salt of uranium into a deep mine, where it was protected by 850 meters of earth from any external radiations, and the salt was still active. When kept in a high vacuum the radiation was still emitted, so that the energy cannot be due to either a mechanical or chemical action of the air. They found that the impact of X-rays or kathode rays on the radiant substance produced no change in its radiation.

Elster and Geitel[3] suggested that the energy may come from an unstable molecule gradually passing over into a stable state. This idea was supported also by Behrendsen.[4] Until the discovery of the magnetic deflection of Becquerel rays many of the investigators had come to the conclusion that these rays were X-rays of feeble intensity. As the rays belonging to the deflectable group are now known to carry negative charges, they probably consist of small particles of matter, matter and electrical charges being, according to our experience, inseparable. According to the theory used to explain the Zeeman effect, each molecule has revolving around it at a high velocity one or more small satellites negatively charged. Measurements based on this theory have shown that the ratio of the charge to the mass, e/m, of a satellite is of the same order of magnitude as that obtained for the kathode ray particle or for the deflectable Becquerel rays. In the molecular groupings of the radioactive elements there may be conditions of instability such that some of these satellites could be lost from their molecules, and moving off with their high velocity become the deflectable Becquerel rays.

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[1] C. R., 126, p. 1101.

[2] Wied. Ann., 69, p. 83, 1899.

[3] Beib., p. 443, 1899.

[4] Wied. Ann., 69, p. 220.

The theory that Becquerel rays consist of extremely short ultra-violet light waves is now almost abandoned. In the case of the deflectable rays it is entirely dropped and it is believed that they, like kathode rays, are negatively charged particles of matter moving at extremely high velocity. Radiations of this character may differ among themselves in the size of the charge, the mass of the particles and their velocity. J. J. Thomson has shown that the ratio of charge to mass for the negatively charged particles in gases at low pressures is a constant. This together with results obtained from the Zeeman phenomena indicate that the ratio of charge to mass in these small particles is always a constant. If this is true it is the velocity which varies. The difference in penetration of Becquerel rays and kathode rays is then readily explained by a greater velocity for the Becquerel ray particle. The measurements of mv/e by Becquerel give values ranging approximately from 400 to 3000. The upper limit is higher than obtained by kathode rays and as it is the rays that give the larger values which are the more penetrating, it is safe to assume that the more penetrating rays have a greater velocity than kathode rays.

All results indicate that the atomic charge e is invariable and as in the case of the negative ion in the kathode ray or ultraviolet light discharge one infers that the mass m of the particle is much smaller than an atom, being only about one-thousanth of the size of the hydrogen atom or the ion found in electrolysis.

A theory of X-rays which is now gaining in popularity is that they are the small kathode ray particles after they have lost their charges. The absence of a charge accounts for the greater penetration. On this basis one might expect to obtain X-rays from radium rays. Villard[1] has detected the presence of such a radiation, an effect being obtained through 1 cm. of glass. This has as yet not been confirmed.

Cornell University.

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[1] C. R., 130, p. 1010, April 9, and p. 1178, April 30, 1900.

THE PROBLEM OF THE STRESSES AND STRAINS IN A LONG ELASTIC HOLLOW CYLINDER SUBJECTED TO INTERNAL AND EXTERNAL PRESSURE, AND TO TENSION.

BY W. S. FRANKLIN.

INTRODUCTORY.

IN March, 1899, the writer came upon an unsatisfactory treatment of this problem in reading a well-known treatise upon Elasticity and the Strength of Materials. The unsatisfactory character of the treatment was due, in part, to unavoidable brevity and in part to what seemed to be erroneous statements concerning the relation of the formulas of Lamé, of Clavarino, and of Birnie, which formulas have been used in the designing of guns. The present writer, having some interest in the problem, worked it out in full as outlined below. The result of this solution is a pair of formulas (one for stresses the other for strains) which include the three above mentioned formulas as special cases. There is no reason to believe, however, that there is anything new in this present solution, although the solution given in Lamé's "*Leçons sur la Theorie Mathematique de l'Élasticité des Corps Solides*" is extremely brief, and no helpful references could be found in the Todhunter-Pearson *History*.

FUNDAMENTAL EQUATIONS.

Let P, Q and R be the principal pulls of the stress at a given point in the elastic cylinder and let p, q and r be the principal stretches of the strain at the same point. Then for any isotropic substance which conforms to Hooke's Law we have:

$$P = ap + bq + br \qquad (1)$$

$$Q = bp + aq + br \qquad (2)$$

$$R = bp + bq + ar \qquad (3)$$

in which a and b are constants.

STATEMENT OF THE PROBLEM.

A long tube of inside radius $\rho_{\prime}$ and outside radius $\rho_{\prime\prime}$ is subjected to an internal pressure P', an external pressure P'' and a total end thrust T. The elastic constants a and b of the material are given and it is required to determine the stress and strain at each point throughout the material.

GENERAL CONSIDERATIONS AND NOTATION.

The principal axes of the stress (and strain) are, at each point, *in the direction of the radius of the cylinder, in the direction of the axis of the cylinder, and tangential (perpendicular to the first two)* respectively. This is evident from considerations of symmetry, the tube being endless.

Consider an element of the material of the tube, distant x from the axis of the tube. Let P be the radial pull (force per unit area), Q the longitudinal pull, and R the tangential pull at the element; and let p, q, and r be the corresponding stretches at the element. It is evident that every portion of the tube is equally elongated so that q has the same value throughout the material of the tube. Considerations of symmetry show that p, r, P and R are the same all along the tube at a given distance from the axis, that is they are functions of x alone. It will appear later that Q has the some value throughout the material of the tube.

BASIS OF SOLUTION.

The problem is to be solved with the help of the equations (1), (2) and (3), using the conditions of internal equilibrium, the condition of continuity of strain (and stress), and the boundary conditions which arise from prescribed external and internal pressure and prescribed total end thrust.

THE CONDITION OF EQUILIBRIUM.

The condition of equilibrium (internal) gives

$$Pdx + xdP = Rdx \qquad (4)$$

THE CONDITION OF CONTINUITY.

The condition of continuity gives

$$pdx = rdx + xdr \qquad (5)$$

INTERNAL AND EXTERNAL BOUNDARY CONDITIONS.

At the inner surface of the tube the radial pull P is equal to the inside pressure P' and at the outer surface of the tube the radial pull P is equal to the outside pressure P''.

Remark: Considered as pulls P' and P'' are of course *negative.*

TOTAL END THRUST.

The distribution of the end thrust over the section of the tube (value of Q at each point) will be determined later. This determination is subject to the condition of a prescribed total end thrust T. The relation between T and Q is:

$$T = 2\pi \int_{\rho_{,}}^{\rho_{,,}} Qx dx \tag{6}$$

SOLUTION OF EQUATION (4), ETC.

Let

$$Px = Y \text{ or } P = \frac{Y}{x}.$$

Then from equation (4)

$$R = \frac{dY}{dx}.$$

Substituting these values of P and R and the value of p from equation (5) in equations (1), (2) and (3), we have:

$$\frac{Y}{x} = a\left(r + x\frac{dr}{dx}\right) + bq + br \tag{7}$$

$$Q = b\left(r + x\frac{dr}{dx}\right) + aq + br \tag{8}$$

$$\frac{dY}{dx} = b\left(r + x\frac{dr}{dx}\right) + bq + ar. \tag{9}$$

Substituting the value of Q from (8) in (6) we have after reducing

$$T = 2\pi \int_{\rho_{,}}^{\rho_{,,}} \frac{d(brx^2)}{dx} \cdot dx + 2\pi \int_{\rho_{,}}^{\rho_{,,}} aqx dx$$

or, since b, a and q are constant:

$$T = 2\pi b(r_{,,}\rho_{,,}^2 - r_{,}\rho_{,}^2) + \pi aq(\rho_{,,}^2 - \rho_{,}^2) \tag{10}$$

in which $\rho_{\prime\prime}$ and $\rho_{\prime}$ are the outer and inner radii of the tube, and $r_{\prime\prime}$ and $r_{\prime}$ are the values of the stretch r (circumferential) at outside and inside of tube respectively.

Remark: Equation (10) shows that the length of the tube will change (q other than zero) when T is zero.

SOLUTION OF EQUATIONS (7), (8), AND (9), ETC.

Differentiating equation (7) with respect to x and substituting the resulting value of $\frac{dY}{dx}$ in equation (9) we have

$$x\frac{d^2r}{dx^2} + 3\frac{dr}{dx} = 0, \tag{11}$$

whence by integration

$$r = C' - \frac{C''}{2x^2} \tag{12}$$

in which C' and C'' are constants.

Substituting this value of r in equation (5) we have

$$p = C' + \frac{C''}{2x^2} \tag{13}$$

Substituting the value of r from (12) in (10) we have

$$T = 2\pi bC'(\rho_{\prime\prime}^2 - \rho_{\prime}^2) + \pi aq(\rho_{\prime\prime}^2 - \rho_{\prime}^2)$$

or

$$q = \frac{T}{\pi a(\rho_{\prime\prime}^2 - \rho_{\prime}^2)} - \frac{2bC'}{a} \tag{14}$$

Remark: Equations (12), (13) and (14) contain the general solution of our problem (since P, Q and R are given by equations (1), (2) and (3) in terms of p, q and r) and it only remains to determine the constants C' and C''. For this purpose we need to substitute r, p and q from (12), (13) and (14) in (1). The resulting expression for P is general. In this expression put $P = P'$ and $x = \rho_{\prime}$ and then put $P = P''$ and $x = \rho_{\prime\prime}$. The two equations thus obtained may be solved for C' and C''. In this manner we obtain

$$C'' = \frac{2(P'-P'')\rho_{\prime}^{2}\rho_{\prime\prime}^{2}}{(a-b)(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})}, \tag{15}$$

and

$$C' = \frac{\pi a(P'\rho_{\prime}^{2}-P''\rho_{\prime\prime}^{2})+bT}{\pi a(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})\left(a+b-\frac{2b^{2}}{a}\right)}. \tag{16}$$

FINAL RESULTS.

Using equations (15) and (16) in conjunction with (12) (13) and (14) we have:

$$p = \frac{\pi a(P''\rho_{\prime\prime}^{2}-P'\rho_{\prime}^{2})-bT}{\pi a(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})\left(a+b-\frac{2b^{2}}{a}\right)} + \frac{(P'-P'')\rho_{\prime}^{2}\rho_{\prime\prime}^{2}}{x^{2}(a-b)(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})} \tag{17}$$

$$q = \frac{T}{\pi a(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})} - \frac{2\pi ab(P''\rho_{\prime\prime}^{2}-P'\rho_{\prime}^{2})-b^{2}T}{\pi a^{2}(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})\left(a+b-\frac{2b^{2}}{a}\right)} \tag{18}$$

$$r = \frac{\pi a(P''\rho_{\prime\prime}^{2}-P'\rho_{\prime}^{2})-bT}{\pi a(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})\left(a+b-\frac{2b^{2}}{a}\right)} - \frac{(P'-P'')\rho_{\prime}^{2}\rho_{\prime\prime}^{2}}{x^{2}(a-b)(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})} \tag{19}$$

Substituting these values of p, q and r in equations (1), (2) and (3) we have:

$$P = \frac{P''\rho_{\prime\prime}^{2}-P'\rho_{\prime}^{2}}{\rho_{\prime\prime}^{2}-\rho_{\prime}^{2}} + \frac{(P'-P'')\rho_{\prime}^{2}\rho_{\prime\prime}^{2}}{x^{2}(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})} \tag{20}$$

$$Q = \frac{T}{\pi(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})} \tag{21}$$

$$R = \frac{P''\rho_{\prime\prime}^{2}-P'\rho_{\prime}^{2}}{\rho_{\prime\prime}^{2}-\rho_{\prime}^{2}} - \frac{(P'-P'')\rho_{\prime}^{2}\rho_{\prime\prime}^{2}}{x^{2}(\rho_{\prime\prime}^{2}-\rho_{\prime}^{2})} \tag{22}$$

These equations (17) to (22) involve all the formulas hitherto published in connection with the problem under discussion.

A UNIVERSAL LAMP–SUPPORT FOR THE PHOTOMETRY OF GLOW–LAMPS.

By Clayton H. Sharp.

THE problem of determining the mean spherical intensity of glow-lamps by a method at once exact, simple and convenient has never received a satisfactory solution. The methods which are in use require a long series of measurements, and the greater the accuracy which is demanded, the more numerous (within limits, of course) must the measurements be. On account of this difficulty the common technical practice has been to deduce the mean spherical intensity from the intensity measured in a certain direction, using a reduction-factor which holds, of course, only for the type of lamp in question, and which represents a mean value for this type. While this method yields average results which are satisfactory, it cannot be maintained that the value so obtained for any single lamp can make claim to any large degree of trustworthiness.

In order to make the matter entirely clear let us consider for a moment the theory on which the determination of mean spherical intensity is based. Suppose the source of light to be placed at the center of a sphere of unit radius. The mean spherical intensity is equal to the total luminous flux Φ through this sphere divided by the area of the sphere, that is $I_m = \frac{1}{4\pi}\Phi$. If we take a zone of width $\delta\theta$ cut on the sphere by two planes perpendicular to the axis of the lamp, and if this zone lies θ radins from the intersection of the sphere by the axis of the lamp, we have for the flux through this zone,

$$\delta\Phi = 2\pi I_\theta \sin\theta\delta\theta$$

in which I_θ is the mean luminous intensity in the direction of the entire zone; that is, it is the mean intensity for the polar-distance θ.

It follows, therefore, that

$$\Phi = 2\pi \sum_{\theta=0}^{\theta=\pi} I_\theta \sin\theta\delta\theta$$

and

$$I_m = \tfrac{1}{2} \sum_{\theta=0}^{\theta=\pi} I_\theta \sin\theta\delta\theta.$$

The value of I_m can therefore be determined as soon as one knows the various values of I_θ. With the ordinary method, however, the determination of I_θ is a rather tedious process, since one must make for each value of θ a series of measurements along the entire length of the zone.

As regards the determination of the mean horizontal intensity of a glow-lamp, the state of affairs is quite different, since there are two perfectly good methods for determining the quantity by means of a single photometer setting. These are Crova's method of the rotating lamp, and the method of the two rotating mirrors used by Dr. Liebenthal[1] in the Physikalische-technische Reichsanstalt in Charlottenburg. The accuracy of the first method has been amply proved by Matthews[2] who obtained concordant results for measurements made by this method and also by the ordinary method of the horizontal distribution of an old-fashioned Maxim lamp giving a very irregular distribution of intensity.

It naturally occurs to one to apply this simple method to the measurement of mean spherical intensity by inclining the rotating lamp at various angles with respect to the photometer and so, by means of one photometer-setting, to determine the mean intensity in the zone in question.[3] The mechanical difficulties which stand in the way of this procedure need scarcely to be pointed out. It would be much easier to place a mirror in the direction in question and, by means of it, to send the beam of light from the rotating lamp along the photometer-bar. The amplification of this idea has lead me to the design of the instrument which is here to be described. Professor Wiener has very kindly had the instrument constructed after my drawings for the Physical Institute of the University of Leipzig in the instrument works of Dr. Emil Stoehorer & Son, in Leipzig.

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[1] Zeitschrift für Instrumentenkunde, Vol. XIX., p. 193, 1899.

[2] PHYSICAL REVIEW, Vol. VI., p. 55, 1898.

[3] Since writing the above I have been informed that this method is actually in use in a large lamp factory in America.

The lamp is fixed to a vertical axis and is turned at the rate of four or five revolutions per second by means of a cord running on a pulley at the lower end of the axis. The current is brought in by means of the two carefully insulated brushes *K*, *K′*. The light sent out by the lamp in two directions 180° apart is received by two

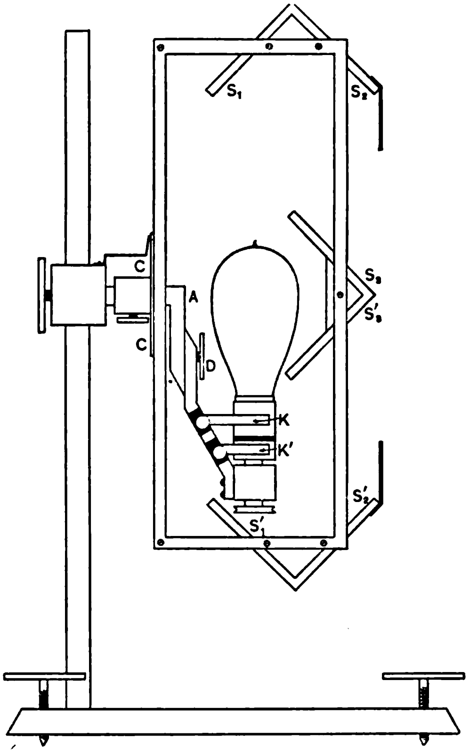

Fig. 1.

sets of mirrors S_1, S_2, S_3 and S_1', S_2', S_3', and, after triple reflection, is sent along the photometer-bar. These mirrors are set between two rectangular frames which can be adjusted at any angular position on a horizontal axis *A*, the prolongation of which passes through the mid-point of the lamp filament. The inclination of the frames with respect to the vertical lines can be read from the divided circle *C* to which they are affixed. The lower part of the lamp-holder is adjustable in a vertical direction so that the filament of a lamp of any size can be brought into the proper position with respect to the axis A. D is the screw by means of which this part is clamped in place.

The whole arrangement is shown in Fig. 2. For the sake of convenience in driving the lamp, a special counter-shaft is attached to the instrument as Fig. 2 shows. A steel shaft is fixed in a vertical position in a metal plate which can be fastened by a clamping screw to either side of the base of the instrument, and which is slotted so that the driving cord can be tightened. A brass tube fitted with steel ring bearings runs on this shaft and carries a large pulley for the cord from the motor to a small one for that running on the lamp-pulley. The position of both pulleys on the tube is adjustable according to the position of the motor and the height of the lamp-pulley above the base of the instrument. The other, less important details of the construction of the apparatus will be understood directly from a study of the figures.

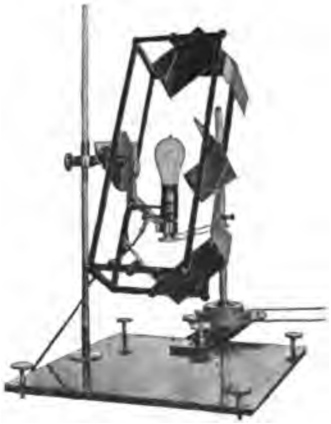

Fig. 2.

It is at once clear that when one looks from where the photometer is placed into the mirror S_3 and S_3' one sees two images of diametrically opposite sides of the lamp. It follows therefore that when the lamp is rotating and the mirrors are set at an angle θ, one gets as the result of a photometer setting the sum of the mean intensities in two zones θ and $\theta + \pi$, diminished, of course, by the amount which is lost by absorption in the mirrors. Therefore, one needs only to make a sufficient number of measurements between $\theta = 0$ and $\theta = 90°$ to get all the data required for the computation of the mean spherical intensity of the lamp.

The results of such a set of measurements made on a lamp with a single-loop filament are shown plotted in polar coördinates in Fig. 3.

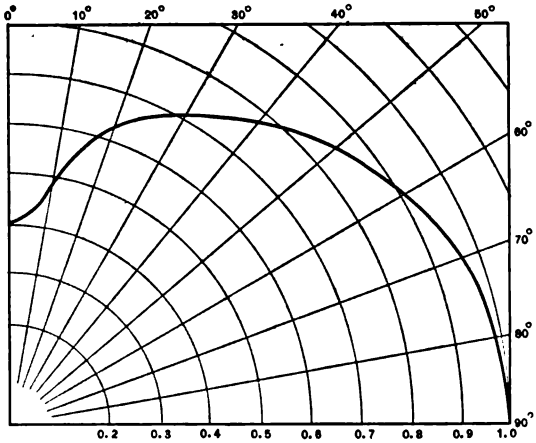

Fig. 3.

From this curve one could deduce the mean spherical intensity by means of the purely graphical method of Rousseau[1] or the method of Liebenthal[2] which is, in principle, quite like that of Rousseau.

If one desires to dispense with graphical methods, one can compute the value of I_m as follows: The unit sphere is divided into zones of such width that one may assume that the mean intensity for the whole width of the zone is given, to the required degree of approximation, by a measurement made in the middle of the zone. Then, for the zone whose boundaries are $\theta = 1$ and $\theta = 2$,

$$\delta\Phi_{1,2'} = 2\pi I_\theta \int_1^2 \sin\theta d\theta = 2\pi I_\theta (\cos\theta_1 - \cos\theta_2).$$

Therefore,

$$I_m = \tfrac{1}{2}\{(\cos\theta_1 - \cos\theta_2)\, I_{1,2'} + (\cos\theta_2 - \cos\theta_3)\, I_{2,3'} + \cdots\}\,{}^{\theta=\pi}_{\theta=0}.$$

In case $I_{1.2'}$, $I_{2.3'}$, etc., refer to measurements made with this apparatus, we have,

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[1] Palaz. Photometry, Petterson's tr., p. 20.

[2] Elektrotechnische Zeitschrift, Vol. X., p. 337, 1889.

$$I_m = \{(\cos\theta_1 - \cos\theta_2)I_{1,2}, + (\cos\theta_2 - \cos\theta_3)I_{2,3}, + \cdots\}_{\theta=0}^{\theta=\frac{1}{2}\pi}.$$

In order to make the numerical computation of I_m more convenient I have computed tables for four different cases, namely for zones 10°, 15°, 18° and 30° wide, beginning and ending the measurements in each case at 0° and at 90° respectively. In these tables, θ is the direction in which the intensity is measured, and κ is the constant with which I_θ is to be multiplied. Designating by r the coefficient of reflection of the mirrors, we have as a working formula,

$$I_m = r\{\kappa_1 I_1 + \kappa_2 I_2 + \cdots\}_{\theta=0}^{\theta=90^\circ}.$$

A 30° zones.		B 18° zones.		C 15° zones.		D 10° zones.	
θ	κ	θ	κ	θ	κ	θ	κ
0°	0.034	0°	0.012	0°	0.009	0°	0.004
30	.259	18	.097	15	.067	10	.030
60	.448	36	.184	30	.131	20	.060
90	.259	54	.253	45	.184	30	.087
		72	.298	60	.226	40	.112
		90	.156	75	.253	50	.133
				90	.1305	60	.151
						70	.164
						80	.172
						90	.087

In order to get at a basis for comparison I have computed I_m for the lamp mentioned above with the help of tables A, B, C and D, that is from 4, 6, 7 and 10 measurements, with the following results, in which I_m is exposed as a fractional part of the mean horizontal intensity.

Width of zones.	Number of Measurements.	I_m
30°	4	0.868
18	6	.863
15	7	.864
10	10	.8683

From the above results one may conclude that the result obtained from measurements in only four directions can not well, in any similar case, be far wrong. If we assume that the ordinary method had been followed and that measurements had been made in twelve different positions 30° apart in each zone, the number of measurements would increase from 4 to 74.

The speed of rotation of the lamp is without influence on the result so long as it is not so slow that too strong a flickering is noticed on the photometer disc. If the speed is too high, the filament may be thrown from its position and the lamp endangered.

As to the length of the path which the light must traverse within the apparatus, we need only to notice that this is equal to the distance between the apexes of the two angles which the mirrors S_1, S_2 and S_1', S_2' enclose; that is taking the apex of the angle between S_3 and S_3' as the line of reference. The equivalent length of the path in the glass of the mirrors can be computed, assuming the value 1.5 for the index of refraction of the glass.

In order to determine the reflecting power of the mirrors, the mean horizontal intensity is measured with and without the mirrors, that is to say, one measures the rotating lamp with the mirror-frame horizontal, and again with the front mirrors S_3 and S_3' removed. The reflecting power can at once be computed from these data. For the present instrument this is 71.0%, that is, 10.8% per mirror.

It may be pointed out that this lamp standard may be used for making all the other usual measurements on glow-lamps. To measure the vertical distribution of intensity, one set of mirrors is covered over and the stationary lamp is viewed through the other set in the various positions between 0° and 180°. In order to make it possible to measure the horizontal distribution of intensity, the little pulley beneath the lamp is divided in 10° divisions. One can therefore properly designate this apparatus as a universal lamp-support for glow-lamp photometry.

I wish also to point out the fact that a single set of mirrors like those described above would be employed in the photometry of arc-lamps as Krüss[1] has already done in gas-flame photometry. As a substitute for the ordinary cumbersome apparatus, this arrangement would possess decided advantages. Whether the principle of the rotating lamp would not also be advantageouly employed in arc-lamp photometry, would be decided by a simple experiment which the means at present at my disposal do not permit me to carry out.

Physical Institute of the University of Leipzig, June, 1900.

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[1] Journal für Gasbeleuchtung, Vol. 41, p. 4, 1898.

NOTES.

Thomas Preston.—Thomas Preston, whose death occurred in Dublin, March 7, 1900, was born in County Armagh, Ireland, in 1860. He was a graduate of Trinity College, Dublin, and also of the Royal University of Ireland, having received from the latter institution the degree of Doctor of Science. At the time of his death he held a fellowship in the Royal University. He was elected a Fellow of the Royal Society in 1898.

It is probably through his "Theory of Light" that Preston is most widely known. This well-known text-book, which first appeared in 1890, was from the first received with universal favor. During the past ten years it has probably been used as an introduction to the study of optical theory by the great majority of advanced students of physics, both in Great Britain and America. Preston's "Theory of Heat" appeared in 1895 and has also been very favorably received.

Since the completion of his two text-books Preston had devoted his time more largely than before to scientific investigation. His earlier work had to do with problems connected with the theoretical side of the subjects of light and heat. We may mention, for example, his papers on "The Continuity of Isothermal Transformation from the Liquid to the Gaseous State,"[1] and the "General Extension of Fourier's Theorem."[2] But his more recent work, and also his most important work, namely, that dealing with the Zeemann effect, was chiefly experimental. Preston was the first to obtain photographs of the Zeemann doublets and triplets suitable for enlargement and lithographic reproduction. All who have followed the development of the Zeemann phenomena will remember with pleasure the plate accompanying Preston's first article on the subject in 1898.[3] In this article he called attention also, for the first time, to the more complicated modifications of the spectral lines produced by a magnetic field, namely, the quartet and sextet types.[4]

For his work in connection with the Zeemann effect Preston was recently awarded the Boyle medal of the Royal Dublin Society. E. M.

American Association for the Advancement of Science.—The forty-ninth meeting of the association was held in New York City, June 23–30, 1900. The early date of the meeting was adopted in order to make it possible for those contemplating a visit to the Paris Exposition to attend the meeting of the American Association before sailing. The same consideration was influential in deciding the place of the meeting. If we may judge by the attendance, which was larger than the average of recent years, the temporary change in the time of the meeting was received with general satisfaction. The fact that the date conflicted with that of the commencement exercises at several of the New England universities is however much to be regretted. In accordance with previous custom the next meeting will be held in the month of August.

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[1] Philosophical Magazine, 42, 231, 1896.

[2] *Ibid.*, 43, 281, 1897.

[3] Philosophical Magazine, 45, 325, 1898.

[4] Indications of a quartet type had, I believe, been previously obtained by Cornu.

For Section B the meeting was especially significant on account of the coöperation of the American Physical Society. The sessions of Section B and of the Physical Society were held on alternate days. But although the programs were kept separate, there was little else to distinguish the two. The holding of such a joint meeting, for the first time in the history of the Section B, was very properly regarded as an experiment. It is to be hoped that its undoubted success will lead to the permanent enrollment of the Physical Society among the numerous organizations already affiliated with the Association. The great increase in the scientific activity of our American physicists has now made the existence of two societies not only possible, but desirable. Certainly the coöperation of the two societies can scarcely fail to be of mutual benefit.

An important step as regards the general policy of the Association was taken by the Council in deciding to send *Science* free of charge during the coming year to all members of the Association. In a sense, *Science* will become the official organ of the American Association, in which its announcements and proceedings will be promptly published. It is hoped that this action will tend to increase the membership of the Association and increase its usefulness. If the results justify this expectation the policy will be continued. The annual volume of proceedings will be published as usual.

The program, while not so crowded as has often been the case in recent years, contained as many papers as could be properly treated in the time available. A brief discussion was possible in all cases, and the feeling of hurry, so conspicuous at the Boston meeting, was happily absent. But still more time for discussion is much to be desired. Sessions were held up to the close of the last day of the meeting, with the attendance almost as large at the last session as at the first.

In addition to the address of Vice-President Merritt on Kathode Rays and some Related Phenomena, the program of Section B included twenty-five papers, a list of which follows:

Temperature Effects on a Tuning-fork, John O. Reed and E. C. Woodruff; Some Lecture-room Methods in the Elementary Theory of Elasticity, W. S. Franklin; Velocity of Ions in the Electric Arc, C. D. Child; An Observation Upon the Surface Viscosity of Mercury, William

A. Anthony ; Note on the Flow of Energy Around a Conducting Screen Near a Current Sheet, W. S. Franklin ; A new Theory of the Electromagnetic Rotation of Light, Reginald A. Fessenden ; Application of the Schlieren Methods to the Microscope, R. W. Wood ; Photography of Sound Waves, R. W. Wood ; Copper Saving in the Joint Transmission of Direct and Alternating Currents, Frederick Bedell ; On the Theory of the Coherer, K. E. Guthe and A. Trowbridge ; The Surface-tension of Water Above 100° C., Charles Tobias Knipp ; The Effect Upon the Persistence of Vision of Exposing the Eye to Light of Various Wave-Lengths, Frank Allen ; A Method of Measuring Surface-Tension, James S. Stevens ; Notes on the Use of the Capillary Electrometer, A. D. Cole ; On the Visible Radiation From Carbon, Edward L. Nichols ; On the Development of Kathode Rays by Ultra-Violet Light, Ernest Merritt and O. M. Stewart ; Diffusion of Light, Edward Atkinson ; A Method for Measuring Surface Tension, S. Stevens ; The Percentage Bridge and its Applications, Herschel C. Parker ; On the Energy of the Cathode Rays, Walter G. Cady ; Power Curves Drawn from Alternating Current Circuits, Edward B. Rosa ; The Nature of Sound, E. M. Souveille ; Photographs of Electric Phenomena, T. B. Kinraide ; Circuit-Breakers and Induction Coils, Reginald A. Fessenden ; Some Experiments in Electric Lighting, Reginald A. Fessenden.

The following papers, fifteen in number, were read before the Physical Society :

The Anomalous Dispersion of Cyanine, R. W. Wood and C. E. Magnusson ; The Optical Properties of Thin Carbon Films, R. W. Wood ; A Mica Echelon Grating, R. W. Wood ; False Spectra from the Rowland Concave Grating, Theodore Lyman ; The Zeemann Effect, H. M. Reese ; Electrical Absorption in Condensers, L. M. Potts ; Polarization of the Solar Corona, N. E. Dorsey ; A Comparison of the Brightness of Prisms and Gratings in Spectrum Work, N. E. Dorsey ; Preliminary Tests on the Efficiency of the Acetylene Flame as a Means of Illumination, E. L. Nichols ; On the Characteristic Equation of Water, A. D. Risteen ; Some Simple Apparatus for the Study of Aerial Vibrations, F. L. Tufts ; The Use of the Bicycle Wheel in Illustrating the Principles of the Gyroscope, C. T. Knipp ; The Effect of Solid Dielectrics in the Production of Electrical Sparks, W. J. Humphreys ; Magnetic Observations on the Day of Eclipse, May 28, 1900, L. A. Bauer ; A Method of Obtaining Brilliancy of Concave Grating Spectra, W. J. Humphreys.

The next meeeting will be held in Denver, August 24–31, 1901, under the presidency of Chas. S. Minot. The officers of Section B for the Denver meeting are : Vice-President, D. B. Brace, Lincoln, Neb.; Secretaty, J. O. Reed, Ann Arbor, Mich.

NEW BOOKS.

Optical Activity and Chemical Composition. By DR. H. LANDOLT, translated by JOHN MCCRAE, Ph.D. London and New York, Whittaker & Co., 1899. 16mo. Pp. 158.

In the original German this small book constitutes the eighth chapter of the first volume of Graham-Otto's "Lehrbuch der Chemie." The translator has greatly enhanced its value by introducing notes on several important investigations of which the results have appeared in the literature since the middle of the year 1897.

The main object of the book is to point out, by means of numerous illustrative examples, the connection between the phenomenon of circular polarization, as manifested by many organic compounds, and certain theories—chiefly stereochemical—as to the structure of the molecules of these substances. In order to fully appreciate the validity of the course of reasoning pursued one should read the work in its entirety, for much of the evidence is cumulative. It may be said, moreover, that the literary style is so simple and direct as to make it scarcely an effort to read. A more available summation of the results of recent research in this line is hardly conceivable.

An idea of the uses which the book is fitted to serve may perhaps be best gained by a few statements of detail. Apparatus for measuring circular polarization is not described. A list of all known circular-polarizing crystals is given, together with the crystal-symmetry of each and the amount of rotation of the D line. The 700 or more substances which are optically active in solution or in fluid condition are classified in 30 groups. The names of nearly two hundred of these substances are mentioned in the statement of the thirty groups, but no data are here given as to the *amount* of rotation. The meaning of specific rotation is explained and illustrated by examples. Many cases of the variation of specific rotation are cited. These variations are attributable to change of concentration, to difference in temperature, to change of solvent, to inactive substances in the solution, or to changes which take place on allowing the solution to stand for a time. The influence of inactive substances in the solution with the active is in some cases so remarkable that it may not be amiss to quote one example here. Ammonium molybdate, under conditions carefully stated in the text, causes the rotary power of malic acid to increase from 0.2° to 5.32°, but on further addition of the molybdate the amount of rotation decreases, passes through zero, and then goes up to a maximum of 72.8° in the opposite sense! When the last point is reached there are two molecules of malic acid in the solution to one of molybdate.

The second (and largest) part of the book deals with the "connection between the Rotatory Power and the Chemical Composition of Carbon Compounds." This is in the main stereochemistry, dealing, among other things, with the possible number of active isomers and the methods of resolving "racemic" compounds into their active components or "antipodes." Many of the facts adduced are extremely interesting merely as matters of general information.

Part three is devoted to the "Connection between Degree of Rotation and Chemical Constitution." One feature of this final chapter is the attempt to show a dependence of the rotatory power on the masses of the four radicles which are united to the asymmetric carbon atom.

In conlusion, the book may be commended as an unusually clear, suggestive, and up-to-date exposition of the subject in hand.

A. C. GILL.

A Brief History of Mathematics. An Authorized Translation of Dr. Karl Fink's Geschichte der Elementar-Mathematik. By W. W. BEMAN and D. E. SMITH. Chicago, The Open Court Publishing Company, 1900. Pp. xii + 333.

While there is no extensive history of mathematics in the English language, such as Marie's in French and Cantor's in German, yet a number of brief histories have appeared in recent years. The principal ones of these are: Ball, A Short History of Mathematics, 1888; Cajori, A History of Mathematics, 1895; Cajori, A History of Elementary Mathematics, 1896. The present work differs from all of these in arrangement, giving the history of each of the elementary subjects, arithmetic, algebra, geometry and trigonometry separately. It differs still more in the choice of matter, omitting biography entirely and referring the reader to a brief table in the appendix or to the encyclopedias.

Fink's history is a somewhat hasty compilation from the larger works. Cantor points out a large number of errors in his review published in *Zeitschrift für Mathematik und Physik*, vol. 36, 1891, pp. 75–77. The translators have corrected a number of unquestionable errors and added occasional references. They have also rewritten the biographical notes, so that the translated work seems much more valuable to the teacher than the original. While the style is not a fluent one yet the work is quite readable and it seems to be unusually free from typographical errors.

G. A. MILLER.

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THE

PHYSICAL REVIEW.

THE EFFECT OF TEMPERATURE, OF COLLOIDAL FERRIC HYDRATE, AND OF A MAGNETIC FIELD ON THE HYDROLYSIS OF FERRIC CHLORIDE.

By H. M. Goodwin and Frederick W. Grover.

THE present investigation is a continuation of one published by one of us[1] in 1896, on the changes which take place in a neutral ferric chloride solution when greatly diluted. Such solutions when first prepared of a concentration of about 0.001 normal are nearly colorless but rapidly change to a deep reddish brown, and this gradual color reaction is accompanied by a corresponding increase in the electrical conductivity of the solution. In the paper cited, the hydrolytic reaction giving rise to this phenomena was studied by following the changes in the conductivity of the solution, and the main results of the investigation may be summarized as follows :

1. The electrical conductivity of dilute ferric chloride solutions increases with the time.

2. The rate of increase, increases very rapidly with the dilution.

3. The increase in conductivity does not begin at once on dilution, but only after the lapse of a certain time.

4. This initial lag or time required to start the reaction increases rapidly with the concentration, being for example, one minute for a

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[1] Goodwin, Phys. Rev., 9, 251, 1896.

0.0006 normal solution, 15 minutes for a 0.0012 normal solution and 45 minutes for a 0.0024 normal solution.

5. The time required for a solution to reach a state of equilibrium increases enormously with the dilution,—a 0.0001 normal solution requiring for example only three hours while a solution six times as strong requires over a week.

6. The hydrolytic dissociation was computed and found to be very large in dilute solutions.

The explanation offered for this peculiar behavior was the gradual formation of Graham's colloidal ferric hydrate from the electrolytically hydrolyzed ferric chloride, according to the reaction

$$x\,\overset{+}{\mathrm{Fe}}\overset{+}{\mathrm{O}}\mathrm{H} + x\,2\overset{-}{\mathrm{O}}\mathrm{H} = (\mathrm{FeO_3H_3})x.$$

The formation of colloidal hydrate accounted for the color changes observed, while the removal of the $\overset{+}{\mathrm{Fe}}\overset{+}{\mathrm{O}}\mathrm{H}$ ions from the solution displaced the hydrolytic equilibrium thus causing more chloride to be hydrolyzed with accompanying increase of conductivity. The initial lag observed in the reaction and the remarkable acceleration of its rate when once started was explained on the hypothesis that the rate of formation of the colloidal hydrate was dependent on the presence of a certain amount of the colloid itself, analogous to the rate of crystallization being a function of the number of crystallizing nuclei around which the crystals can form.

The present investigation was undertaken first to study the effect of temperature on the rate of the reaction, the former measurements all having been made at 25° ; second to test the above hypothesis regarding the cause of the accelerated rate of the reaction after the lapse of a certain time ; and third to see if any effect could be observed on the rate of the reaction of a strong magnetic field.

APPARATUS AND METHOD.

The conductivity measurements were made by the familiar Kohlrausch method. The solutions were diluted as in the previous investigation by adding with a pipette a few cubic centimeters of a strong solution of neutral ferric hydrate to a previously weighed amount of distilled water brought to the proper temperature of the experiment. This water was contained in 250 cc. or 500 cc. thoroughly steamed wide necked glass bottles which served also as measuring cells. It was found very advantageous to use "plunge electrodes" in cells of this kind (Kohlrausch & Holborn's Leitvermōgen der Electrolyte, p. 19) instead of the usual Arrhenius type previously employed. The considerable care and patience required in the preparation of these electrodes was more than compensated for by their absolute constancy and particularly the independence of their "constant" from the position of the electrodes in the bottle. Electrodes about one square centimeter in area platinized with a Lummer-Kurlbaum solution gave an excellent minimum for most dilutions investigated. The solutions were investigated at 0°, 15° and 25° C. The first temperature could of course be maintained indefinitely constant by a large ice bath packed in hair felt in which the bottles were completely immersed. A temperature of 15° C. was maintained constant to within 0.1° during the day and 0.2°–0.3° during the night by means of a large tank of water, insulated from outside temperature changes by being packed in a box with six inches of finely divided cork and covered with four inches of air felt. A temperature of 25° was maintained indefinitely constant to a few hundredths of a degree in the large 5x3x3 foot automatically regulated and electrically heated thermostat of the laboratory.

SOLUTIONS.

Ferric Chloride.—The stock ferric chloride solution used in the following experiments was made from a preparation of sublimed ferric chloride from Eimer and Amend. A quantity weighed from a weighing tube was made up to one gram-equivalent in a liter and then analyzed for chlorine. Three analyses gave an equivalent normality of 0.966, 1.001 and 0.999, the mean being 0.9987. This solution had an equivalent conductivity of 43.6 at 24.96° C. after standing about a week. This stock solution was then diluted ten or a hundred times for the secondary stock solutions used in the subsequent great solutions.

Graham's Colloidal Ferric Hydrate.—A solution of Graham's soluble colloidal hydrate was prepared by treating a saturated solution of pure (unsublimed) ferric chloride with ammonium carbonate with constant stirring, until the light colored precipitate which formed just refused to dissolve on further stirring. The color of the solution rapidly turned to an intense garnet color. It was then subjected to dialysis for thirty-two days. The concentration of the resulting solution was determined by evaporating to dryness and igniting portions of 10 cc. each and weighing the ferric oxide. The mean of two determinations showed it to be 0.3085-equivalent normal with respect to $Fe(OH)_3$. It still contained some ammonium salts as shown by a slight visible volatilization on first heating and from the fact that the solution had a considerable conductivity which had it contained only colloidal hydrate should not be the case. At 18.1° its specific conductivity was 0.00304, which corresponds to that of an ammonium chloride solution about 0.02 normal.

Experimental Results.

In the following Tables are tabulated the results obtained at 25°, 15.5°, and 0°, with solutions of the indicated concentrations, C always being expressed in *gram equivalents* in a litre. The conductivities are those observed at the indicated time intervals elapsing from the time of dilution. The values are expressed as equivalent conductivities in reciprocal Siemen's units for convenience of comparison with former measurements.

Table I.

$C = 0.000150$. $t = 24.92°$. *Diluted from* $C = 0.0909$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
1 40	295.3	37 25	363.6
2 35	310.2	58 50	364.8
4 25	323.1	1 28 45	366.9
6 15	326.8	2 1 15	366.9
7 30	328.3	....	....
14 35	350.9	20 19	364.8
18 35	353.0	21 21	364.8
27 35	358.7		

TABLE II.

$C = 0.0004545$. $t = 25.12^\circ$. *Diluted from* $C = 0.0909$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
2 0	260.2	13 10	319.8
2 30	267.8	14 45	322.1
3 0	272.5	18 10	326.8
3 45	280.3	21 45	330.2
4 20	284.9	27 40	333.8
4 50	287.8	33 40	335.6
5 30	295.7	37 35	338.6
6 0	297.9	1 42 0	351.1
6 35	300.6	2 10 0	352.4
7 5	302.2	4 50 0	352.4
8 30	306.6	....	....
10 40	315.5	27	351.
12 15	318.7		

TABLE III.

$C = 0.000906$. $t = 25^\circ.09$. *Diluted from* $C = 0.0909$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
2 40	214.6	12 30	277.0
3 20	223.5	15 0	283.3
3 55	229.1	21 0	294.8
4 40	236.8	24 30	300.1
5 30	242.5	32 15	309.3
1 20	252.2	45 30	318.7
6 50	255.6	59 30	326.4
7 40	259.4	1 15 30	331.7
8 20	263.0	1 39 30	337.7
9 40	266.8	2 51 0	344.6
10 30	270.7		
11 0	272.4		
12 0	275.4		

Table IV.

$C = 0.001335$. $t = 25°.04$. *Diluted from* $C = 0.0909$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent Conductivity.
2 0	200.6	23 45	274.2
2 55	209.1	27 45	279.9
3 25	212.6	44 0	294.0
4 25	222.9	50 0	297.7
5 15	228.7	1 30 0	316.6
6 45	234.5	2 6 0	325.1
9 45	246.3	2 24 0	328.1
14 30	259.2	2 35 0	329.6
17 5	266.3		

Table V.

$C = 0.00360$. $t = 24°.96$. *Diluted from* $C = 0.999$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
1 45	183.5	41 20	212.3
4 20	18[illegible].3	56 0	222.2
5 0	185.3	1 9 0	229.7
7 55	186.9	1 26 20	239.3
8 45	187.6	1 38 0	243.0
10 10	188.2	1 51 0	248.0
13 0	190.3	2 11 0	254.5
16 0	193.2	2 36 0	261.6
23 15	198.7	2 49 0	264.5
26 25	201.1	2 58 0	266.4
34 45	207.4	3 3	268.1

Table VI.

$C = 0.00750$. $t = 24.87°$. *Diluted from* $C = 0.999$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
4 0	171.9	42	176.0
8 0	172.8	56	178.0
11 30	172.8	1 2	180.2
15 0	173.4	3 7	206.7
17 0	173.2	3 9	207.5
20 0	173.7	3 14	208.9
23 0	174.6	3 20	210.0
27 0	174.3	....	
30 0	175.6	21	260.
35 0	175.3		

TABLE VII.

$C = 0.000150$. $t = 15°.5$. *Diluted from* $C = 0.0100$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
3 15	249.3	18 20	281.7
4 15	255.3	19 45	283.6
5 0	259.0	26 10	287.1
6 45	264.1	51 0	292.8
8 50	269.6	1 1 0	294.7
10 10	273.7	1 31 0	296.5
11 40	274.8	2 6 0	297.7
12 10	276.0	....	....
14 10	279.3	18 23	301.6

TABLE VIII.

$C = 0.00033$. $t = 15°.4$. *Diluted from* $C = 0.100$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
2 35	207.4	22 0	256.2
3 30	210.2	31 30	260.3
5 5	220.8	38 10	263.2
6 50	231.8	49 50	267.6
7 30	233.8	1 0 35	269.4
9 20	238.6	1 27 50	274.7
12 35	245.0	2 9 10	279.1
14 10	246.8	....	....
17 50	251.4	25 54	286.0

TABLE IX.

$C = 0.0004545$. $t = 15.9°$. *Diluted from* $C = 0.100$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
3 40	182.2	25 40	233.3
4 25	186.6	31 0	238.7
5 20	191.1	1 0 35	256.1
6 40	197.2	1 19 20	262.0
7 55	201.5	1 38 0	268.3
10 25	209.4	1 49 0	270.9
13 55	217.4	1 55 0	272.2
16 40	221.9	....	....
21 35	228.7	19 28	301.9

TABLE X.

C = 0.000909. *t* = 15.7°. *Diluted from* *C* = 0.100.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
2 15	167.0	54 0	206.2
2 50	167.5	1 9 0	214.1
3 15	168.2	1 29 0	222.1
3 45	168.3	1 39 0	225.6
4 15	168.7	1 57 0	231.5
5 30	169.6	2 28 0	238.9
6 0	170.0	2 47 0	243.0
7 45	170.9	3 24 0	249.9
14 30	177.1	23 — —	298.3
25 45	187.1	———	———
38 15	196.5	5 d. 23 h.	299.5

TABLE XI.

C = 0.001335. *t* = 15°.5. *Diluted from* *C* = 0.100.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
2 0	167.8	1 13	188.6
2 50	168.0	1 43	203.3
5 35	168.9	2 13	212.4
10 10	170.0	2 52	222.4
14 20	170.8	22 49	285.9
27 50	175.8	———	———
38 30	179.7	7 d. 20 h.	300.7

TABLE XII.

C = 0.00500. *t* = 15°.6. *Diluted from* *C* = 0.999.

Time elapsed. d. h. m.	Equivalent conductivity.	Time elapsed. d. h. m.	Equivalent conductivity.
7	146.4	2 38	149.0
10	146.5	2 53	149.3
18	146.6	3 6	149.1
28	147.0	8 10	153.1
32	147.1	1 2 22	198.0
41	147.2	3 5 48	219.3
57	147.3	4 5 32	223.1
1 28	147.8	6 4	223.4

Solution colorless during first 8 hours.
“ deep yellow after 1 day.

TABLE XIII.

$C = 0.00750$. $t = 15^\circ.5$. *Diluted from* $C = 0.999$.

Time elapsed. d. h. m.	Equivalent conductivity.	Time elapsed. d. h. m.	Equivalent conductivity.
4	139.5	21 58	140.0
1 41	140.8	3 1 51	178.1
1 52	140.8	4 0 59	188.9
2 3	141.0	6 0 31	199.1
4 2	140.4		

Solution colorless during first 21 hours.
" deep yellow after 3 days.
" slightly turbid after 4 days.
" very " " 6 "

TABLE XIV.

$C = 0.000150$. $t = 0^\circ$. *Diluted from* $C = 0.00999$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
4 0	122.6	3 11	171.7
6 30	129.1	4 14	174.6
8 0	131.1	6 4	178.4
10 0	135.0	6 54	178.9
14 40	140.5	7 28	179.8
26 0	145.4	24 44	183.0
1 24 0	162.8	3 d. 2 h.	183.0
2 37 0	170.6		

TABLE XV.

$C = 0.00033$. $t = 0^\circ$. *Diluted from* $C = 0.00999$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
2 30	121.1	34 30	124.6
3 15	121.6	56 0	127.6
5 20	121.7	1 16 0	131.1
6 30	122.1	1 50 0	136.6
9 0	122.1	2 34 0	142.5
11 10	122.6	3 1 0	145.4
17 0	123.2	1 d. 20 h.	194.1
25 15	124.6	3 d. 40 m.	600.3

TABLE XVI.

$C = 0.0004545$. $t = 0°$. *Diluted from* $C = 0.090$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
6 0	113.9	6 8	130.8
7 0	114.3	6 53	135.5
7 30	114.7	7 10	137.0
9 0	115.3	7 36	138.4
12 0	116.1	d. h. m.	
23 0	117.3	1 3 30	167.6
1 26 0	123.2	1 4 14	168.6
3 11	121.1	1 6 13	169.5
4 4	125.9		

TABLE XVII.

$C = 0.000909$. $t = 0°$. *Diluted from* $C = 0.0999$.

Time elapsed. d. h. m.	Equivalent conductivity.	Time elapsed. d. h. m.	Equivalent conductivity.
6	113.1	1 40	113.9
25	112.0	2 20	115.0
31	112.1	3 16	116.1
51	112.6	1 5 20	143.3
$C = 0.000909$.		*Diluted from* $C = 0.0909$.	
37	110.0	1 6 30	144.5
2 54	113.0	4 1 40	160.0
6 19	113.3	5 1 28	159.9
22 23	114.2		

TABLE XVIII.

$C = 0.001335$. $t = 0°$. *Diluted from* $C = 0.0999$.

Time elapsed. d. h. m.	Equivalent conductivity.	Time elapsed. d. h. m.	Equivalent conductivity.
27	107.9	1 19 11	123.4
1 24	107.9	2 2 7	130.0
1 54	107.9	2 19 13	136.0
1 1 15	107.8	4 19 6	145.1
1 17 32	121.3	8 2 32	147.6

Solution deep yellow.

Table XIX.

$C = 0.00250$. $t = 0°$. *Diluted from* $C = 0.0999$.

Time elapsed. d. h. m.	Equivalent conductivity.	Time elapsed. d. h. m.	Equivalent conductivity.
32	101.4	5 1 23	106.7
2 0	99.9	6 1 53	112.6
3 1 19	99.7	6 21 18	117.8
3 22 20	101.2	10 — —	125.1
4 20 27	105.6	12 — —	128.5

Discussion of Results.

The data in the preceding tables is represented graphically in the following plots (1, 2, 3 and 4). Considering first plot 1, con-

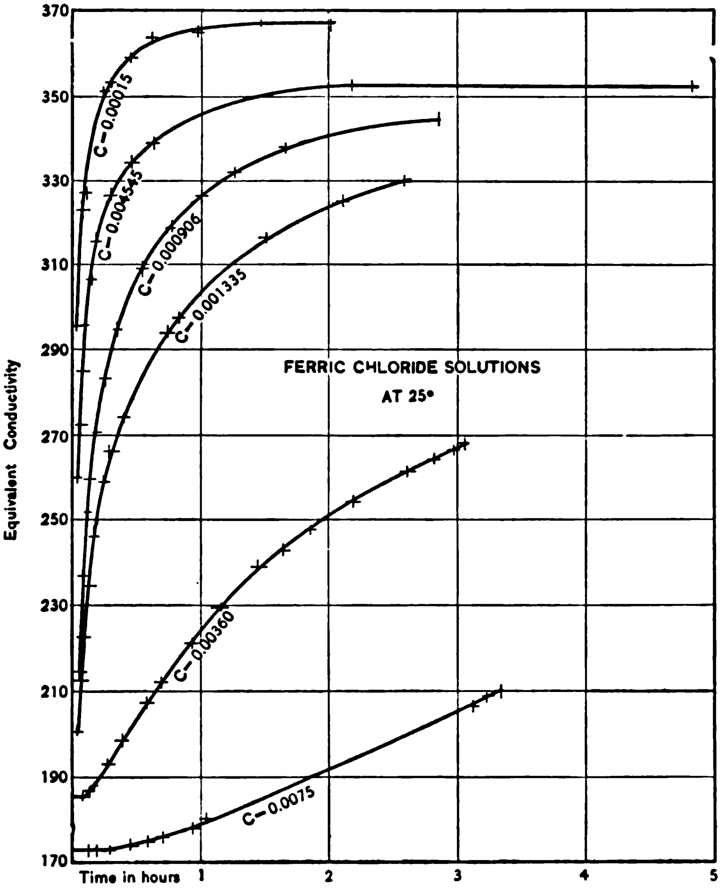

Fig. 1.

taining the results at 25°, we find that the earlier results at this temperature are in general well confirmed. The initial values for $t = 0$ and the form of the curves agree well with those previously obtained, but the initial lag in the present series is less than in the former. This may be due to the fact that in the former experiments, the "stock" solution from which the dilutions were made, was three times as strong as the present "stock" solution. The "lag" is, however, unmistakable in the solution $C = 0.0075$.

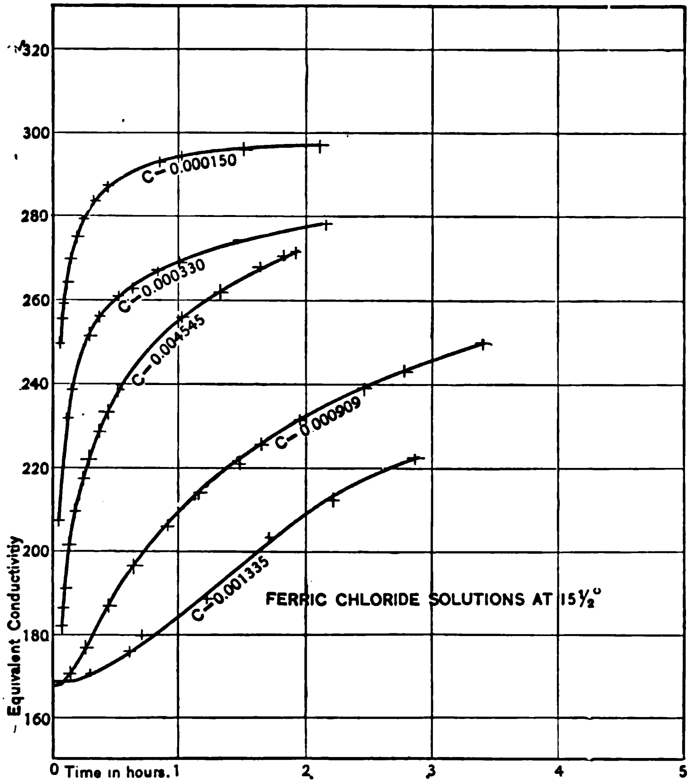

Fig. 2.

On comparing plot 1 with plots 2, 3 and 4 the enormous effect of temperature on the reaction is at once evident. Whereas, the hour was a convenient unit for representing the data at 25°, the day must be used to represent the data at 0° and the more concentrated solutions at 15°. It is seen, as was expected, that lowering the temperature diminishes the rate of the reaction, increases its initial lag, and the time required to reach equilibrium. The form of the curves for equal concentration thus flatten out with decreasing temperature; the curves for the solutions at 25° approach in form, in fact almost coincide with, the curves for greater dilutions at lower temperatures. Thus compare:

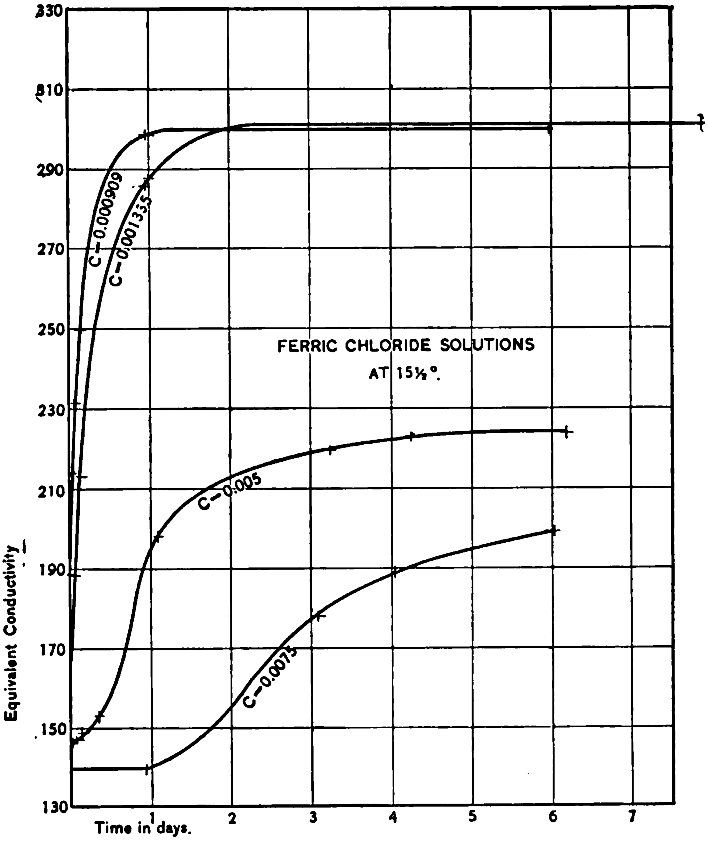

Fig. 3.

at 25°	at 15°
$C = 0.0004545$	$C = 0.00015$
$C = 0.001335$	$C = 0.0004545$
$C = 0.00360$	$C = 0.000909$

A decrease of temperature of 10° from 25° has nearly the same effect on retarding the reaction as increasing its concentration about three times; lowering the temperature to 0° retards the reaction as much as increasing the concentration fifteen to twenty times.

It is interesting to note how the solution $C = 0.001335$, which exhibits no lag at 25°, does not change appreciably until after about five minutes at 15.5°, and does not change until after a day at 0°.

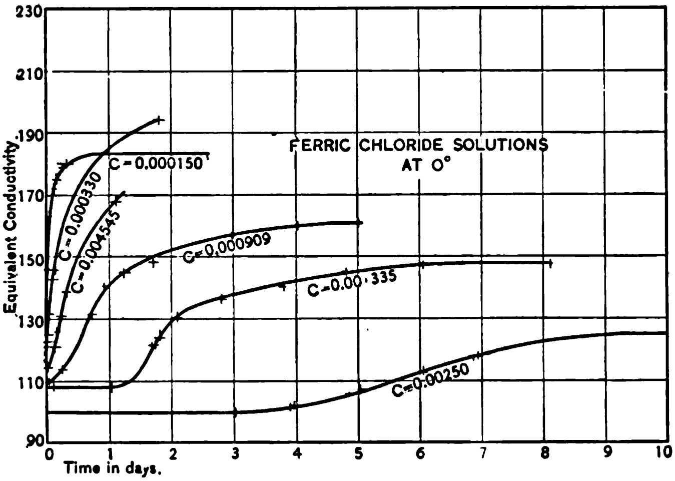

Fig. 4.

The solution $C = 0.0025$ remains unchanged at 0° until after four days, and it is probable that a solution of $C = 0.005$ which has a lag of a few hours at 15° would remain practically unchanged at 0°. The general expression for the rate of change as function of the temperature cannot, however, be determined from the data yet at hand.

In the course of the experiments a few observations were also made on the effect of temperature on the turbidity of ferric chloride solutions of certain concentrations when allowed to stand. This turbidity, which is probably due to the formation of oxychloride was found by Goodwin to be accompanied by a corresponding change in conductivity of the solutions and to have a "lag" dependent on the concentration of the solution.

Thus a 0.09 normal solution became turbid only after the lapse of two weeks while a solution ten times as dilute became turbid in about a day. On the other hand, a solution 0.0057 normal remained perfectly clear even after two weeks thus showing that the precipitate was formed at 25° only within narrow limits of concentration.

In the present investigation, solutions of strengths 0.02, 0.01, 0.004 and 0.002 normal were found to remain perfectly clear for at least two weeks when kept in ice. A 0.0075 normal solution on the other hand, became very turbid at 25° in less than four days, slightly turbid in the same time at 15° (very turbid after six days), while it remained clear at 0°.

Lowering the temperature thus greatly tends to diminish the separation of oxychloride from the solution as well as to prevent the formation of colloidal hydrate.

II.

EFFECT OF COLLOIDAL FERRIC HYDRATE ON THE REACTION.

To test whether the hypothesis, previously advanced to account for the accelerated rate of the reaction when once started, was verified by experiment, various amounts of previously prepared Graham's colloidal ferric hydrate were added to dilute solutions of ferric chloride, and the change in conductivity then followed as in the previous experiments. A solution 0.0075- equivalent normal and a temperature of 25° were chosen, as the preceding experiments showed that at this temperature a solution of this strength had a marked initial lag of about 20 minutes. A known amount of best distilled water was weighed out in a 250 cc. bottle, brought to a temperature of 25°, and a known small amount of the colloidal ferric hydrate added with a pipette. The specific conductivity of this solution was then determined. At a noted time a measured amount of ferric chloride solution was added, the whole vigorously shaken, the "plunge" electrodes replaced, and the change in conductivity then followed at known intervals of time.

In the following Tables, XX, XXI, XXII and XXIII, are given the results obtained. Under "equivalent conductivity" is given the conductivity of the ferric chloride in the mixed solution after correcting for the conductivity of the added colloidal ferric hydrate in the solution. This had an appreciable conductivity, as already stated, due to the presence of some crystalloids still remaining in it. In Table XXIII, however, the values given are the uncorrected conductivity values for the mixture—the specific conductivity of the colloidal solution not having been in this case accurately determined. A comparison of the initial corrected values in Tables XX, XXI and XXII, namely, 172.6, 171.2 and 172.6, with the initial value found for the pure ferric chloride solution 172.8 (Table VI), shows that they agree almost exactly whence it is safe to conclude that the conductivity of the mixture may be assumed additive. We have therefore, for convenience of comparison in plotting, corrected all the values given in Table XXIII by − 8, thus reducing the initial values of the conductivity of the ferric chloride to that found in the other solutions.

Table XX.

$FeCl_3$ $C = 0.00750$. $Fe(OH)_3$ $C = 0.000116$. $t = 25°.0$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
1 20	172.6	13 40	174.9
2 20	172.8	34 20	177.8
3 50	173.3	36 0	178.1
5 15	173.6	46 10	179.5
6 45	173.9	54 0	180.8
8 20	174.1	1 5 0	181.7
9 30	174.2		

Table XXI.

$FeCl_3$ $C = 0.00750$. $Fe(OH)_3$ $C = 0.000232$. $t = 24°.96$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
1 35	171.2	54 50	186.1
3 5	172.4	1 15 0	192.6
4 30	172.7	1 36 0	199.0
6 35	173.2	1 40 0	200.0
7 40	173.6	3 31 0	222.2
10 50	174.3	3 36	223.2
25 20	177.8	23 51	262.6

TABLE XXII.

$FeCl_3$ $C = 0.00750$. $Fe(OH)_3$ $C = 0.00116$. $t = 24^\circ.96$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
2 30	172.6	29 50	200.7
4 45	176.0	40 30	207.3
5 30	176.9	46 30	211.9
13 0	184.8	....	....
20 30	192.5	19 5	277.1

TABLE XXIII.

$FeCl_3$ $C = 0.00750$. $Fe(OH)_3$ $C = 0.00580$. $t = 25^\circ.0$.

Time elapsed. h. m. s.	Equivalent conductivity.	Time elapsed. h. m. s.	Equivalent conductivity.
50	180.4	11 15	222.0
1 15	187.0	13 45	225.8
2 15	197.0	16 40	229.7
2 35	198.6	19 20	232.5
3 5	200.7	22 10	235.8
3 35	203.0	31 0	244.0
4 5	204.8	1 2 20	257.5
4 30	206.3	1 41 0	267.4
5 15	208.3	4 15 0	284.6
6 15	211.2	....	....
7 35	214.7	21 32	312.1

The effect of the presence of increasing amounts of colloidal hydrate on the course of the reaction is most readily seen from plot 5, made from the date in the preceding tables.

It is seen at once from this plot that, as anticipated, the presence of colloidal hydrate accelerates the reaction to a very marked extent. In all cases the reaction starts immediately upon diluting the chloride, the initial lag of 20 minutes observed in the neutral solution being thus completely absent. Moreover, the increase in the initial rate of the reaction is approximately proportional to the concentration of the added colloidal hydrate. This in the four solutions investigated is present in the ratio, 1 : 2 : 10 : 50, while the increase of conductivity per minute is approximately in the ratio 0.2 : 0.4 : 1.4 : 10, or 1 : 2 : 7 : 50. The first three values are the rates during the first five or six minutes of the reaction; the last, during a still shorter interval of two and a half minutes, as this last reaction slows up very quickly after the first few minutes. It seems probable, therefore, that other things being equal, the presence of colloidal hydrate exerts a catalytic action nearly proportional to its concentration.

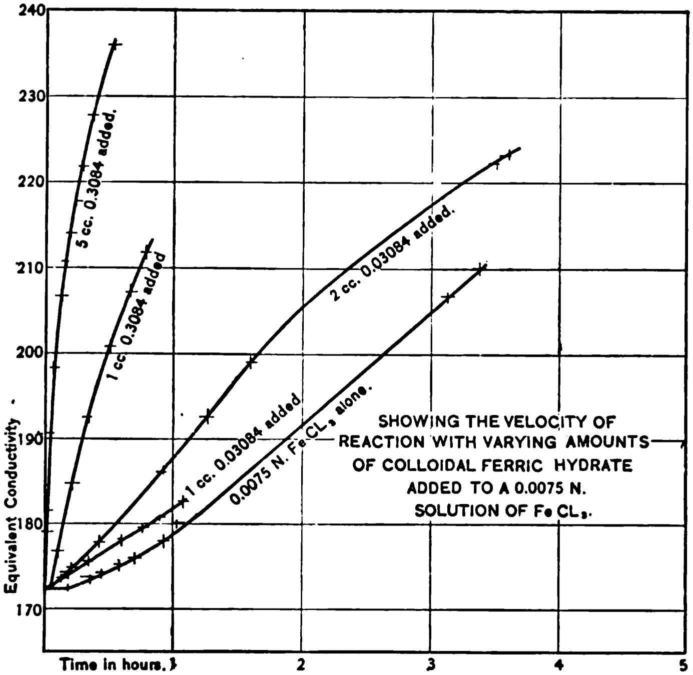

Fig. 5.

III.

The Effect of a Magnetic Field on the Rate of the Reaction.

It has now been pretty well established[1] that chemical reactions may in certain cases be modified when they are caused to take place in a powerful magnetic field. Wiedemann[2] has further shown that the magnetic moment of a ferric chloride solution is much diminished by the formation of colloidal ferric hydrate. It was therefore thought to be not without interest to investigate whether or not the velocity of the colloidal formation studied above, was in any way affected by a magnetic field. As our experiments gave only negative results, the numerous preliminary experiments and arrangements of apparatus will be passed over and only the final arrangement briefly described.

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[1] Wiedemann's Lehre von der Electricität for the early literature, vol. 3, p. 1129; Liesesang, Beibl. 15, 123; Braham, Beibl. 16, 304; Andrews, Proc. Roy. Soc. 52, 114, 1892; Nichols, Am. J. of Sc. 31, 372.

[2] Lehre von der Elect., vol. 3, p. 964.

It was desired to compare the rate of the colloidal formation in two exactly similar solutions at exactly the same temperature, the one placed in a strong magnetic field, the other removed from it. Any ordinary form of thermostat was of course out of the question, on account of the small distance between the poles of the electromagnet between which it was desired to place the cell. We arranged, therefore, as thermostats, two duplicate Beckmann boiling-point tubes, provided with condensers and containing pure ether, boiling at 33° C., and used test-tubes fitted into these ether jackets and provided with plunge electrodes as conductivity cells.

TABLE XXIV.

$C = 0.0040$ *approximately.*

h. m. s.	$k \times 10^{-4}$ I.	$k \cdot 10^{-4}$ II.	h. m. s.	$k \times 10^{-4}$ I.	$k \times 10^{-4}$ II.
3 59 30	Diluted.		4 20 30		8.01
4 4 45		6.95	21 10	8.04	
5 30	7.12		23 30		8.10
6 50		7.22	24 0	8.12	
7 15	7.29		26 35		8.16
7 50		7.30	29 0	8.26	
8 50	7.38		32 5		8.29
9 25		7.50	32 50	8.33	
10 5	7.53		39 20		8.43
10 50		7.60	40 15	8.44	
12 0	7.66		47 50		8.53
13 0		7.69	48 30	8.54	
14 20	7.78		5 6 5		8.70
14 50		7.79	8 0	8.71	
16 15	7.87		11 20		8.72
17 0		7.88	13 20	8.75	
17 35	7.93		14 30		8.75
18 55		7.96			
19 40	7.99				

TABLE XXV.

$C = 0.0040$ *approximately.*

h. m. s.	$k \times 10^{-4}$ I.	$k \times 10^{-4}$ II.	h. m. s.	$k \times 10^{-4}$ I.	$k \times 10^{-4}$ II.
1 37 0	Diluted.		2 3 50	8.14	
1 40 45	6.50		4 20		8.12
41 50		6.63	6 20	8.19	
42 25	6.78		7 0		8.20[1]
42 55		6.81	10 15	8.29	
43 25	6.91		10 55		8.30[1]
44 0		6.94	16 55	8.43	
44 40	7.04		17 35		8.43[1]
46 35		7.22	24 0	8.54	
47 15	7.29		24 35		8.55[1]
47 45		7.32[1]	27 35	8.59	
48 55	7.42		28 5		8.60[1]
49 30		7.46[1]	41 50	8.75	
50 55	7.56		42 25		8.76[1]
52 20		7.61[1]	45 55	8.79	
54 40	7.79		46 15		8.80[1]
55 10		7.79	3 6 25	8.93	
58 30	7.95		3 7 0		8.94[1]
2 0 0		7.99			

One of these was set up between the poles of a powerful electromagnet, and the other at some distance away. Direct experiment then showed that the temperatures of aqueous solutions placed in the cells could be kept the same to 0.01° for several hours. The procedure was to allow the ether to boil for some minutes in both thermostats in order to bring the dry cell and electrodes placed therein to the proper temperature ; to then dilute the ferric chloride to the desired strength in a separate flask containing water previously weighed and heated to the boiling point of ether ; and then to quickly transfer the solution thus prepared to the two measuring cells.

The current in the electromagnet could be regulated and thrown on or off at will. The strength of the field in which the solution was placed was determined by the change of resistance of a bismuth coil of wire placed in the field. The strength of field was thus found to be about 1850 lines per sq. cm. Table XXIV contains the results of a blank run, both cells being unmagnetized. The values given are specific conductivities at the temperature of boiling ether (33°). They are shown in plot 6, where it is seen that the two curves practically coincide throughout, thus confirming the accuracy of the cell constants and the constancy of temperature of the ether jackets.

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[1] Signifies that II was in the magnetic field.

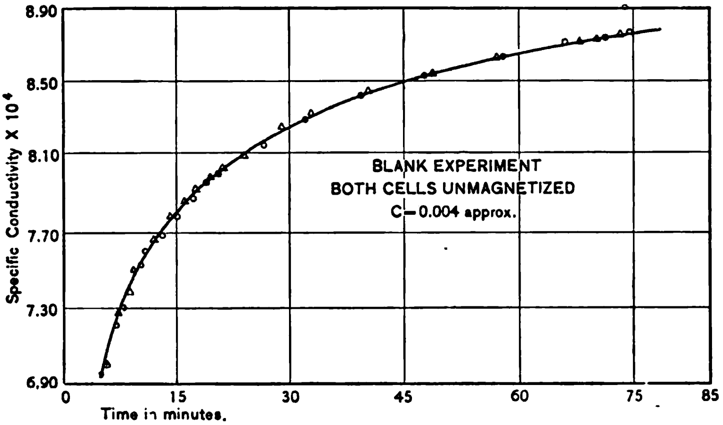

Fig. 6.

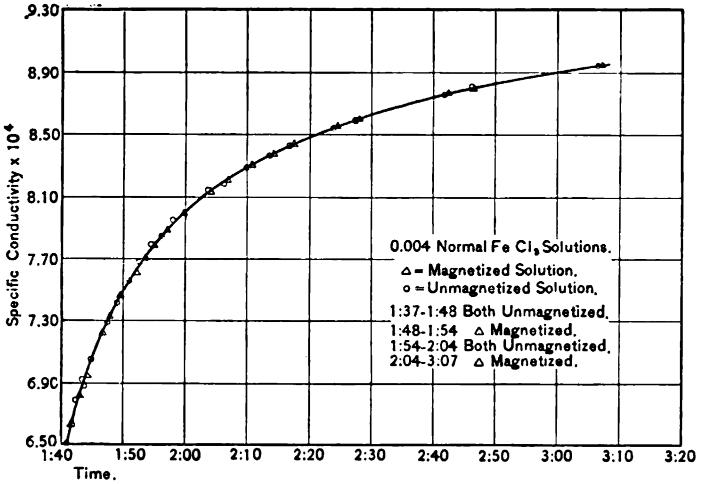

Fig. 7.

Table XXV contains the results of a run in which one cell remained unmagnetized, the other alternately magnetized and unmagnetized.

The results of both series are shown in plot 7. The curves coincide throughout, showing that no change in the rate of reaction was produced by the magnetic field. Numerous other runs confirmed this result, so that the conclusion may be drawn that the rate of formation of colloidal hydrate in ferric chloride solutions is not affected by the presence of a magnetic field of strength not greater than about 2000 lines per sq. cm.

Conclusion.

The principal results of this investigation may be summed up as follows:

1. The effect of lowering the temperature is to very greatly diminish the rate of formation of colloidal ferric hydrate, and to increase the initial lag in the reaction.

2. The hypothesis previously advanced to account for the acceleration of the reaction when once started is confirmed. The presence of colloidal hydrate exerts a very marked catalytic action causing the reaction to be accelerated by an amount approximately proportional to the amount of colloidal hydrate present in the solution.

3. No effect of a magnetic field could be detected on the course of the reaction.

Rogers Laboratory of Physics,
Massachusetts Institute of Technology, June, 1900.

PRELIMINARY NOTE ON THE EFFICIENCY OF THE ACETYLENE FLAME.[1]

By Edward L. Nichols.

In the study of flames or of glowing bodies of any kind, considered as devices for the production of light, and it is upon such devices that artificial lighting depends, the question of efficiency is of great importance. The complete solution of the problem involves the determination of the ratio of the light-giving radiation to the total energy expended in producing it; and the measurement of this quantity, *the total efficiency*, usually involves as an intermediate step, the determination of *the radiant efficiency*, which is the ratio of the luminous or light-giving radiation to the total radiation.

In the measurement of radiant efficiencies at least four distinct methods have been employed.

Julius Thomsen,[2] who was a pioneer in this field of research, read a paper at Stockholm in 1863, entitled *The Mechanical Equivalent of Light*, in which he applied the method of Melloni to the separation of the light-giving from the non-luminous radiation. Thomsen used a water cell containing a layer of liquid 20 cm. in thickness, by the interposition of which between the source of light and the thermopile used in measuring its radiation, he sought to cut off the dark heat. He studied the flames of spermaceti candles, of ordinary illuminating gas, and of the moderator lamp; determined the energy, expressed in calories per minute, emanating by radiation from these sources, and compared it with the calories of luminous energy. Blattner[3] in 1886 applied the second method under consideration, that of immersing the source of light in a calorimeter of glass to incandescent lamps under varying supplies of current. Merritt and Ryder[4] in the same year applied both methods to incandescent lamps. They used a solution of alum instead of water, and made the very necessary correction for the infra-red radiation which passes through the cell by the interposition of a second cell containing a solution of an opaque solution of iodine in carbon disulphide. They checked the results obtained by this method by submerging the lamp in a calorimeter through which a constant flow of water was maintained, and measuring the temperature of the water upon entering and leaving the vessel. The non-luminous radiation was in great part retained by the water surrounding the lamp, while the light-giving rays were largely transmitted by the liquid and by the glass walls of the calorimeter. The necessary corrections were determined by a study of the transmitting power of the glass and water for dark rays, and their absorbing power for light. The total radiation was computed by measurements of the electro-motive force at the terminals of the lamp and the current traversing the filament. This quantity was likewise checked by placing the lamp in a calorimeter with opaque walls and measuring the heat energy delivered to the water directly.

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[1] A paper read before the American Physical Society, June 27, 1900.

[2] Thomsen, Poggendorff's Annalen, 125, p. 348.

[3] Blattner, Das optische Nutzeffect der Glühlampen, Fraunfeld, 1886.

[4] Merritt, American Journal of Science, vol. 37, p. 67.

Staub[1] in 1890 studied the radiant efficiency of the illumination from vacuum tubes when excited by means of the electrical discharge; for which purpose he employed a modified form of the Bunsen ice calorimeter.

Determinations of radiant efficiency have likewise been carried out by the method of the thermopile and water-cell by Nakano[2] and by Marks[3] in the case of the arc lamp, by Rogers[4] in his study of magnesium as a source of light, by Miss Crehore[5] (Mrs. F. Bedell) who investigated in this manner the efficiency of the lime light, and by others.

A fourth and far more rigorous method (that of obtaining the distribution of energy throughout the visible and invisible spectrum and comparing the areas) of determining the radiant efficiency is due to Langley[1] and has been applied by him to sunlight, to various artificial illuminants, and to the light from the Cuban fire fly. The difficulties in the way of carrying out this method properly are, however, extremely great and it is frequently a question whether under the ordinary degree of command over the conditions of the experiment, the method yields a better result than the more convenient procedure invented by Melloni and developed by Tyndall and by the various subsequent workers just cited.

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[1] Staub, Inaugural Dissertation, Zürich, 1890; also Beiblätter zu den Annalen der Physik, 14, p. 538.

[2] Nakano, Transactions of the American Institute of Electrical Engineers, vol. 6, p. 308 (1889).

[3] Marks, Transactions of the American Institute of Electrical Engineers, vol. 7, p. 175 (1890).

[4] Rogers, American Journal of Science, vol. 43, p. 301 (1892).

[5] Nichols and Crehore, PHYSICAL REVIEW, vol. 2, p. 161 (1894).

Upon the problem of the total efficiency of sources of light, much less has been done than upon the radiant efficiency. Thomsen in the article already cited, estimated that in the case of the candle and of the moderator lamp the total development of heat was about three hundred and fifty times as great as the luminous energy of the flame, and that in the case of gas flames it was approximately one thousand times as great. Rogers determined the heat of combustion of magnesium, and, by means of a modification of Thomsen's method, ascertained the relation of the luminous radiation from a mass of the burning metal to the total heat produced by the reaction. Various other writers,[2] in the course of discussions upon the relative merits of electric lighting and of artificial illumination by means of flames, have attempted to express in one way or another the efficiency of the various methods, but actual measurements by direct experiment have been very few.

Experiments are in progress under the direction of the writer, by means of which the efficiency of the acetylene flame is to be directly determined with as high a degree of accuracy as the nature of the case may permit. For this purpose, the radiant efficiency is to be computed in two ways. First, by the method of Melloni, and second by the exploration of the spectrum and the comparison of the areas enclosed by the curve within and without the limits of visibility. The heat of combustion of acetylene gas, carefully prepared and thoroughly purified, is to be determined, and the total radiation from a flame of this gas found by a modification of Thomsen's method. Measurements of the amount of gas supplied to the flame will then make it possible to ascertain with considerable accuracy the total efficiency of this source of light.

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[1] Langley (see the well-known report of the Mt. Whitney expedition, and subsequent memoirs).

[2] See in particular, Preece; Nature, vol. 38, p. 496, (also B. A. Report (1888), or the table computed from data by Preece and published by the present writer in the Transactions of the American Institute of Electrical Engineers, vol. 6, p. 175 (1889).

Preliminary measurements from which it is possible to compute the value of the efficiency of acetylene flames obtained from the ordinary gas, such as is produced from the usual form of generators in which calcium carbide is thrown into water, are already completed and since the interest in this source of illumination is at present widespread, it seemed worth while to report these preliminary results without waiting for the completion of the more refined experiments which are to follow.

RADIANT EFFICIENCY.

These preliminary experiments naturally divide themselves into three parts, of which those upon the radiant efficiency were carried out by Messrs. Stewart and Hoxie. Their apparatus differed in several respects from that used by most of the earlier observers. Instead of a thermopile, they employed a bolometer of iron wire. The acetylene flame was of the flat type produced by the well known *Naphey* burner. Between the flame and the bolometer, a water cell was moved in and out. This cell was much smaller than those commonly employed, the dimensions being such that the rays traverse 1 cm. of water and 0.4 cm. of glass. The use of water instead of solutions of alum frequently employed in such work is justified by the investigations of Ernest Nichols[1] upon the perviousness of these and other substances to the infra-red rays of the spectrum. It will be seen by reference to Fig. 10 in that article, that water, potassium alum, and ammonium alum, all possess approximately the same transmitting power between the red end of the spectrum and wave lengths 1.5 μ where they all become opaque.

From thirty-nine sets of readings Stewart and Hoxie obtained the value 0.207 for the ratio of the light transmitted through this cell to the total energy reaching the thermopile when it was interposed. That this ratio does not express the true relation between the light-giving and the total energy from the flame is obvious.

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[1] Nichols, PHYSICAL REVIEW, vol. 1, p. 1, 1893.

Water and glass, on the one hand, absorb a certain proportion of the light and there are four reflecting surfaces at which losses occur. On the other hand, water transmits a considerable amount of non-luminous energy. The degree of this perviousness to rays lying beyond the limits of the visible spectrum is known to be considerable[1] and it is clear that, contrary to the view expressed by Thomsen in his article on the *Mechanical Equivalent of Light*, a large correction must be applied. This correction was determined in the usual way. Iodine was dissolved in carbon disulphide, the solution being of such strength as to be opaque to visible rays. The water cell and the cell containing the iodine were set up in such positions that the rays from the acetylene flame passed through both in the order named, on their way to the bolometer. The iodine cell was of the same size as that which contained the water. It was moved in and out, and twenty-two sets of readings were taken from which it appeared that the iodine cell absorbed fifty-one per cent. of the energy which passed through the water cell. Upon the assumption that the iodine cell is entirely transparent to the rays of the infra-red spectrum and entirely opaque to visible rays, we obtain for the radiant efficiency of the acetylene flame

$$0.51 \times 0.207 = 0.105.$$

This value can only be considered as approximate since no corrections were made for reflection from the surfaces of the glass, nor for the absorption of light by water, nor of infra-red energy by the iodine. From the following statement of the theory of these corrections, worked out by the observers, it will be seen that the losses in the water and iodine cells tend to annul each other and that consequently they may be ignored altogether without serious error.

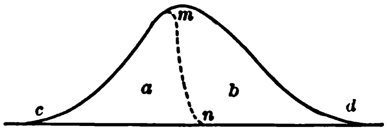

Fig. 1.

Let the curve *cd* in Fig. 1 represent the distribution of energy in the spectrum of the flame, let the line *mn* mark the limit of the spectrum transmitted by water, and let the total energy of the spectrum from the flame be A; then

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[1] See Ernest Nichols, l. c.

$$a + b = A \tag{1}$$

By moving the water cell in and out of the path of the rays, we are able to determine the ratio

$$a = \frac{a'}{A}$$

where a' is the value of a, less losses due to reflection and to absorption by water. Let the area d (Fig. 2) be that portion of the energy transmitted by the iodine solution and let e be that which will traverse both solutions. If the water cell be kept in the path of the rays and the iodine cell be moved in and out, we are able to obtain from observation the ratio

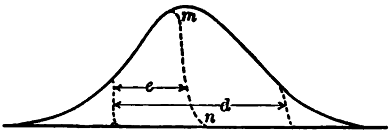

Fig. 2.

$$\varepsilon = \frac{e'}{a'}$$

in which e' is e, minus the losses by reflection and by absorption in the water and in the iodine. If c be the visible energy of the spectrum and c' the part of the same transmitted by water; then, since for a given surface and solution, the loss by reflection and by absorption is proportional to the intensity, we have

$$e' = (a' - c')(1 - k_i) \tag{2}$$

where k_i is the coefficient for the reflection and absorption by the iodine cell in the infra-red. We may write in like manner

$$c' = c(1 - k_w) \tag{3}$$

where k_w is the coefficient for the water cell in the visible part of the spectrum. From these, by substitution, we have

$$e' = a' - c - (a' - c')k_i + ck_w \tag{4}$$

and substituting this value of e', we have an expression for the energy absorbed by the use of water and iodine cells

$$\varepsilon = \frac{a' - c - (a' - c')k_i + ck_w}{a'}. \tag{5}$$

Now the approximate radiant efficiency 0.105, given as the result of these preliminary experiments is

$$\underset{\text{(approx.)}}{E} = \left(1 - \frac{c'}{a'}\right)\frac{a'}{A} = \frac{c + (a' - c')k_i - ck_w}{A}, \qquad (6)$$

while the true radiant efficiency is

$$E = \frac{c}{A}.$$

In equation 6, $(a' - c')k_i$ is the loss due to reflection and to absorption of the infra-red rays in the iodine cell, while ck_w is the loss due to reflection and to absorption of the light rays by water. If these two losses are equivalent, the method gives us the true radiant efficiency c/A without attempting any further corrections. In precisely similar cells, the losses by reflection will be nearly equivalent. Data for the absorption losses in water are available but the curves

Table I.

Radiant efficiency of various sources of light.

Source.	**Observer.**	**Luminous radiation. Total radiation.**
Sperm candle.	Thomsen.	0.0210
Paraffine candle.	Rogers.	0.0153
Moderator lamp.	Thomsen.	0.026
Illuminating gas.	Thomsen.	0.0197
“ “	Langley.	0.0240
“ (bat's wing).	Rogers.	0.0128
“ (argand).	Rogers.	0.0161
Incandescent lamp.	Blattner.	0.05 to 0.06
“ “	Merritt.	Various values up to 0.06
“ “	(From integration of Abney and Festings' curves).	0.055
Welsbach mantle.	Stebbins.	0.02 to 0.07
Lime light (new).	Crehore.	0.14
“ (old).	“	0.084
Acetylene.	Stewart and Hoxie.	**0.105**
Arc light (Foucault).	Tyndall.	0.104
“	Nakano.	0.104
“	Marks.	0.08 to 0.13
Magnesium light.	Rogers.	0.125
Geissler tubes.	Staub.	0.32

for solutions of iodine have, so far as I know, never been obtained. Since the two quantities tend to annul each other, there is probably less error involved in neglecting corrections altogether than in correcting for the water cell and in attempting without requisite data to correct for the losses in the iodine. While the nature of these losses is not accurately known, it is probable that the uncorrected result is at least approximately true.

These preliminary measurements give to the acetylene flame a position in the scale of radiant efficiencies very nearly equal to that of the arc light. The relation which different sources of light bear to each other in this respect may be seen from the table on page 221, which contains a list of previous determinations of radiant efficiencies.

HEAT OF COMBUSTION.

The preliminary experiments upon the heat of combustion of acetylene, necessary to the computation of the efficiency of the flame, were undertaken at the writer's suggestion by Mr. H. A. Rands. The gas used was the ordinary unpurified acetylene obtained from a generator in which the carbide of calcium was dropped into a large mass of water. The gas produced by this so-called wet process contains a larger percentage of acetylene and is less subject to polymerism than where water is applied in small quantities to the carbide. Unless special processes of purification are employed, however, the product is by no means pure acetylene. Analyses of several samples of the gas generated from the same lot of carbide as that used in these experiments showed from 20% to 25% of other gases, chiefly inert. The Favre and Silbermann calorimeter was used, the combustion jet being slightly modified so as to secure complete and smokelesss burning of the acetylene. Various samples of the gas taken on different days, gave a mean result of approximately 10,000 calories for the heat of combustion. This is considerably below the theoretical value of 12,200 as given by Pellissier, but the difference is essentially what one would expect from the analysis of the gas itself. In the completion of the investigation, it is proposed to make an extended series of determinations of the heat of combustion of acetylene after the gas has been subject to thorough purification, and to give the analyses of the gas employed.

RATIO OF TOTAL HEAT GENERATED TO TOTAL RADIATION.

Upon the relation between the total heat generated in the flame and the heat dissipated in the form of radiation, our experiments are still in progress. This portion of the subject is being studied by Mr. Albert Ball. His method, the results of the preliminary application of which only are given in the present note, is as follows:

A thermopile is connected in closed circuit with a galvanometer of suitable sensitiveness. At a convenient distance from the face of the thermopile, the acetylene flame is mounted, and between the two is a shutter by means of which the aperture in an intervening screen can be opened and closed by an observer situated at the telescope of the instrument. Readings are made by removing the shutter from the aperture for an interval of time corresponding to that of the first swing of the galvanometer. As soon as the needle makes its first turning point, the aperture is closed so that further heating of the face of the thermopile may be avoided. It is well established that the deflection, counting from the original position of the needle to its first turning point, is, under these conditions, always proportional to the total deflection which the galvanometer would take.[1] The method is one which enables a great saving of time and also reduces the risk of error from the shifting of the zero point. To calibrate a thermopile and galvanometer thus used, the following modification of Thomsen's procedure was employed. A spherical flask with its outer surface blackened was filled with water and its center was placed exactly in the position occupied by the flame in the operation just described. In the bottom of the flask, a coil of German silver wire was mounted, the terminals of which were brought out through the neck by means of copper lead wires of low resistance. By means of this coil, the water in the flask was heated to a temperature sufficient to produce upon the galvanometer an effect comparable to the radiation from the flame, and to maintain it at that temperature permanently. The shape and position of this coil was found to be a matter of considerable importance, and it was only after several trials that a form of coil was obtained which gave a satisfactorily even distribution of temperature throughout the flask. By giving this coil the form of a spiral wound upon a cylinder with axis horizontal and placing the same as near the bottom of the flask as possible, and centering it carefully, it was found that the convection currents were of such a character as to maintain the water in the flask at a constant temperature throughout. Exploration of the temperature in different portions showed variations amounting only to a small fraction of a degree.

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[1] See Merritt, American Journal of Science, vol. 41, p. 417.

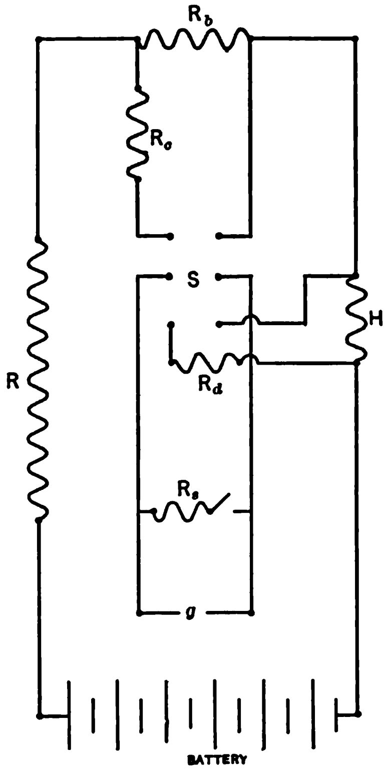

Fig. 3.

The electrical connections, by means of which the current from the coil was adjusted and measured, and the fall of potential between its terminals was determined are shown in Fig. 3.

The source of the current was a storage battery in circuit with which in series were an adjustable resistance (R), the heating coil (H), and a standard coil (R_b) of manganin wire, having a value of .1 ohm. The current was measured by placing a sensitive d'Arsonval galvanometer (g) in multiple with this standard coil and measuring the fall of potential through the same. The galvanometer was calibrated by means of a Clark cell. Wires connected with the terminals of the heating coil were carried to the same galvanometer, a six point switch (S) being so arranged that the instrument could be used alternately for measuring the fall of potential in the heating coil and in the standard coil (R_b) already described. Suitable resistances, (R_c) and (R_d), were placed in the galvanometer circuit and the instrument itself was shunted around a resistance

R_e by means of which its deflection could be reduced to the proper size. The delicacy of this galvanometer was such that in order to secure the desired reading, R_d was given a value of 1,000,000 ohms, and R_e 9,930 ohms, which latter, together with the resistance of the galvanometer itself, made the total resistance of that circuit 10,000 ohms.

The energy imparted to the flask was computed from the electrical measurements and to determine what proportion of this energy was lost by radiation and what by convection, the flask was placed in a bell-jar from which the air was subsequently removed. The loss by conduction was, for the purpose of these preliminary determinations, assumed to be negligible.

Assuming the loss of heat by radiation to be the same for a given temperature at the surface of the flask in air and in vacuo, and that all the heat developed in the flask by the action of the electric current is, in vacuo, dissipated by radiation, we may plot a curve having as abscissæ the units of heat radiated at different temperatures as computed by means of these electrical measurements when the flask is in vacuo, and the deflection produced when the thermopile is exposed to the flask *in the air* at the same temperatures. This curve serves to enable us to find *directly* the total amount of radiation from *any source* radiating equally in all directions, which may be placed in the position of the center of the flask, by means of the deflection of the galvanometer exposed to it under conditions similar to those made with the flask in situ. It is true that there is a small error due to the fact that the flask is not a perfect sphere and that the radiation from it is not strictly the same in all directions. It is likewise true that the distribution of radiation from the acetylene flame is not perfectly uniform, but these errors are so small that it was not necessary to consider them, in this preliminary determination of the efficiency.[1] It is obvious that the ratio of total radiation to total energy of combustion must be a function of the gas pressure and that it will reach a maximum for some particular size of the flame. With increasing pressures, the tendency is for the gases to be hurried away from the hot center of the flame so that there will be increasing losses of heat by convection with increasing pressure. Up to a certain point, this loss is overbalanced by the improved conditions under which combustion takes place, and there will be a value for the pressure at which, for any given burner, the efficiency will be greatest. Mr. Ball's work in these preliminary experiments divided itself, as will appear from the foregoing, into two parts. First, a calibration of the flask for convection losses, and secondly, the reading of the galvanometer when the thermopile was exposed to the action of the flame. The range of temperatures of the flask lay, as will be seen by inspection of the following table, between 81.8° and 96.1°C.

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[1] For a curve of horizontal distribution of intensities, see a paper on the acetylene flame read by the present writer before The Franklin Institute in March, 1900.

TABLE II.

Relation between temperatures of flask, galvanometer throws, and the total radiation of heat.

Temperatures of flask.	Total heat radiated (in calories per second).	Throw of the galvanometer.	Temperatures of flask.	Total heat radiated in calories per second).	Throw of galvanometer.
81.8°C.	6.976	10.05 s. d.	90.1°C.	7.619	12.35 s. d.
84.8	7.143	11.15	92.6	7.917	12.70
86.1	7.238	11.35	94.2	8.143	13.10
88.0	7.393	11.78	96.1	8.452	13.70

The total heat radiated from the flask at these temperatures is computed from the electrical measurements already described. The throw of the galvanometer when exposed to the flask at the same temperature in air is given in column 3. From these, a curve of the sort already alluded to for the radiation ·between heat radiation per second and galvanometer throws was drawn (Fig. 4).

To use the method just described, for the determination of the ratio of radiated to total heat, the acetylene flame was substituted for the heated flask, the deflection produced when the thermopile was exposed to its action being interpreted by means of the curve given in Fig. 4. Gas for the supply of this flame, passed first into an automatic regulator, the construction of which has been given in a previous paper.[1] From this regulator, it passed through an experimental gas meter reading in liters per minute, thence to a tube provided with a side arm leading to an open tube manometer, and thence to the flame. The flame was set up, as the flask had been in the calibration experiments, in an inner room quite free from drafts, through the wall of which an aperture was cut, with a double shutter capable of being raised and lowered. Opposite this opening the thermopile was mounted in such a position that one of its faces would receive the radiation from the flame. This thermopile, as has already been indicated, was held in closed circuit with the galvanometer and the reading was made by alternately raising and lowering the shutter. In computing the results of these measurements, the heat of combustion found by Mr. Rands in the experiments already described (namely, about 10,000) was assumed to be correct. Two determinations were made, the data for which together with the results obtained are given below.

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[1] Nichols, l. c.

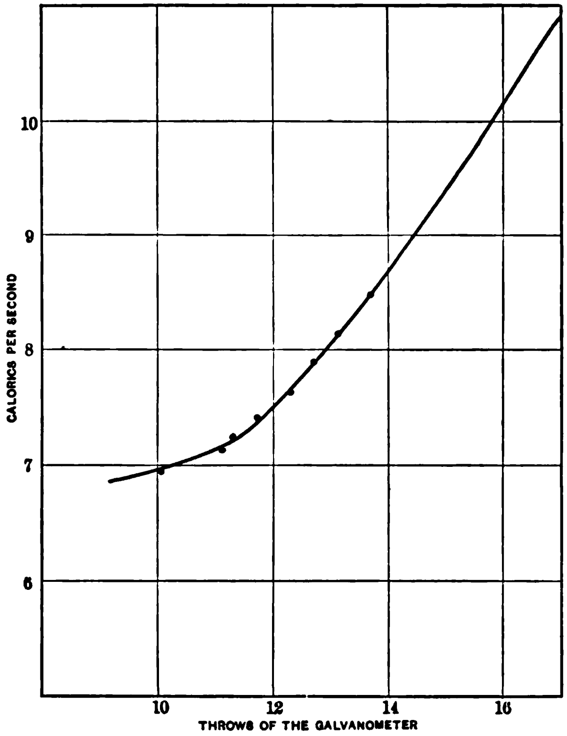

Fig. 4.

Experiment A.

Molecular weight of acetylene (C_2H_2), 26.

Molecular volume of acetylene (760 mm. and 273° abs.), 22.4.

Pressure at which the gas was supplied to the burner, 766 mm.

Temperature of the gas, 293° (abs.).

Volume of 26 grams of the gas at above pressure and temperature, 23.85 liters.

Rate of delivery of gas, 0.174 liters per minute, or 0.00316 grams per second.

Total heat produced by the reaction per second, $0.00316 \times 10{,}000 = 31.6$.

Deflection of galvanometer, 11.0 s. d.

Calories indicated by flame per second, as indicated by above deflection, 7.13.

$$\text{Ratio}: \frac{\text{total radiation}}{\text{total heat}} = \frac{7.13}{31.6} = 0.225.$$

TOTAL EFFICIENCY; assuming the radiant efficiency to be 10.5 %; $0.225 \times 0.105 = 0.0236$.

In experiment *A* the flame was below normal size.

Experiment B.

Rate of delivery of gas, 0.297 liters per minute, or 0.00539 grams per second.

Total heat produced by the reaction, per second, $0.00539 \times 10{,}000 = 53.9$.

Deflection of the galvanometer, 16.12 s. d.

Calories radiated by flame per second, as indicated by above deflection, 10:25.[1]

$$\text{Ratio}: \frac{\text{total radiation}}{\text{total heat}} = \frac{10.25}{53.9} = 0.190.$$

TOTAL EFFICIENCY, assuming the radiant efficiency to be 10.5%, $0.190 \times 0.105 = .0199$.

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[1] This value is much less reliable than that used in experiment *A* because it is taken from a portion of the curve not covered by the calibration observations. It is unlikely, however, that the extrapolation is sufficiently at fault to vitiate the conclusion that the efficiency of this flame, which was of fully normal size, was lower than that of the foregoing.

These values are many times greater than the corresponding values for ordinary gas and oil flames. They are at least double the corresponding value for the arc light under the most favorable conditions for the latter source.

The magnesium flame alone, which, according to Rogers, has a total efficiency of 0.1025, is, on account of the very small loss by convection and the very high radiant efficiency, superior to acetylene in this respect.

Physical Laboratory of Cornell University;
June 20, 1900.

THE DEVELOPMENT OF KATHODE RAYS BY ULTRAVIOLET LIGHT.[1]

By Ernest Merritt and Oscar M. Stewart.

IT has long been known that the discharge of negative electricity under the influence of ultraviolet light is the result of convection. All observers agree in the conclusion that negatively electrified particles of some sort act as the carriers of the escaping charge; whether these particles come from the surrounding air, or from the layer of condensed gas upon the electrified surface, or from the material of the charged body itself, is not yet definitely known.

In air at ordinary pressures the motion of the particles is relatively slow, even for a large potential gradient. But as the pressure is reduced the speed rapidly increases, being approximately proportional to the reciprocal of the pressure. The experiments[2] upon which this law is based extend only to a pressure of 34 mm.; but that the increase in speed continues with further exhaustion is indirectly shown by the earlier experiments of Righi[3], who found that at low pressures the paths followed by the electrified particles no longer coincided with the electrostatic lines of force, as is the case at ordinary pressures, but became straighter and more nearly coincident with the normal to the charged surface.

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[1] A brief account of the experiments here described was published in the Physikalische Zeitschrift, May 5, 1900. Since that time an article by Lenard, dealing with the same subject, has appeared in the Annalen der Physik (Vol. 2, p. 359, 1900), being republished from the Wiener Berichte (Vol. 108, Abth. IIa, p. 1649, 1899). The experiments of Lenard upon the magnetic deflectability of the photo-electric rays are similar to those here described, and, like the latter, give a *direct* proof that the rays are deflected. Some differences in method will, however, be noticed, and we have therefore left the present article unaltered. Except in the case of the magnetic deflectability of the rays the work of Lenard deals with a different phase of the subject from that here considered. We regret that the delay in the appearance of the Wiener Berichte prevented us from referring to Lenard's article in our original communication.

[2] Rutherford, Camb., Phil. Soc. 9, p. 401, Feb. '98.

[3] Acc. de Lincei 6, p. 81; Acc. di Bologna 10, p. 85; Wiedemann's Beiblätter 14, p. 1167.

This peculiarity of the photo-electric discharge at low pressures brings to mind the similar behavior of the negatively charged corpuscles which form the kathode rays. In each case we have to deal with a stream of negatively charged particles repelled from the kathode surface, and moving so rapidly, and with so little frictional resistance, that the path followed is only slightly influenced by the form of the electric field. In recognition of the analogy between the two phenemona, and for the sake of brevity, we shall refer to the stream of charged particles in the case of the photo-electric discharge as photo-electric kathode rays, or *photo-electric rays.*

The photo-electric discharge, while unaffected by magnetic influences at ordinary pressures, may be greatly diminished, or even suppressed, by a magnetic field when the discharge occurs in vacuo. In order to produce this effect the field must have a component parallel to the charged surface, and therefore at right angles to the path of the charged particles. It has been suggested by Elster and Geitel[1] that the effect is due to a deflection of the charged particles or ions in passing through the magnetic field. If this view is correct the phenomenon must be regarded as closely analogous to the well known magnetic deflection of the kathode rays.

Considered in connection with certain speculations regarding the mechanism of the discharge produced by illumination, the facts just mentioned have led us to believe that at low pressures the photo-electric discharge is accompanied by the development of true kathode rays: in other words, that the photo-electric rays are identical in character with the kathode rays produced in an ordinary vacuum tube. Since the experiments to be described in this paper were begun, this view has received strong support from the work of J. J. Thomson.[2] Using a form of apparatus capable of quantitative work, Thomson has not only found very convincing indirect evidence of a magnetic deflection of the photo-electric rays, but has also been able to determine the ratio of the charge carried by each particle to its mass. The values obtained for this ratio range from 5.8×10^6 to 8.5×10^6. In the case of the corpuscles which constitute the kathode rays the values found for the corresponding ratio lie between 6.3×10^6 and 18.65×10^6. When we recall the experimental difficulties of such determinations the agreement is to be regarded as a good one, and it seems highly probable, as suggested by Thomson, that the carriers of the negative charge are the same in both classes of phenomena.

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[1] Wied. Ann. 41, p. 166, 1890; Wien. Ber. 97, Abth. IIa, p. 1175.

[2] Phil. Mag. 48, p. 547.

The experiments described in the present communication were undertaken in order to test the conclusion that kathode rays may be developed, at low pressures, by the action of ultraviolet light. Taking up in turn the well known properties of the ordinary kathode rays, we have tried to show experimentally that similar properties are possessed by the photo-electric rays. In the majority of cases the attempt has proven successful. Thus we have been able to show that the rays are magnetically deflected; that they ionize the gas through which they pass; and that their motion is modified by electrostatic forces. We have not been able to observe fluorescence produced by the photo-electric rays; nor do they appear to develop Röntgen rays. The apparent absence of these two phenomena is sufficiently explained by the feeble intensity of the rays used by us, and by the crude character of the experiments made to detect them.[1]

APPARATUS.

The general plan of the vacuum tube used is shown in Figure 1. The tube was mounted vertically with the kathode K at the bottom and the electrodes L, M, and R at the top. The kathode was a short zinc cylinder with its upper end carefully polished. Being mounted in a glass stopper it could be readily removed for polishing. K was connected to the negative terminal of a dry pile and thus kept permanently charged. Immediately above the kathode K was the anode A, which consisted of a ring of rather large aluminium wire. The ring is shown only in projection in the figure, the plane of the ring being perpendicular to the plane of the figure. The three electrodes *L*, *M* and *R* were held in place by glass tubes which passed through rubber stoppers. These were made air tight by filling the space above with mercury. The figure, being drawn to scale, shows relatively the various dimensions. The distance between *K* and *M* was 12 centimeters.

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[1] Since the kathode potential in our experiments was only about 1000 volts, we had little hope of detecting any Röntgen rays. Only one exposure, lasting an hour, was made. The conditions for observing fluorescence were also unfavorable on account of the light from the spark used to produce ultraviolet rays. Appreciating the weakness of the photo-electric rays, as compared with the kathode rays in an ordinary tube, we had little hope of success in this case also, and did not really give the matter a serious trial.

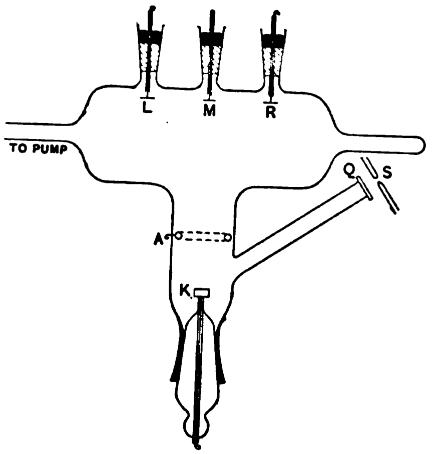

Fig. 1.

In order to admit ultraviolet light into the interior of the tube the quartz plate *Q* was mounted on the side tube.[1] The end of this tube, which was ground plane, was rubbed with a mixture of tallow and beeswax and the quartz plate laid upon it. After the pump was started this mixture was applied in rather large quantities to the outside of the joint between the tube and the quartz plate. The pressure of the outside air forced the mixture into any small leak due to the imperfect fitting of the plate.

The ultraviolet light that fell on the polished zinc, came through the quartz window, from a spark between zinc terminals. The spark was from two to three millimeters long and was placed two or three centimeters from the quartz window. By suitably arranged screens the ultraviolet light was cut off from all parts of the tube except the zinc kathode, and from all the connecting wires.

The induction coil, condensers, and all connections to the high potential secondary circuit of the coil, including the spark itself, were enclosed in a metallic box, so that the electrodes in the tube were screened from the electrostatic inductive action of the high potential charges. In addition to this the tube itself and the wire leading to the electrometer were surrounded by electrostatic screens. As a further protection to *L*, *M*, and *R*, the tube near the anode was wrapped with a fine mesh wire gauze. When this and the anode were grounded any inductive action of the kathode on the terminals *L*, *M*, and *R* was entirely cut off.

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[1] Instead of being in the plane of the figure as shown, this side tube was in a plane perpendicular to the figure and projected toward the observer instead of toward the one side.

A sensitive Kelvin electrometer, kept at approximately the same degree of sensitiveness by its inductive gauge, was used to determine the potential of the electrode to which it was connected. Usually it was the current passing into the electrode that was wanted. The potential was then observed every five or ten seconds. The difference in successive readings gave the rate of change of potential of the electrode. These values are proportional to the current.

The tube was permanently connected to the vacuum pump. Although the tube had four mercury seals, one ground glass stopper, and the quartz glass joint in addition to the cocks and seals in the pump, it was found that the tube would hold a high vacuum for weeks at a time. No measurements of the pressure in the tube were made, but a rough estimate of the degree of exhaustion was obtained by noting the size of the bubble forced out at each stroke of the pump, and by occasionally observing the character of the discharge produced between *L* and *R* by an induction coil. Most of the experiments were performed at a high Crookes' vacuum, but in some cases the pressure was about 1 mm., in which case it could be estimated by a rough gauge.

No attempt was made in any of our experiments to obtain quantitative results. Our intention was merely to show that the properties of the photo-electric rays are qualitatively the same as those of the kathode rays.

MAGNETIC DEFLECTION.

In view of the experiments of Elster and Geitel, Righi, and J. J. Thomson it might at first appear that further evidence of the magnetic deflection of the photo-electric rays is superfluous. It must be remembered, however, that these experiments contain only an indirect proof that the rays are magnetically deflected.[1] The results might be equally well explained in other ways. Thus the experiments of Elster and Geitel showed that the discharge rate was diminished by a strong field. If we assume that the negative ions which carry the discharge are deflected by the field, the results are satisfactorily explained; but the same result would be produced if the magnetic field, instead of acting on the ions themselves, served merely to so modify the phenomena at the charged surface as to alter the number of ions developed.

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[1] Direct proof has been given by Lenard. See foot note p. 230.

In Thomson's experiments an insulated metal plate of practically infinite extent was placed parallel to the charged surface. Upon illuminating the latter the insulated plate received a negative charge. The rate at which this charge was received was found to be diminished by a field parallel to the charged surface, provided the strength of the field exceeded a certain value. The assumption that the photo-electric rays are subject to a magnetic deflection gives an excellent explanation of the effect, since for some critical value of the field strength the rays would be bent through 180° and return to the charged surface.[1] But here too it seems clear that other equally plausible explanations might readily be found.

In fact, both the experiments of Elster and Geitel and those of Thomson are subject to the disadvantages common to all indirect demonstrations. To one who is already inclined to accept the hypothesis upon which they are based, the results are conclusive; but for others they are often far from convincing. For this reason the experiments described below, which afford direct evidence of a magnetic deflection of the photo-electric rays, seem not without interest.

The electrode *M* (Fig. 1) was permanently connected with a quadrant electrometer, and was brought to zero potential by grounding. Upon removing the ground connection and illuminating the zinc kathode by means of the spark at *S* the electrometer showed that *M* was gradually acquiring a negative charge. The rate at which the charge was received depended upon the potential of the kathode, the brilliancy of the spark, and the pressure in the tube. That the charge was not due to induction, or to any ordinary leakage, was readily proven by a number of obvious tests. It can scarcely be doubted that the charge was carried to *M* from the kathode chiefly by the photo-electric rays, although reasons will be given later for believing that the effect is complicated to some extent by the ionization of the residual gas. The illumination of the kathode did not serve merely to *start* a discharge, which afterwards continued by itself. The potential of *K* was far below that necessary to maintain a discharge. Upon preventing the ultra-violet rays from falling upon the kathode the discharge of the latter ceased at once.

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[1] The theory of Thomson's experiment requires that when the critical field strength is reached the transfer of charge to the insulated plate shall entirely cease. In reality the decrease in the rate was found to be quite gradual, and the determination of the critical field strength was attended with considerable uncertainty. It seems clear that the phenomenon is more complicated than the simple theory would lead us to expect. Certainly the discrepancy is sufficiently serious to lead to some question as to the correctness of the conclusions, especially in the mind of one not inclined to regard Thomson's general theory with favor. We shall return to this subject later.

A magnetic field at right angles to the line joining *Z* and *M* diminished the rate at which *M* was charged, and if sufficiently strong suppressed it entirely. This diminished rate was *not* due to any falling off in the rate at which the negative electricity was discharged from *Z;* this was shown by connecting *Z* to an electroscope and noting the rate at which the latter was discharged, first with, and then without a magnetic field. Doubtless a sufficiently strong field would have diminished the discharge rate, as in the original experiments of Elster and Geitel; but the relatively weak fields employed in our own experiments were unable to produce any measurable diminution.

With a rather weak field, having its positive direction normal to the plane *ZLR* and directed away from the observer, the charge received by *R* was *increased* when the field was established, while the charge received by *M* was diminished. Upon reversing the field its effect upon the charge received by *M* was the same as before; but the increased charge was now found at *L* instead of at *R*. In order that the effect should be sharply marked, it was necessary that the strength of the magnetic field should lie between rather narrow limits. If the field was too weak, the increase in the charge received by *R* or *L* was so small as to be scarcely observable; while if the strength of filed was increased beyond its critical value, the charge diminished again, and finally disappeared altogether.

It is clear, from what precedes, that the effect of a magnetic field is to produce a *deflection* of the photo-electric rays; the path followed by the escaping charge is altered, while its amount remains practically unchanged.

The method of taking observations was usually as follows: One of the electrodes, say *R*, being connected with the electrometer, readings of the latter were recorded every ten seconds so as to obtain the rate of charge. Except with a very strong magnetic field this rate was never zero so long as the kathode was illuminated. Doubtless this fact is explained by the diffuse character of the photo-electric rays; for the form of the apparatus was unfortunately not such as to give a compact beam. After taking enough observations to determine the rate of charge without a magnetic field, the electro-magnet used in developing the field was excited, and the rate was again determined. In some cases the effect of reversing the field was also tested. The results of several sets of observations of this kind are given in Tables I, II, and III. In the first case (I) the strength of field had been adjusted by trial so as to give the maximum effect. It will be observed that the rate at which *R* was charged was more than doubled by the magnetic field. When the field was reversed (II) a diminished rate resulted. In this case the photo-electric rays were deflected in the opposite direction, and some

Table I.

Electrode R. Magnetic field direct (+).

Time after beginning of charge.	Charge rate.	Magnetic field.	Time after beginning of charge.	Charge rate.	Magnetic field.
10 sec.	10	off.	60 sec.	9	off.
20 "	11		70 "	7	
30 "	12		80 "	22	on.
40 "	29	on.	90 "	19	
50 "	29		100 "	8	off.

Table II.

Electrode R. Magnetic field reversed (—).

Time after beginning of charge.	Charge rate.	Magnetic field.
10 sec.	14	off.
20 "	6	on.
30 "	7	
40 "	12	off.
50 "	11	
60 "	7	on.
70 "	7	

TABLE III.

Electrode R. Strong field direct (+).

Time after beginning of charge.	Charge rate.	Magnetic field.
10 sec.	16	off.
20 "	2	on.
30 "	17	off.
40 "	2	on.
50 "	15	off.

of the diffuse rays which had previously reached *R* now no longer did so. Table III gives the results in the case of quite a strong field, whose direction was the same as in Table I. But instead of increasing the rate, it reduced it nearly to zero.

These results appear to us to prove beyond question that the photo-electric rays are magnetically deflected. From the above and other experiments we conclude that when the field is too weak the deflection is small, and the rays proceed to some point between *M* and *R*; in this case, while the charge received by *M* is diminished, that received by *R* is increased only slightly. Were it not for the fact that the bundle of rays is quite diffuse, it is probable that weak fields would produce no change at all at *R*. As the field is gradually strengthened the deflection of the rays increases. Finally the deflected rays fall upon the electrode *R*, and the rate of charge of the latter reaches a maximum. If the field is made still stronger the rays are deflected so far that they pass beyond *R*, or are intercepted by the walls of the vertical tube. The charge reaching *R* therefore diminishes again, and finally vanishes.

The relation between the direction of the magnetic field and the direction of the deflection was found to be the same as in the case of the kathode rays. Although no quantitative work was undertaken it was easily seen that the magnitude of the deflection was about the same as would be observed with kathode rays developed in a low vacuum.

It was found that the potential of the zinc kathode was an important factor in determining the susceptibility of the rays to magnetic deflection. When the potential of the kathode was high a much stronger field was required to produce a given deflection than when the kathode potential was reduced. In other words the photo-electric rays became "stiffer" as the potential of the kathode was increased. The same is true with the ordinary kathode rays. The photo-electric rays were deflected much more readily than the kathode rays in an ordinary X-ray tube. It is to be remembered that the potential of the zinc kathode was only slightly in excess of 1000 volts.

The deflection was only slightly affected by changes in pressure. In this respect the photo-electric rays would appear at first glance to differ essentially from the kathode rays; for it is well known that by altering the pressure in a vacuum tube we may change the magnetic deflection of the kathode rays very greatly. But it must be remembered that in such cases the change in the magnetic deflection is really a secondary effect. The first effect of altering the pressure is to change the potential difference necessary to produce discharge, and since the magnetic deflectability of the rays is completely determined by the fall of potential in the tube[1] a change in their stiffness necessarily follows.

In addition to the photo-electric rays proper, in which the charged particles are probably identical with those which form the kathode rays, it appears to us that heavier and more sluggish ions are also present, whose motion is practically unaffected by a magnetic field. The experiments leading to this view, which unfortunately give only indirect evidence of its correctness, were as follows:

Having determined the rate at which *M* was charged in the absence of a magnetic field, the rate was redetermined with a field sufficiently strong to deflect the rays far beyond *R*. The latter rate was still far from zero. In one instance, when the pressure in the tube was about 1 mm, the ratio of the two rates was 4.3 : 1. Upon carrying the exhaustion further this ratio was found to steadily increase, and in the set of observations just mentioned it finally reached a value of 8.2 : 1.

It is quite possible that these results might be accounted for by the extreme diffuseness of the photo-electric rays, since the diffusion would naturally be less at high exhaustions. We know, however, from the experiments described in the next section, that the residual gas in the tube is ionized by the photo-electric rays. Ordinary gaseous ions are therefore present in practically all parts of the tube. We are inclined to ascribe the results just mentioned in part at least to the presence of these ions, rather than to a scattering of the photo-electric rays. Under the influence of the electric field between *A* and *K* the negative ions developed in that region will move in the same general direction as the photo-electric rays; but since these ions are not produced at the kathode itself, but throughout the whole region traversed by the photo-electric rays, the direction of their motion need not be even approximately normal to the kathode surface. While most of the negative ions doubtless fall upon the anode and there give up their charge, some will probably pass beyond *A*. In the region above *A* two kinds of negative particles are therefore present: (1) the small rapidly moving corpuscles of the photo-electric rays, forming a fairly compact stream normal to the kathode; (2) the relatively heavy and slowly moving negative ions, which form a stream having the same general direction as the photo-electric rays but much more diffuse. The photo-electric rays are deflected by a magnet; the moving ions on the other hand, on account of their greater inertia, are not appreciably affected by magnetic influences.[1]

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[1] Kaufmann, Wied. Ann. 61, p. 545, 1897; J. J. Thomson, Phil. Mag. 44, p. 193, 1897.

The view here advanced not only gives a satisfactory explanation of our own results, but also explains the partial discrepancy between experiment and theory in the experiments of Thomson.[2]

IONIZATION.

As is now well known, kathode rays in passing through any gas make it for the time being a conductor. Following the ordinary electrolytic hypothesis, it is thought that under the impact of kathode rays some of the molecules are broken up into ions. Part of these ions carry positive, while the rest carry negative charges, the result being that a temporary power of electrical conduction is imparted to the gas. If the photo-electric rays are identical in character with kathode rays, the gas through which this discharge passes should become conducting. We were able to show that this was the case in several different ways.

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[1] As the ionization produced by photo-electric rays has not been previously investigated, the assumption that the ions possess relatively large inertia rests only on analogy with other instances of gaseous conduction. In all cases so far studied there is every reason to believe that the ions produced are much larger than the kathode ray corpuscles. It is natural to suppose that the same is true in this case.

[2] See foot-note p. 235.

The electrode *M*, Figure 1, lies directly in the path of the negatively charged particles that compose the photo-electric stream. As these particles impinge on *M* they give to it a negative charge. *M* will therefore accumulate a negative charge, and, if the gas were not a conductor, the potential of *M* would increase until the electrical repulsion prevented more negative particles from approaching *M*. This potential would have to be high, as the ions from *K* approach *M* with considerable velocity. Observation shows, however, that *M* instead of reaching a high potential under the impact of the rays becomes charged to a potential of only a few volts. The simplest explanation of this is that the photo-electric rays ionize the gas. As soon as *M* becomes charged negatively it attracts the positive ions. The final potential reached by *M* represents the state of equilibrium between the inrush of the negative corpuscles of the photo-electric rays and of positive ions from the surrounding gas.

If one of the side electrodes, say *L*, was charged by a dry pile while *M* connected to the electrometer, a current passed through the gas between *M* and *L*. The direction of this current could be reversed by changing the sign of the charge on *L*. To eliminate errors due to the inductive effect of charges that may accumulate on the glass walls of the tube or to charges in the gas itself, the electrodes, *L*, *M*, and *R* were incased in metallic cylinders. In the final experiments the electrode, *M* was built in slightly different form from the others, so that a high potential difference could be maintained between the cylinder *B* and the electrode *M* (Fig. 2) without an appreciable leakage. The electrode *M* was an aluminium wire supported by the glass tube *ab*, and was permanently connected to the electrometer. The cylinder *B* was held in place by the polished ebonite block *e*. The glass tube *ab* passed through the rubber stopper *d* and the mercury seal *c*. Being connected by a small wire to *R*, Fig. 1, the cylinder could be charged to any desired potential.

The usual method of observing the current between *B* and *M* was to first ground the electrometer and then, having insulated it, to note the rate of change of the potential of *M*. This rate is proportional to the current going through the gas to *M*. When *B* was charged positively, *M* took a positive charge; when *B* was negative, a negative charge was given to *M*. That this conductivity was due to the passage of the photo-electric rays through the gas was easily shown. The vacuum was so high that for the potentials used it was a perfect insulator. It was only when the photo-electric rays passed near *B* that the conduction occurred. Screening off the ultra-violet light, or removing the negative charge from the zinc, stopped the effect. When a magnet deflected the rays from *K* against the walls of the tube near *K* the conductivity was destroyed and no current passed from *B* to *M*. Upon removing of the magnet, current again flowed. Anything that stopped the photo-electric rays destroyed the conductivity.

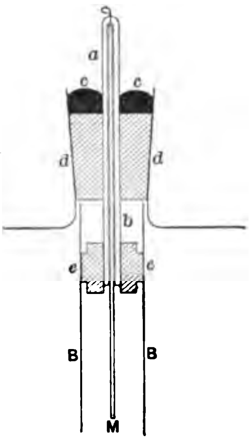

Fig. 2.

In observing the current between the wire *M* and the cylinder *B*, the latter being charged positively, rather striking effects were noticed, both when the ultraviolet light was first allowed to fall on the kathode, and when the light was screened off. The current instead of gradually increasing to a maximum started in the reversed direction with a rather large value, then slowly decreased to zero and reversed.

Figure 3 shows graphically the change in potential of the electrode *M*.[1] Time in seconds is counted from the instant the ultraviolet light is started. In this case the ground connection of the electrode *M* was broken before the spark producing the ultraviolet light was started.

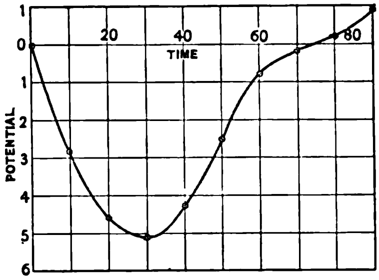

Fig. 3.

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[1] The slope of the curve at any point is proportional to the current at that instant.

In explaining this effect it must be remembered that the process of ionizing a gas is not an instantaneous one. As soon as an ion is created it comes at once under the influence of the electrostatic field, moves over to one of the conductors and is destroyed. For some time the ions are generated by the rays faster than they are thus destroyed. Therefore during this time the number of ions is increasing. The first rush of negatively charged corpuscles in the photo-electric rays imparts a negative charge to the electrode *M*, and until the time when the number of positive ions reaching *M* exceeds the number of negative corpuscles the current will flow in the negative direction.

When the stream of particles from the kathode was cut off the current was for a short time greatly increased. This was true only when *B* was charged positively and when the electrometer was taking a positive charge. As has already been shown the current measured is the result of two opposing currents, one due to the negative charges brought by the photo-electric rays, and the other to the positive ions moving under the electrostatic forces. Therefore, if one is suppressed, as in this case the negative ions, the resultant is increased. The effect was the same whether the rays were stopped by breaking the current in the induction coil that furnishes the spark giving the ultraviolet light, or by screening off the ultraviolet light, or by deflecting the rays with a magnet.

In Figure 4 is shown graphically the effect on the change of potential of *M*, both of starting and stopping the ultraviolet light.

It will be noticed in the curve that the electrode *M* continued taking a positive charge for about thirty seconds after the spark was stopped. This can hardly be due entirely to the positive ions left in the gas, as they probably move too rapidly to last longer than a few seconds. The complete explanation of this phenomenon is not clear to us.

When *B* is negative, negative ions under the electrostatic forces go from the gas to *M*. There is therefore no tendency to oppose the effect of the negative corpuscles of the rays which impinge on *M* as is the case when *B* is positive. Hence no reversal of the current is found after the rays are started. When the ultraviolet light is suppressed the current died down to zero in a few seconds.

It was found for high vacua that the conductivity decreases as the pressure of the gas in the tube decreases. This is to be expected, since it is known that the amount of ionization imparted to a gas depends among other things upon the absorption of the rays by the gas. As the vacuum is increased the absorption decreases and the number of ions present is diminished. This decreased conductivity is easily shown by observing the permanent deflection acquired by the electrode *M* while *B* was grounded. This depends, as has been previously explained, on the conductivity of the air as well as on the intensity of the photo-electric rays. When the pressure of the gas was decreased the potential acquired under the impact of the photo-electric rays was increased.

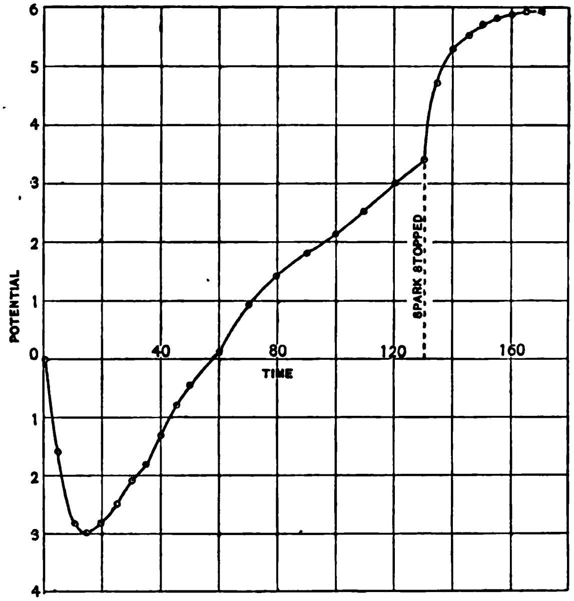

Fig. 4.

Electrostatic Effects.

Since the residual gas is rendered conducting by the photo-electric rays, the direct proof of an electrostatic deflection of these rays is subject to considerable experimental difficulty. The difficulty is in fact the same as that which so long concealed the existence of the corresponding phenomenon in the case of the kathode rays. For this reason we had not intended to undertake a study of the electrostatic properties of the rays without apparatus especially constructed for the purpose. But indirect evidence of electrostatic deflection was accidentally obtained which is not without interest.

In the course of our experiments upon the conducting power imparted to the air by the photo-electric rays, numerous observations were made to determine the relation between the difference of potential between *B* and *M* and the current flowing. In all other cases of electrolytic conduction in gases it has been observed that Ohm's law does not hold, but that with increasing electromotive force the current approaches a limiting value; in other words the current becomes "saturated." In the present case the phenomenon is more complicated, and the results obtained seemed at first difficult of explanation.

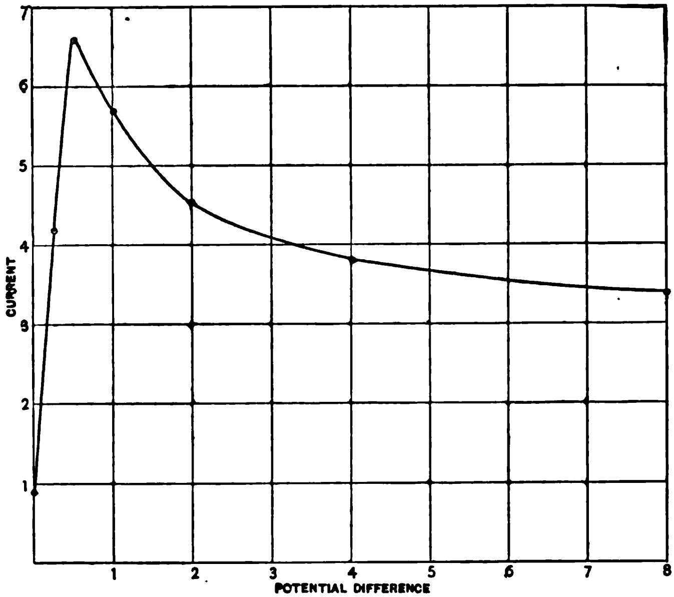

Fig. 5.

With the cylinder *B* (Fig. 2) charged to different potentials, the current passing between *B* and *M* was determined as in the experiments described in the last section. It was found that as the potential increased the current at first increased nearly in the same ratio. Indications of approaching saturation were however soon noticed, and when a potential of about 100 volts was reached, the current had attained its maximum value. If *B* was charged negatively a further increase in the potential difference caused a *diminution* in the current. For a potential difference of about 100 volts the current was nearly twice as great as for a potential difference of 1,000 volts. The results are shown graphically in Fig. 5.

When *B* was positive this peculiar behavior was not observed, and the saturation curve was found to have the usual form.

It appears to us that the results observed when *B* was negative are due to the electrostatic repulsion between the charged cylinder and the negative particles of the photo-electric rays. When the potential of the cylinder attains a value that is at all comparable with the potential of the kathode, this repulsion either deflects the rays, so as to produce a decrease in the number of particles passing near *B*, or reduces the speed of the particles. In either case the ionization in the region near *B* and *M* would be diminished, and the current between them might become less, in spite of the increased potential difference. Except for the low potential of the kathode, which was rarely above 1,000 volts, the effect would probably have been inappreciable.

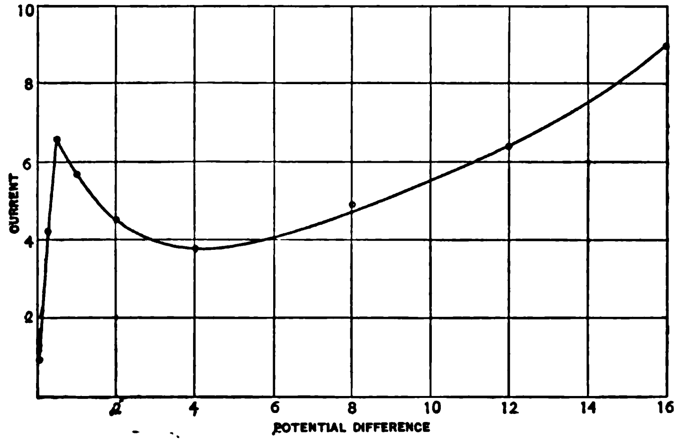

Fig. 6.

It appeared interesting to determine what would happen if the cylinder *B* was made more strongly negative than the kathode. In this case it would clearly be impossible for particles which owed their motion to repulsion from *K*, to proceed as far as *B*. At first thought we should therefore expect the current between *B* and *M* to fall to zero. Instead of this the saturation curve took the form shown in Fig. 6. The potential of the kathode in these experiments was, as usual, about 1,000 volts. Two considerations not mentioned in the preceding paragraph appear sufficient to explain these results. First, it seems probable that the current between *B* and *M* is not carried solely by the ions that are developed in the region immediately between the two, but that ions produced at a distance from *B* may also take part in the conduction. This might result in part from a slow diffusion of the ions, although the effect of a magnetic field in suppressing the current shows that the diffusion is quite small. The field due to the charge on *B* extends, however, throughout the whole of the region above *A*. It seems not unlikely that some of the lines of force

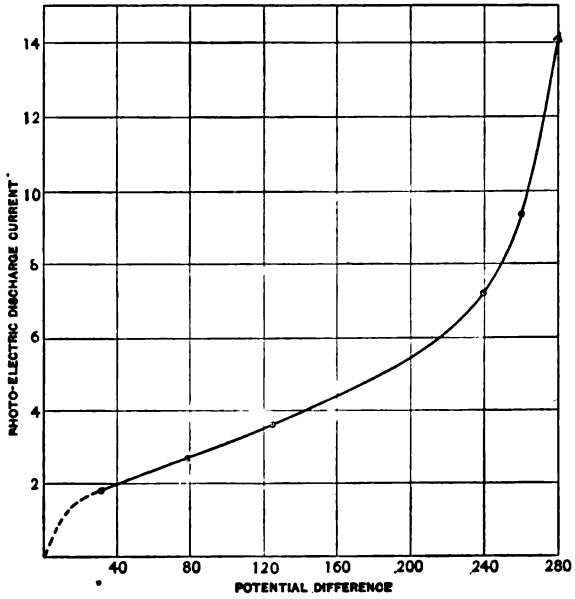

Fig. 7.

extending from *B* to *M* may pass through a region where ionization has occurred. In this case a current would flow, even if the space immediately surrounding *M* was absolutely non-conducting. The fact that the current not only does not vanish for high negative potentials, but actually increases, is probably related with the phenomena observed by Kreusler[1] and v. Schweidler,[2] who found that for potentials approaching that of normal discharge the photo-electric current increased with great rapidity. In Fig. 7 we reproduce one of the curves given by v. Schweidler. The air pressure in this case was about 17 mm. The results of Kreusler and v. Schweidler, together with those obtained in our own experiments, appear to us to indicate that when a gas is subjected to strong electric stress its susceptibility to the ionizing action of the photo-electric rays is greatly increased.

DISCUSSION AND THEORY.

The preceding experiments show that the photo-electric rays, like kathode rays, are deflected in passing through a magnetic field; that they render the air through which they pass conducting; and that in all probability they are deflected by electrostatic forces. That they carry a negative charge has long been known. The experiments of Thomson[3] and Lenard[4] show that the ratio e/m is the same, within experimental errors, as in the case of ordinary kathode rays. The fact that we have not yet been able to detect any fluorescence produced by the photo-electric rays is readily accounted for by the small intensity of the rays used in these experiments; and their failure to produce Röntgen rays is doubtless due to a similar cause.

In view of these facts the conclusion seems to us warranted that the photo-electric rays are identical in character with ordinary kathode rays.

The mechanism of the process by which the photo-electric rays are produced is still far from clear. It is not even known whether they come from the material of the charged body itself or from the layer of gas at the surface. In the theory of the the photo-electric discharge advanced by Elster and Geitel[5] the rays are assumed to originate in the gaseous layer. Lenard, on the other hand, having found that photo-electric rays are developed even at the highest vacuum attainable, is of the opinion that the effect is independent of the presence of air ; in this case the material of the kathode is the only remaining source. It does not appear to us that Lenard's results are conclusive on this point; for although he probably worked with a vacuum as nearly perfect as any that has ever been obtained, yet an *absolute* vacuum is of course out of the question. Again, it is known that there is a tendency for a layer of gas to form over a metallic surface which the most persistent pumping and heating fail to entirely remove. We are, therefore, of the opinion that the source of the corpuscles that form the photo-electric rays must still be regarded as unknown. It may be mentioned in passing that the same uncertainty exists in the case of ordinary kathode rays.

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[1] Über den photo-elektrischen Effect in der Nähe des Entladungpotentiales, Verh. der phys. Ges., Berlin, Vol. 17, No. 8.

[2] Sitzungsberichte der k. Akad. zu Wien., 108, IIa, p. 273, 1899.

[3] Phil. Mag. 48, p. 547.

[4] Annalen der Physik, Vol. 2, p. 359. 1900. § 3.

Although there appears to be little doubt that real kathode rays may be developed, at sufficiently low pressure, merely by illuminating a suitable surface with ultra-violet light, there is no experimental evidence that the rays can be developed in this way at atmospheric pressure. Yet the photo-electric discharge occurs at atmospheric pressure as well as in a vacuum. There are important differences between the two cases ; but it appears to us that the phenomenon must be essentially the same in both. The following way of looking at the matter seems to us sufficiently plausible to deserve consideration.

We assume first that the primary effect of illumination is to cause the development of kathode rays, and that these rays are produced not only at low pressures, where it has been possible to detect them directly, but also under ordinary atmospheric conditions. Remembering that the rays make the air a conductor, it is seen that the photo-electric current will in general consist of two parts : (1) the current carried by the photo-electric rays themselves ; (2) the current carried by the ionized air. At high vacua practically the whole current will be carried by the photo-electric rays. But as the pres-

[5] Wiedemann's Annalen 55, p. 697.

sure of the surrounding air increases, the rays will be more and more completely absorbed, and since this absorption results in ionization, the conduction current through the air will become more important. At atmospheric pressures the rays are probably absorbed in the immediate neighborhood of the charged surface. In this case the discharge is almost wholly carried by electrolytic conduction; but since the ions are developed very close to the surface the case differs essentially from that of the ionization produced in gases by other causes. In most cases both positive and negative ions are present throughout the gas. But in the photo-electric discharge the positive ions pass at once to the kathode and are destroyed.

Physical Laboratory of Cornell University.

A NEW METHOD OF MEASURING THE RESISTANCE OF A GALVANOMETER.

By Wm. S. Day.

THE following method of measuring the resistance of a galvanometer I have not seen described before, although it has very likely occurred to others. It is not always more accurate nor more convenient than other methods, but as it could be used with a supply of apparatus which would not allow of using the usual other methods, it may be worth putting on record. It might also be of use as a laboratory experiment for students.

The method is based on the simple notion that if the terminals of a galvanometer are brought to a certain potential difference, and the resulting current through the galvanometer gives a certain constant deflection, when the needle comes to rest, then, if we double this potential difference, in order to keep the same current through the galvanometer and the same deflection, it will be necessary to double the resistance of the galvanometer circuit by adding a resistance equal to that of the galvanometer. In order to be able to double the potential difference, use may conveniently be made of the uniform fall of potential along a wire of uniform resistance per unit length, such as the wire of a Wheatstone wire bridge, when it is carrying a constant current. The galvanometer terminals may be placed in contact with points on such a wire a suitable distance apart, and the constant deflection noted. This distance may then be doubled and at the same time a resistance may be inserted in series with the galvanometer and varied until the deflection becomes the same as at first. Then supposing the resistance of the part of the bridge wire between the terminals is negligible in comparison with the galvanometer resistance, the resistance inserted will be the same as that of the galvanometer. If, however, it would not be proper to overlook the resistance of this portion of the wire in comparison with the galvanometer resistance, its effect may be allowed for by taking a third position of the terminals three times as far apart on the bridge wire as their original position, and observing what resistance is then necessary to keep the galvanometer current the same. The manner of doing this will be shown presently. Of course, it is not even necessary and sometimes may be impossible to double and treble exactly the distance apart of the points of contact. The formula covering this more general case from which the simple cases follow, can be deduced by writing down the condition that the current through the galvanometer, each time the contact is made, is the same. The diagram represents the state of affairs. There is a battery cell, a suitable but not necessarily known resistance, and a bridge wire, in series. The two galvanometer wires make contact with the points P and Q_1, in the first part of the experiment. The dotted lines represent the arrangement of the galvanometer wires in the second and third parts of the experiment, in which while one terminal is kept at P, the other one makes contact successively at the points Q_2 and Q_3. The notation is

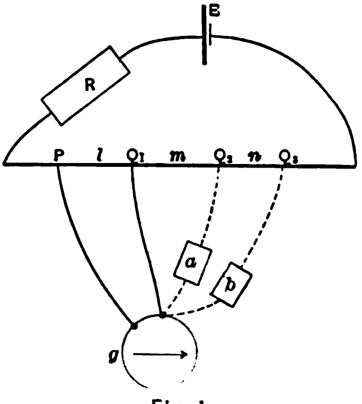

Fig. 1.

E = the electromotive force of the battery, supposed constant.

R = the whole resistance of the battery circuit up to the extreme points P and Q_3 at which contact is made.

ρ = the resistance of the bridge wire per unit length.

l = the length of the segment of the wire PQ_1.

m = the length of the segment of the wire Q_1Q_2.

n = the length of the segment of the wire Q_2Q_3.

g = the galvanometer resistance.

a = the resistance added to the galvanometer circuit when the terminals are at P and Q_2.

b = the resistance added when they are at P and Q_3.

In addition let

C = the constant current through the galvanometer.

C_1, C_2, C_3 = the currents in the three cases flowing through the main battery circuit. The current through the battery circuit in the three cases will be given by the three expressions

$$C_1 = \frac{E}{R + (m+n)\rho + \dfrac{\rho l g}{\rho l + g}}$$

$$C_2 = \frac{E}{R + n\rho + \dfrac{\rho(l+m)(g+a)}{\rho(l+m) + g + a}}$$

$$C_3 = \frac{E}{R + \dfrac{\rho(l+m+n)(g+b)}{\rho(l+m+n) + g + b}}.$$

The galvanometer current will be the fractional part of these currents represented by the equations

$$C=\frac{\rho l}{\rho l+g}C_1=\frac{\rho(l+m)}{\rho(l+m)+g+a}C_2=\frac{\rho(l+m+n)}{\rho(l+m+n)+g+b}C_3.$$

Substituting in these the expressions for C_1, C_2 and C_3 we get the following two equations:

$$\frac{\rho lE}{(\rho l+g)[R+(m+n)\rho]+\rho lg}$$
$$=\frac{\rho(l+m)E}{[\rho(l+m)+g+a](R+n\rho)+\rho(l+m)(g+a)}$$
$$=\frac{\rho(l+m+n)E}{[\rho(l+m+n)+g+b]R+\rho(l+m+n)(g+b)}.$$

It will be seen that the factor ρE can be divided out at once and on eliminating R it will be found that ρ appears merely as a factor. Dividing it out and solving for g we get the expression

$$g=\frac{l[(l+m+n)(m+n)a-m(l+m)b]}{mn(m+n)} \qquad (1)$$

which gives g in terms of the measurable quantities.

If in (1) $l=m=n$, the equation is reduced to the simple form

$$g=3a-b. \qquad (2)$$

If ρl were negligible in comparison with g it would not be necessary to find the resistance b, and the equation would be

$$g=\frac{l}{m}a. \qquad (3)$$

Or, if $l=m$,

$$g=a \qquad (4)$$

So far as the formulæ show there does not appear to be any reason why the method could not be used with galvanometers of any resistance or of any sensitiveness. With sensitive galvanometers, however, thermoelectric effects may give irregular deflections, although these are fully as troublesome in using Lord Kelvin's method. To avoid them as much as possible, care must be taken to touch the wires and connections with the warm hand as little as possible. The chief difficulty to be met with in applying the method to sensitive d'Arsonval galvanometers using a phosphor bronze suspension ribbon is the tendency for the zero to change owing perhaps to elastic fatigue or variation of some kind of the suspending wire. This can be partly avoided by taking the readings quickly and always observing the actual zero point before each deflection is taken. The method was applied to the following five galvanometers, whose resistances are given and the number of the formula applied:

1.	D'Arsonal, made by Queen & Co.	1097.	ohms.	(4)
2.	Rowland d'Arsonval, made by Willyoung.	1204.	"	(4)
3.	Astatic galvanoscope, made by T. W. Gleeson.	10.39	"	(1)
4.	" " " L. E. Knott.	0.86	"	(1)
5.	Voltmeter (without multiplier) Weston.	1.08	"	(1)

The resistance of each of these instruments as given was measured with the help of another galvanometer by the ordinary Wheatstone bridge method, the temperature of the room being observed at the same time. After making a number of trials so as to discover the source of the errors likely to arise and the precautions necessary to avoid them, it was found possible to measure the resistances by the new method described, within 2 per cent. or 3 per cent. Kelvin's method[1] seemed less accurate than this for the Queen instrument, and about the same for the Rowland and Weston instruments. The Queen instrument gave a deflection of 1 millimeter on a scale 1 meter distant, for a current of about 1.5×10^{-9} amperes. The Rowland instrument required about four times as much current for an equivalent deflection. The wire used for the measurements was one on a cheap form of Wheatstone wire bridge for students' use. It was 1 meter long, and had a resistance of about 6.5 ohms. As any piece of uniform wire stretched on a board could be used, the only other piece of apparatus necessary beside the battery would be a resistance box, and a few pieces of wire. Lord Kelvin's method could not easily be used with such a supply of apparatus.

The unknown contact resistances where the galvanometer terminals are placed in contact with the wire, may become an important cause of error in measuring a very low resistance galvanometer.

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[1] Maxwell, Elect. and Mag., 3d ed., vol. 1, p. 489.

THE VERIFICATION OF BOYLE'S LAW.

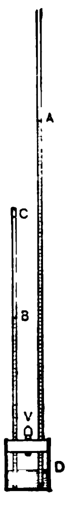
By N. H. Williams.

THE value of the experimental verification of Boyle's law as a laboratory exercise in elementary physics is unquestioned. It is usually the first inverse relationship that the student meets in his experimental work; it furnishes him a new study in the representing of relations by curves, and is on the whole one of the most valuable experiments in the elementary course.

The apparatus that has usually been employed for this experiment needs no description. It is acknowledged to have some defects, among which might be mentioned the stretching of the rubber tube connecting the two glass tubes. This usually causes considerable error in the work. The size, usually two metres long, is another objection to it, as is also the expense if many pieces are needed.

With the old type of apparatus, the rubber tube soon deteriorates, a leak develops, and several pounds of mercury are lost.

These are a few of the reasons why an effort has been made to improve the apparatus for this experiment.



In the figure *D* represents a small glass bottle with a wide mouth fitted with a rubber stopper. Two glass tubes about an eighth of an inch in internal diameter pass through the stopper and dip into the mercury in the bottle. The tubes are about eighteen and thirty inches in length respectively. The short one is closed at the upper end by fusing a glass plug into it. If this seems difficult to do, a little wax melted in over the plug will make it perfectly tight without fusing. This gives a square cut end from which to measure the length of the air column. The amount of air in the tube is adjusted by placing the apparatus in a horizontal position and allowing mercury to run into the tube till a column of air about 20 cms. in length remains.

V is a bicycle valve, which passes through a third hole in the stopper. The valve and the tubes are moistened with liquid glue before they are inserted. The stopper is also glued into the bottle.

When the apparatus is ready for use, a bicycle pump is employed to produce a pressure in the bottle sufficient to force the mercury nearly to the top of the long tube.

The apparatus is then placed before the mirrored scale of a Jolly balance, and the positions of *A*, *B*, and *C* accurately read. A little air is then allowed to escape from the bottle for the purpose of varying the pressure, and another set of readings is taken. This operation is repeated until ten or fifteen sets of readings have been taken.

$C = 18.00$ *Barometric reading* 74.20 *c. m.*

B	*A*	*B–A*	*P*	*V*	*PV*
30.43	13.40	17.03	91.23	12.43	1134
31.00	17.90	13.10	87.30	13.00	1135
31.55	21.90	9.65	83.85	13.55	1136
32.10	25.80	6.30	80.50	14.10	1135
32.63	29.15	3.48	77.68	14.63	1136
33.34	33.45	− 0.11	74.09	15.34	1136
34.02	37.11	− 3.09	71.11	16.02	1139
34.73	40.90	− 6.17	68.03	16.73	1138
35.39	44.00	− 8.64	65.59	17.39	1141
35.98	46.70	−10.72	63.48	17.98	1141
36.70	49.70	−13.20	61.00	18.70	1141

The difference, $B - C$, is taken as the volume of the confined gas, and the pressure is found by adding the barometric reading to the difference $B - A$. This difference becomes negative as A falls below B, thus giving pressures less than atmospheric.

The last column consists of the products of the volumes by the corresponding pressures. It shows an extreme variation of a trifle over one-half of one per cent.

It will be seen at once that this apparatus possesses advantages over the other. It is small and light, is inexpensive and easily made. The mercury does not come in contact with rubber. Errors due to the stretching of rubber are avoided and the results are considerably better than those usually obtained by the other apparatus.

The gradual increase in the products with decreasing pressure is an error due largely to the presence of moisture in the enclosed gas.

CENTRAL HIGH SCHOOL, DETROIT, June 16, 1900.

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THE PHYSICAL REVIEW.

EFFECT UPON THE PERSISTENCE OF VISION OF EXPOSING THE EYE TO LIGHT OF VARIOUS WAVE LENGTHS.[1]

By Frank Allen.

IN the early part of this century Plateau[2] engaged in some researches upon the persistence of vision. A disc composed of alternate black and white sectors was rotated at a speed just sufficient to give the surface the appearance of a uniform illumination, the velocity then giving the measure of the duration on the retina of the images of the white portions. By replacing the white sectors in turn with others of red, yellow and blue, he found that the duration of the images was different for the different colors, being least for the yellow and greatest for the blue. In a much more recent paper[3] the same writer extended his investigations, but used no method capable of giving more accurate or more comprehensive results.

Shortly after the publication of this latter paper, E. L. Nichols[4] introduced the method of using the spectrum instead of pigmentary colors. In the path of the ray of light producing the spectrum was placed a disc with alternate sectors cut out, the rotation of which gave flashes of light followed by equal periods of darkness causing a flickering of the portion of the spectrum in the field of view. The measurement of the velocity of rotation of the disc, at the point when the rapid flickering gave way to a continuous band of color, determined the duration of the maximum intensity of the sensation. A series of observations throughout the spectrum showed that the duration of the color sensation was least in the neighborhood of the *D* line, and increased towards both ends of the spectrum, the duration at the extreme violet being much greater than at the extreme red.

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[1] Read before the American Association, New York, June 27, 1900.

[2] Plateau, Dissertation sur quelques propriétés des impressions produits par la lumière sur l'organe de la vue. Liege, 1829.

[3] Plateau, Sur une loi de la persistance des impressions dans l'oeil, Bruxelles, 1878.

[4] Nichols, On the duration of color impressions upon the retina. Am. Jour. Sci., Vol. 28, 1884.

Taking advantage of this method, Ferry[1] made a more extended study of the persistence of vision, in which he increased the accuracy of the measurements by using an electric motor to rotate the disc the speed of which was recorded electrically on paper placed on the cylinder of a chronograph. With this apparatus Ferry more accurately investigated the law of the duration of retinal impressions, showing that it was a function of the luminosity and entirely independent of the wave length. In addition to this he obtained the persistency curves for a number of color blind persons. Comparison of their curves with that obtained for a perfectly normal eye disclosed the fact that in the regions to which the eye was color blind, the values of the persistence were greater than in the normal, while not differing in other parts of the spectrum.

In his investigations, Abney[2] has shown that the luminosity and persistency[3] curves of persons naturally color blind, or made so by disease, or by continued use of narcotics, invariably differed from similar curves for the normal eye.

Very recently Burch[4] has studied the spectrum by producing temporary blindness to the different colors in turn. Sunlight was allowed to stream through glass of the desired color and focused on the eye by means of a lens. After exposure for some minutes to this intensely bright color, the retina becomes irresponsive to that particular sensation, remaining so for a length of time depending on the wave length of the fatiguing color. The recovery of the retina was most rapid from the red and least from the violet. By this method, Burch confirmed the fundamental color sensations of the Young-Helmholtz theory, but in addition claimed evidence for believing blue to be a fourth sensation.

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[1] E. S Ferry, The Persistence of Vision. Am. Jour. Sci., Vol. 44, 1892.

[2] Capt. Abney, Colour Vision. Tyndall lectures at Roy. Inst., 1894.

[3] By "persistence" Abney means the reciprocal of the intensity of the light at the point of extinction, which is different from the meaning of the word in this paper.

[4] G. J. Burch, On artificial temporary color blindness. Proc. Roy. Soc., Vol. 63, 1898, p. 35. (Abstract.)

It was suggested to the writer that a combination of this method with that of Ferry would probably lend itself to a more conclusive determination of the fundamental color sensations.

Description of Apparatus and Method of Observation.

The apparatus consisted of a Browning two-prism spectrometer, a small motor with a disc attached, and an arrangement for registering the speed of the latter. An acetylene flame in a "Perfection" burner was the source of light, which gave a brilliant spectrum of considerable dispersion. The gas was supplied to the burner at a constant pressure. As the rotation of the disc caused air currents which disturbed the flame, it was found necessary to place the burner on a shelf attached to the wall at some distance behind the motor, and to focus the light on the slit by means of a lens. No flicker could be seen in the spectrum except that due to the motion of the disc, which rotated in the path of the rays of light between the lens and the slit. As the disc had alternate sectors of 90 degrees cut out, the duration of a single flash of light was therefore one quarter of the time of a complete revolution. The general diagram of the apparatus is shown in figure 1. The speed of the motor was regulated by means of a simple form of brake (see fig-

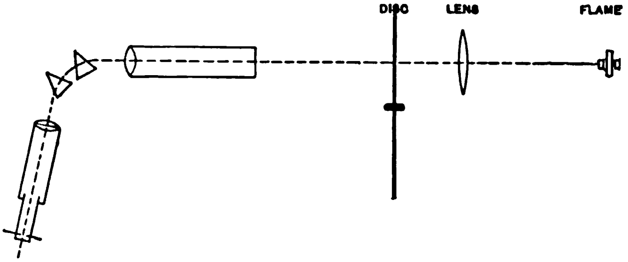

Fig. 1.

ure 2), made by passing a strip of leather over the smooth groove in the thumb screw that held the disc on the axle of the motor. One end of this leather was fastened to a spiral spring screwed to the table; the other had a long cord attached which was wound round a long brass rod turning in a friction socket, also screwed to the table. By turning this rod the string could be wound or unwound, and the friction of the leather on the axle of the motor increased or decreased. With this simple friction brake, the motor could be maintained constant at any speed from zero up to 1800 revolutions per minute for a period of several hundred revolutions.

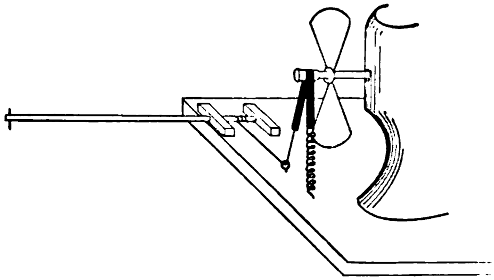

Fig. 2.

To record automatically the speed of the disc a small hand speed-counter was clamped in a position so that its rubber tip pressed firmly against the rear end of the axle of the motor (fig. 3). The rubber tip served to prevent the relative slipping of motor and speed-counter, and to insulate the latter, which was necessary as it carried a current. The small index on the counter was re-

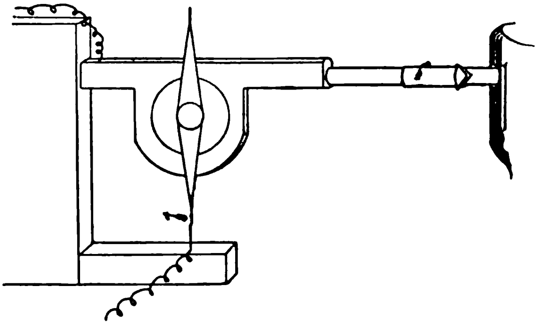

Fig. 3.

placed by a long diametral arm, the ends of which in turn closed the primary circuit of an induction coil at every fiftieth revolution of the motor. The secondary circuit of this coil passed through metal parts of a chronograph, with a narrow spark gap at the surface of the cylinder. At every fiftieth revolution of the disc, therefore, a spark perforated a piece of paper carried on the cylinder. The pendulum of a clock, by a mercury contact, closed the primary circuit of another coil every second, and similar perforations were made at these intervals on the same piece of paper. The time of fifty or one hundred revolutions of the disc could then be easily measured, and the duration of the flashes of light on the retina computed. In order to have the registering apparatus completely and conveniently under the control of the experimenter, the primary circuits of both coils ran to a double mercury key within easy reach. At the proper moment both circuits were closed simultaneously, and the clock and motor automatically made their records for whatever length of time was necessary.

The method of taking observations was as follows: By means of a diaphragm in the eye-piece of the spectrometer a narrow portion of the spectrum was isolated. The motor was then started at such a speed that the flickering of the narrow band became quite distinct. By relaxing the friction-brake, the speed was gradually increased until the flickering just disappeared and a uniform band of color was presented to the eye. The speed of the motor was then kept constant while the circuits were closed and the clock and speed-counter made their records. Generally the time sufficient for one hundred revolutions of the disc was recorded as being most convenient for purposes of computation. A little practice enabled one to complete a setting in half a minute. Where greater accuracy was required, the slight tiring effect was avoided by making the first adjustments with the left eye, after which the setting could be obtained by the right eye with an exposure of less than half the time mentioned. After each observation in the case of the normal curves, the eye was rested in diffused daylight for at least ten minutes. Two and sometimes more settings were made at each part of the spectrum; and so constant were the different adjustments of the apparatus, the intensity of the light, and the sensitiveness of the eye, that observations taken at different times, and even on different days, differed from the mean value by less than two per cent.

The room within which the apparatus was set up was dark, and all extraneous light was excluded; the only illumination being due to such light as escaped through the screens surrounding the acetylene flame.

To reduce the circle readings of the spectrometer and thus identify the parts of the spectrum in which observations were made, readings were made for various Fraunhofer lines and a calibration curve of the usual form was plotted.

In this investigation a large part of the work done was with the eye fatigued with variously colored light. In order to conveniently attain this object, a plan was adopted differing from that described by Burch in the paper already cited. In the dark room an arc light was placed on a stand in front of which hung a dark cloth curtain. Light passing through a hole in this was focused by a lens upon the slit of a large direct vision spectroscope, which also was calibrated, so that the wave length of the fatiguing light could be found. A diaphragm in the eyepiece cut off all the brilliant arc spectrum except a narrow band about two millimetres in width of the required color. The intensity of the light also rendered a very narrow slit possible, thus increasing the purity of the spectrum. The eye was fatigued by allowing a narrow band of any pure color from this spectrum to fall upon the center of the retina for at least five minutes, a length of time found sufficient to give the degree of fatigue desired. This accessory apparatus was so placed that when the eye had become fatigued the observer, by simply turning in his chair, could make the setting for the persistence, as described above, without any loss of time, and before the eye had any time to recover to any appreciable extent from the tiring process. In taking a series of observations this fatiguing operation was repeated before each setting, after the completion of which the recording apparatus required readjustment, which gave the eye one minute rest. No extended rest was considered necessary during the progress of a series of measurements, as Burch found that any fundamental color sensation could be exhausted without affecting the others. That this statement was abundantly justified will be seen upon a comparison of the normal persistency curves with those obtained for the eye when fatigued with various colors. The observations could therefore be taken with great rapidity.

When the eye was fatigued with any color, a piece of white paper placed where the white light from the arc fell upon it, appeared of a tint always complementary to the color affecting the eye. This afforded a convenient means of enabling the experimenter to determine the degree of exhaustion of the retina.

The time allowed for the fatiguing process with all colors was about five minutes. The uniformity of the measurements, for some of which this time was even doubled, seemed to indicate that with light of a given intensity the exhaustion of the color sensation reached a maximum in a few minutes, and that further exposure produced no effect. The following experiment served to confirm this inference· The eye was exposed to a band of pure red, the center of which had a wave length, $\lambda = .680\mu$, and persistence observations were made on a similar band of red the center of which had a wave length $\lambda = .758\mu$. Normal persistence = .0179 sec. The results are as follows:

Time of fatigue.	Persistence.
7 minutes.	.0195 sec.
5 "	.0193 "
10 "	.0188 "
20 "	.0188 "

Less than a minute rest was allowed between the readings except for the last which was made two days later. The eye accordingly was under a continued exhausting stimulus with the object of bringing about a greater degree of fatigue. The maximum exhaustion was apparently reached in seven minutes, though the other values may only differ by an experimental error. It may be, however, that the bright light tires the retina to a certain degree at first, after which the eye adjusts itself to the new conditions and partially recovers its normal condition. The above values would indicate this, though sufficient data are not yet available to establish this supposition. In any case the retina ultimately reaches a point of equilibrium between the destructive action of the external stimulus and the constructive power of the eye. If the luminosity of the source of light is increased, the degree of exhaustion will undoubtedly be greater, and the maximum may be reached in the same or even in less time.

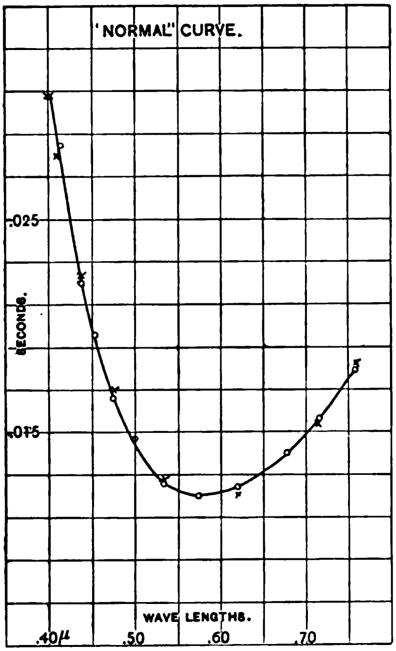

Fig. 4.

Normal Persistency Curves.

The first persistency curve obtained was the normal. As this was the standard with which curves for abnormal conditions of the eye were to be compared, the readings were taken with great care to avoid all sources of error. The eye was allowed to rest in diffused daylight for more than ten minutes before each reading. The values of this series of settings are given in Table I. They are also graphically shown in Fig. 4, in which ordinates represent the persistence of vision for various parts of the spectrum in seconds, and abscissæ are wave lengths.

To guard against error, observations with the eye in its normal condition were frequently made to see if any change had taken place. A series of such settings taken towards the latter part of the investigation is given in Table II. Their relation to corresponding readings upon the original normal curve is indicated by X's in Fig.

Table I.

Normal curve.

Wave length.	Persistence.	Wave length.	Persistence.
.400 μ	.0308 sec.	.533 μ	.0126 sec.
.415 "	.0285 "	.573 "	.0120 "
.438 "	.0220 "	.620 "	.0124 "
.454 "	.0196 "	.678 "	.0140 "
.475 "	.0166 "	.758 "	.0179 "
.500 "	.0147 "		

5. This curve, like the ones given by Nichols and by Ferry in the papers cited, shows that the persistence is least near the *D* line and increases towards both ends of the spectrum. The curves are parabolic in form.

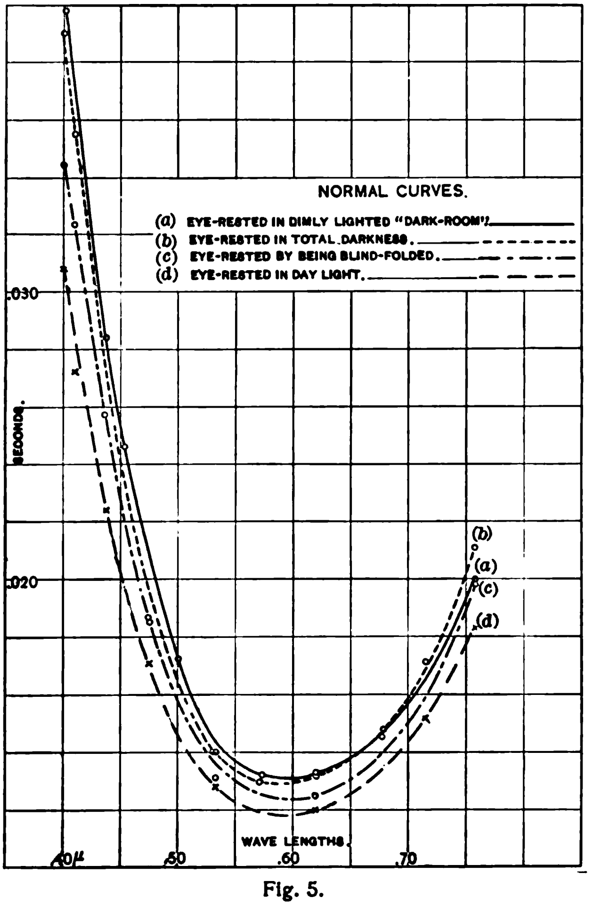

Fig. 5.

TABLE II.

Supplementary normal settings.

Wave length.	Persistence.	Wave length.	Persistence.
.400 μ	.0308 sec.	.533 μ	.0128 sec.
.410 "	.0272 "	.620 "	.0120 "
.438 "	.0224 "	.715 "	.0152 "
.475 "	.0171 "	.758 "	.0183 "

It was accidentally noticed at one time during a series of observations, that the duration of visual impressions was increased when the eye had been resting in the dark. This phenomenon was thereupon studied in some detail. Before each observation the eye was allowed to rest in the dark room in which the experiments were being conducted. This room was not completely dark, as the walls could be dimly seen, owing to the stray light from the acetylene flame. The results are given in column *a* of Table III. After coming out into the light, two of these readings were repeated and the following values obtained:

Table III.

Persistence after exposure to varying degrees of light and darkness compared with "normal" persistence.[1]

	Persistence under various Conditions.			
	a Eye rested in dark room.	*b* Eye rested in total darkness.	*c* Eye blindfolded.	*d*[2] Eye "normal" (exposed to diffuse day light).
.400 μ	.0398 △	.0390	.0344	.0308
.410 "	——	.0355	.0323	——
.415 "	——	——	——	.0285
.438 "	——	.0284	.0257	.0220
.454 "	.0246	——	——	.0196
.475 "	——	.0185	.0187	.0166
.500 "	.0172	——	——	.0147
.533 "	.0140	.0140	.0131	.0126
.573 "	.0132	.0130	——	.0120
.620 "	.0133	.0132	.0125	.0124
.678 "	.0145	.0148	——	.0140
.715 "	.0171	——	——	——
.758 "	.0200	.0211 ▽	.0199 ▽	.0179 ▽

Wave length.	Persistence.
.678 μ	.0132 sec.
.620 "	.0125 "

On comparing these with the values given in Table III (*a*) for the same wave lengths, the difference will be at once noticed.

It was thought that, as the walls of the dark room could be seen indistinctly, the eye was subject to a strain which might be really fatiguing it, and so increase the persistence of the color impressions. The observations were accordingly repeated, this time with the precaution of resting the eye in total darkness. The results are given in column *b* of Table III.

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[1] Arrows indicate the order in which settings were made.

[2] Repeated for convenience of comparison from table I.

These values are a little greater in the red than the corresponding ones in column *a*, but the two curves lie very close together, as will be seen by inspection of Fig. 5, in which these curves are compared. It is quite evident, therefore, that the increased sensitiveness of the retina in the dark increases the persistence of vision, which is just the opposite of the anticipated effect. The increase in the values of the persistence of different wave lengths is not uniform, being about 15 per cent. greater than the normal in the extreme red, 11 per cent. in the yellow, and 30 per cent. in the extreme violet. This result harmonizes with a statement of Parinaud,[1] who found that " the increase in the sensitiveness for light which is produced on a retina placed in the dark, is not equal for rays of different refrangibility." He further states that this is true for all parts of the retina except the yellow spot. As the observations described in this paper extended at the same time over the yellow spot and surrounding portions of the retina, it is not possible to state conclusively whether the above statement is confirmed or not. The persistence of vision being a function of the sensitiveness of the retina as well as of the intensity of the light stimulus, connects what is found in the curves just described with the observation of the French writer.

In these last experiments the eye was kept open while being rested, and it was decided to further test the supposition that the increased values of the duration of the impressions might be due to the strain of adaptation of the retina, or to some kind of muscular strain. For this purpose the right eye, the one used in all these experiments, was rested by being closed and blindfolded. All muscular strain at least was removed in this way, and the results, in column *c*, Table III, show that a different condition was induced in the eye.

These values, shown graphically with the others in Fig. 5, are intermediate between what may be termed the ordinary and extraordinary normal curves. In this comparison it will be seen that the hypothesis that the great increase in the duration of the color impressions is due to some kind of muscular strain is substantiated partially, since the elimination of that possible disturbing factor brought about a decrease in these values. There still remains a substantial increase which must be accounted for, the most obvious explanation being that it is due to the increased sensitiveness of the retina. This curve also shows that the increased sensitiveness of the retina is unequal for waves of different lengths, the increase above the normal being 9 per cent. in the red, 5 per cent. in the yellow and 12 per cent. in the violet. These results may be looked at from another point of view. According to the Hering theory, the sensation, or rather the lack of sensation, black, has the power of assimilation of the white-black visual substance. The eye in its normal condition is under the mild influence of daylight which would dissimilate this substance to a certain extent. Darkness, being the absence of any stimulus of the retina, permits the restoration of the white-black substance to its maximum sensitiveness. In case this is the explanation of the phenomenon, why should the restoration of this visual substance affect the various colors at all, and especially different colors in different degrees? Instead of the visual substances of the Hering theory being independent of each other, some connection must exist between them. Darkness cannot be held to assimilate simply the white-black substance, but also the red-green and yellow-blue. It would at least be supposed that if restoring the white-black substance did affect the others, that it would affect the red-green substance equally in both red and green, and similarly the yellow-blue substance. But the persistence of the red is increased 9 per cent., while the increase in the green is 13 per cent. The discrepancy is far greater in the other case, as the increase in the yellow is only 5 per cent., while in the blue it is 13 per cent. These facts can hardly be said to be in harmony with the theory in question. Similar objections can not be urged against the Young-Hemholtz theory, since it does not group the color sensations into pairs of opposites, nor does it limit the restoring power of perfect rest—darkness—to counteracting any specific stimulus of the retina.

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[1] M. H. Parinaud, De l' intensité lumineuse des couleurs spectrales. Comp. Rendus, Vol. 99, 1884, p. 937.

We have found so far that the persistence of vision is a maximum for all colors when the eye is rested by being kept open in total darkness. The values fall to an intermediate position when the eye is rested by being blindfolded, and reach a minimum when the rest has been given in diffused daylight. It would be expected that a considerable degree of exhaustion of the retina would have an opposite effect to that we have been discussing. But the experiments now to be described show that the opposite is the case—that fatiguing the retina causes the duration of the color impressions to increase again.

Effect of Fatiguing the Retina with Various Colors.

Retina fatigued with red.

The eye was fatigued, as described in the first part of this paper, by looking at an isolated portion of a brilliant arc spectrum for about five minutes, the process being repeated before every observation. The condition of the retina with the red sensation partially exhausted, approximates that of the naturally red-blind person.

In this process a band of color about 2 mm. wide was used, the center of which in the case of red had a wave length $\lambda = .680\ \mu$.

The results of the measurements of the duration of the various color impressions upon the retina in this abnormal condition, are exhibited in Table IV, and graphically in Fig. 6, in comparison with the normal curve.

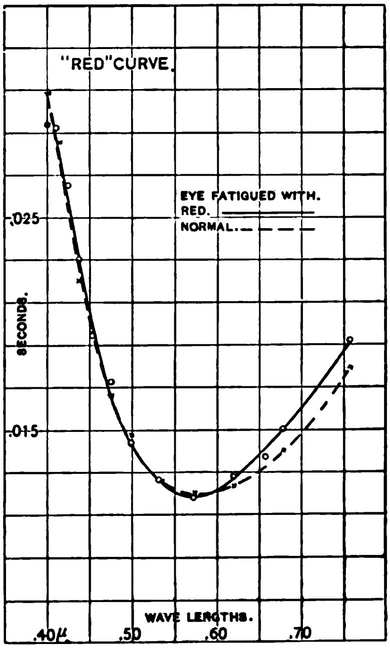

Fig. 6.

Comparing the two curves in Fig. 6, it is seen that fatiguing the eye with red has the effect of increasing the persistence of that color and also of about half of the orange, the remainder of the spectrum being unaffected in any way. The persistence of the red is increased by 8 per cent., while the increase in the same portion of the spectrum when the eye was blindfolded was 9 per cent., the two agreeing within the limits of experimental error. The Hering theory would certainly lead one to suppose that the green part of the curve would show some difference from the normal. A glance at the two curves, however, shows a perfect agreement between them. A similar lack of accordance of the experimental data with the anticipations of the Hering theory will be frequently noticed in the remainder of this paper.

Table IV.

Eye fatigued with red ($\lambda = .680\ \mu.$)

Wave length.	Persistence.	Wave length.	Persistence.
.400 μ	.0293 sec.	.533 μ.	.0126 sec.
.410 "	.0292 "	.573 "	.0118 "
.425 "	.0265 "	.620 "	.0128 "
.438 "	.0230 "	.657 "	.0137 "
.454 "	.0194 "	.678 "	.0150 "
.475 "	.0172 "	.758 "	.0192 "
.500 "	.0143 "		

It is to be noticed that the red sensation can be exhausted without affecting in any way the remaining sensations. This was a matter of some surprise; for upon looking at the acetylene spectrum after tiring the eye with red of the much more brilliant arc spectrum, all colors were apparently equally affected. The measurements showed that in this the eye was deceived. This confirms the statement of Burch in the paper previously cited, who, however, much more completely exhausted the color sensations than the author of this paper.

Retina fatigued with green.

The measurements in this case were made in the manner already described, but with the retina fatigued by a pure green light from the arc spectrum ($\lambda = .520\ \mu.$). The results are given in Table V, and graphically in Fig. 7.

This curve shows an elevation or bunch in the green indicating, as before, an increase of the persistence in that part of the spectrum,

TABLE V.

Eye fatigued with green ($\lambda = .520\ \mu$).

Wave length.	Persistence.	Wave length.	Persistence.
.400 μ	.0311 sec.	.512 μ	.0150 sec.
.415 "	.0253 "	.533 "	.0126 "
.438 "	.0206 "	.573 "	.0120 "
.460 "	.0187 "	.620 "	.0125 "
.475 "	.0176 "	.678 "	.0137 "
.492 "	.0164 "	.758 "	.0185 "

due to the tiring process. The maximum increase of the persistence of the green is about 12 per cent. which again coincides with the increase in the persistence of the same wave length when the eye was blindfolded. The curve coincides with the normal in the yellow and red, but runs under it in the blue and violet. The Hering theory would lead us to expect the curves to differ in the red; the Young–Helmholtz in both violet and red if they differed at all. For if one sensation were exhausted it is possible that the intensity of the remaining sensations might be increased, producing the effect noticed in the blue and violet, and a similar effect in the red. The same phenomenon will be noticed in two other curves (see Figs. 8 and 9), though since it is but slight it may be due entirely to experimental errors.

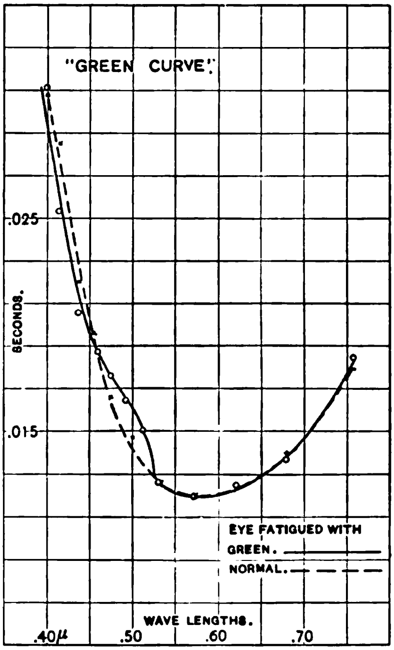

Fig. 7.

Retina fatigued with violet.

The following values were obtained when the eye was fatigued with violet:

TABLE VI.

Eye fatigued with violet ($\lambda = .400\ \mu$).

Wave length.	Persistence.	Wave length.	Persistence.
.403 μ	.0350 sec.	.502 μ	.0122 sec.
.410 "	.0287 "	.533 "	.0110 "
.423 "	.0227 "	.596 "	.0109 "
.434 "	.0198 "	.657 "	.0125 "
.443 "	.0188 "	.715 "	.0149 "
.454 "	.0161 "	.758 "	.0176 "
.468 "	.0149 "		

The data are plotted in Fig. 8 in comparison with the normal curve. The only part of the curve affected is the violet, which shows results similar to those obtained in the two preceding curves. There is an almost complete coincidence of the curves from the blue to the middle of the red, the fatigue curve then falling a little below the normal. This illustrates the remarks made upon a similar ef-

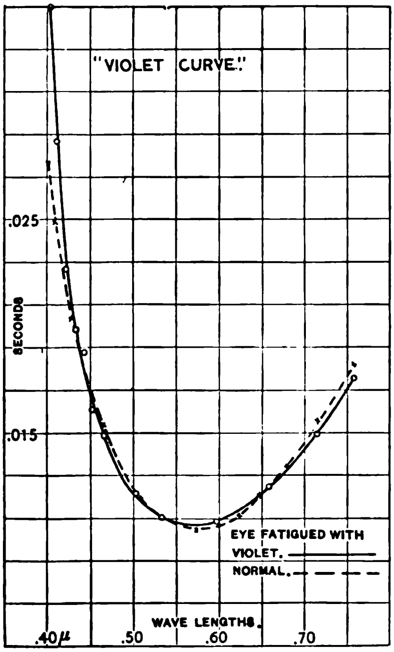

Fig. 8.

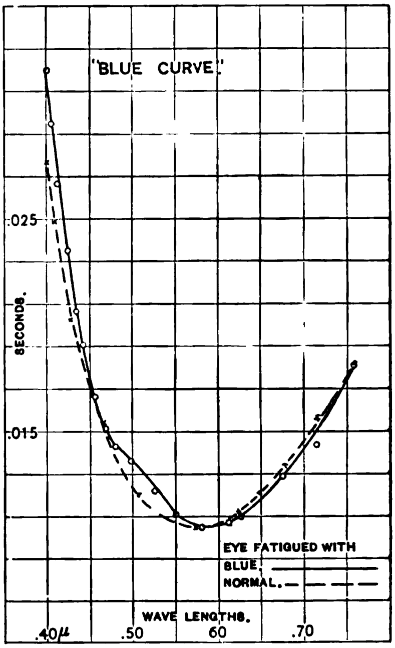

Fig. 9.

fect obtained in the abnormal curve for green. The persistence of the violet is increased by 27 per cent. These three curves for the eye tired with *red*, *green* and *violet* respectively, are of the same type, having one elevation which occurs in the part of the curve corresponding to the fatiguing color.

It would be quite natural to expect, in view of the results already discussed, that if blue and yellow are fundamental sensations in the same sense as are red and green according to either of the two most widely accepted color theories, similar elevations of the curve would be produced by using blue and yellow, respectively, as fatiguing stimuli, and these elevations would occur in the corresponding parts of the curves. In other words if blue and yellow are fundamental sensations, fatiguing the retina with these colors ought to affect these color sensations and no others. On the other hand, if blue and yellow are not fundamental, but compounded of two other sensations, then fatiguing with one of these ought not to show any effect in the part of the curve corresponding to that color, but should cause the elevations in the curve in the parts of the spectrum corresponding to those fundamental sensations which are affected. The latter is found to be the case.

Retina fatigued with blue.

Table VII shows the results obtained when the retina was exposed to blue light.

TABLE VII.

Retina fatigued with blue ($\lambda = .440\ \mu$).

Wave length.	Persistence.	Wave length.	Persistence.
.400 μ	.0320 sec.	.498 μ	.0136 sec.
.405 "	.0295 "	.525 "	.0122 "
.414 "	.0267 "	.550 "	.0111 "
.425 "	.0235 "	.580 "	.0105 "
.435 "	.0207 "	.612 "	.0107 "
.444 "	.0191 "	.625 "	.0110 "
.455 "	.0166 "	.675 "	.0129 "
.467 "	.0151 "	.715 "	.0143 "
.480 "	.0143 "	.758 "	.0182 "

These values will be found plotted in Fig. 9 in comparison with the normal curve. In the part corresponding to the blue, the two curves coincide, showing that in this region the persistence has not been changed by the fatiguing process. The persistence has however been increased in both the green and violet, which is shown by the elevation of those parts of the curve. In this case the increase in the green is 10 per cent. while for the eye fatigued with green it was 12 per cent. The increase in the violet is 16 per cent. against 27 per cent. for the case when the eye was fatigued with violet. The readings in the extreme violet were, however, subject to a greater experimental error, as the illumination of that part of the spectrum was comparatively feeble. The curve obtained for the eye exhausted with blue, is therefore practically the superposition of the green and violet curves. The deduction is obviously that the sensation of blue is the consequence of the excitation of the sensations of green and violet. It will be noticed that the blue curve runs under the normal in the red, a phenomenon observed in two previous curves. This is shown best in the violet curve, where the abnormal runs under the normal in both green and red, indicating that when the retina has been exposed to violet this sensation is exhausted, but the other fundamental sensations red and green, are rendered a little more acute.

Retina fatigued with yellow.

We have now to examine the results obtained when the fatiguing color was yellow. The arc spectrum was dispensed with in this case, and the arc was replaced by a sodium flame. The yellow sodium light was of sufficient intensity to fatigue the eye which was placed at the focus of a lens. The following results were obtained:

TABLE VIII.

Retina fatigued with yellow ($\lambda = .589\ \mu$).

Wave length.	Persistence.	Wave length.	Persistence.
.400 μ	.0312 sec.	.533 μ	.0143 sec.
.415 "	.0302 "	.573 "	.0124 "
.438 "	.0236 "	.620 "	.0124 "
.454 "	.0205 "	.647 "	.0128 "
.475 "	.0177 "	.678 "	.0140 "
.492 "	.0163 "	.715 "	.0164 "
.512 "	.0155 "	.758 "	.0201 "

This curve, plotted in Fig. 10, in comparison with the normal curve, shows the yellow, blue and violet regions to be unaffected, while elevations were produced in the green and red. The whole curve is a little above the normal, which is probably due to a slight change in the pressure of the gas making the spectrum a little less luminous. The maximum increase in the persistence of red impressions in this case is 10 per cent., while in the case of exposure to red it was 8 per cent. This may be compared also with the increase in the red when the eye was blindfolded, which amounted to 9 per cent. In the yellow curve the increase in the green is 12 per cent. which is exactly the same as in the green curve. The position of the maximum in the two cases is however not the same.

Yellow light, therefore, has the same effect on the retina as red and green together, and it must be concluded that the sensation of yellow is due to a simultaneous disturbance of the red and green visual apparatus. Whatever may be the "deliverances of consciousness" as to the purity of blue and yellow, in the light of experiment we are forced to conclude that they are not fundamental but compound sensations, as far at least as their effect on the retina is concerned. Further than that, this paper does not pretend to deal with them.

Retina fatigued with white light.

Having successively fatigued the retina with five well-defined spectral colors with the interesting results described, some curiosity was evinced as to the probable effect of tiring the retina with all wave lengths at once, that is with white light. From the standpoint of the Young-Helmholtz theory, it was thought that the curve in this case should show three elevations; in the red, the green and the violet. On the other hand, assuming the truth of the Hering theory, the white light would affect the white-black visual substance which is independent of the other two. Color sensations affecting the like-wise independent red-green and yellow-blue substances should accordingly show no change in persistence. The result does not confirm either expectation. To accomplish the fatiguing of the retina, the white light from the arc passed through a lens and fell upon a piece of white paper placed at a little distance from the focus. The round spot of light was partly transmitted and fell upon the retina for the usual time of five minutes. A series of after images followed which threatened to interfere with the accuracy of the observations. Upon looking at the portion of the spectrum in the field of view, it appeared as if seen through a hazy light of varying color. For the first few seconds this after image was of a faint yellowish tint which gradually deepened into an intense yellow completely obscuring the spectrum. The first hazy tint could be easily restored by winking, and this enabled the observer to take the readings always under the same conditions. The results given in Table IX, are so uniform as to inspire belief in their accuracy.

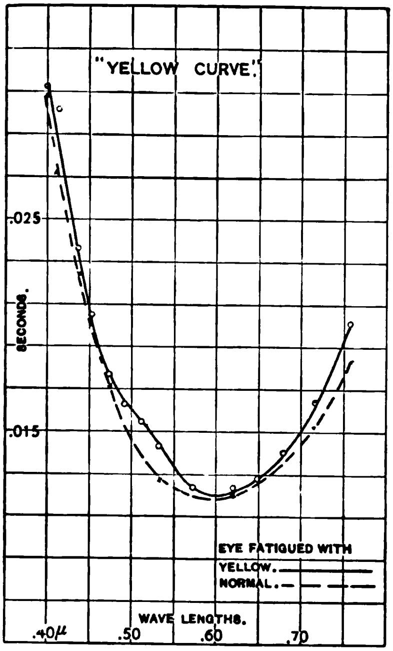

Fig. 10.

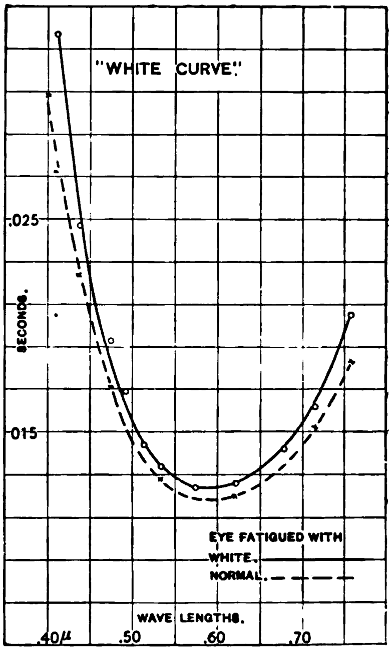

Fig. 11.

These data are plotted in figure 11 in comparison with the normal curve. The curves, it will be noticed, are approximately parallel, indicating no elevations in any part. Apparently this does not confirm the Young-Helmholtz theory of the sensation of white. The fact that the persistence of all the color sensations is increased shows an interdependence of the visual substances not provided for by the

Hering theory. But at any rate the curve seems to indicate the existence of a white sensation partially connected with and partially distinct from the fundamental color sensations. That white is either an independent or semi-independent sensation, is claimed by Ebbinghaus and by Mrs. Franklin in their theories, and Abney[1] has recently obtained experimental results which point to the same conclusion.

TABLE IX.

Eye fatigued with white light.

Wave length.	Persistence.	Wave length.	Persistence.
.410 μ	.0337 sec.	.573 μ	.0124 sec.
.438 "	.0247 "	.620 "	.0126 "
.475 "	.0193 "	.678 "	.0142 "
.492 "	.0169 "	.715 "	.0161 "
.512 "	.0144 "	.758 "	.0205 "
.533 "	.0134 "		

In the discussion of the various curves thus far considered (Figures 4 to 10), the *percentage increase* of the duration of color impressions has been given. For greater convenience of comparison these values have been brought together in the following table :

TABLE X.

Percentage increase in the duration of color impressions when the eye is fatigued by exposure to various colors.

Region affected.	Color of the fatigue stimulus.						
	Red.	Green.	Violet.	Blue.	Yellow.	White.	Eye blindfolded.
Red.	8 %	0 %	0 %	0 %	10 %	9 %	12 %
Yellow.	0	0	0	0	0	5	5
Green.	0	12	0	10	12	13	7
Blue.	0	0	0	0	0	13	10
Violet.	0	0	27	16	0	12	14

When it is remembered that the values in this table represent experimental results obtained at intervals during several months, that no attempt whatever was made to keep the eye in a condition of constant sensitiveness, and that the conditions under which the abnormal states of the retina were induced varied to a considerable extent from time to time, the results, especially in the red and green, show a uniformity that may very properly be considered remarkable. It can be no mere chance agreement of results. Certainly the retina, even under abnormal conditions, cannot have such a varying sensitiveness as is generally supposed. The values do not differ, for each color, by more than the presumable experimental error, except one value in the green and one in the violet, which will be evident on referring to the table. The averages of the tabulated results are as follows:

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[1] Abney, Sensitiveness of the Retina to Light and Color. Phil. Trans., A., 1897, Vol. 190.

Red,	9.75	per cent.
Yellow,	5	"
Green,	10.8	"
Blue,	11.5	"
Violet,	17.2	"

In all these experiments the retina has apparently been brought to a maximum abnormal condition, though the fatiguing light has been of varying intensity. There are not sufficient experimental data yet to show whether greatly increasing the intensity of the light exhausting the retina, will increase this maximum condition or not, though it is anticipated that it will.

It has already been pointed out that, when the retina is made abnormally sensitive by being kept in the dark, the persistences of the various color sensations are unequally increased. The same is true when the retina is fatigued; for when any colors have their persistences affected, the increase is unequal in different parts of the spectrum, in all cases being least in the yellow and reaching maxima in the red and violet.

BEARING OF THE FOREGOING RESULTS UPON EXISTING THEORIES OF COLOR SENSATION.

The Hering Theory.

According to this theory, as is well known, the color sensations are arranged in three pairs, red-green, yellow-blue, and white-black. Since the first sensation in each pair is produced by the dissimilation of a supposed photo-chemical substance, which exists in three forms to correspond to the three pairs of color processes, it would be expected that all being affected in precisely similar ways would show similar results. One would also expect the assimilating colors to show opposite effects. Whatever effect is produced by fatiguing the retina with red, an exactly similar effect should be shown with the yellow. So also for green and blue. Reference to the curves shows that red has an elevation in the red, while yellow gives two, neither of which corresponds to that color, and white gives none, or rather the whole curve is elevated above the normal. Similarly for the others—green, has one, blue two, and black none, if we may consider the curve obtained by blindfolding the eye as the one corresponding to black. We thus have three types of curves for the three pairs of color sensations. The agreement in number is the chief point of harmony with the Hering theory.

Analyzing the curves more particularly, we may observe, first, that acting on the red-green substance with red light has dissimilated it, producing a characteristic result. Assimilating this same substance with green light produces a similar result but in the part of the curve corresponding to green. Certainly the theory would expect each operation to affect both red and green parts of the curve, or else each would affect one and the same part but in different ways. Violet, too, giving a result similar in every way to those produced by red and green, must be placed on an equal footing with them, and whether it be considered as a dissimilation or an assimilation color its opposite must be found in the spectrum. Exhausting the retina with yellow discloses no special substance that is acted upon, but elevations are produced in both red and green. Translating this into the language of the theory, yellow simultaneously exercises the functions of both red and green to the same percentage amount, as we have seen, as each of them separately. Under these circumstances a balance should have been maintained between the opposing actions of red and green, which should have produced a curve differing in no respect from the normal. Supposing however that the action of the yellow stimulus is in harmony with the requirements of the theory, exposure of the retina to its assimilative companion blue ought to counteract the effects previously produced. Experiment shows that the green is affected, but in addition the violet is also affected which was not disturbed at all by the yellow. Further, by stimulating the retina with strong yellow and blue lights alternately, the red-green substance could be overbalanced, and red-blindness induced by the action of certain luminous waves which according to theory should have no effect in this direction. It is quite evident, therefore, that these experimental results are antagonistic to the theory in question. The curves obtained when the eye was rested in the dark and fatigued with white light at first sight seem to confirm the Hering theory; but since in both cases all the color sensations are affected, there must be an intimate connection between white, black, and the color processes. This, too, is not in harmony with the provisions of the Hering theory.

The Ebbinghaus Theory.

The same objections will also apply to the theory of Ebbinghaus,[1] which is practically a modification of that of Hering. The main difference is that while the latter postulates three visual substances which may be regarded as so many independent variables, the former separates these substances into four for the perception of color and one for white, making in all five independent variables. The fundamental color tones in this theory are the same as in that of Hering, viz., red, yellow, green and blue, two of which, yellow and blue, have been shown to be compound.

The Franklin Theory.

The ingenious color theory of Mrs. Franklin[2] is framed partly from the physical and partly from the psychological standpoint. This evolutionary theory regards color-sense as a development out of an earlier existing gray-white sensation. Guided only by the requirements of consciousness the fundamental color tones would be red, green, yellow and blue. But physical experiments have rendered it improbable that yellow is fundamental, so the theory is resolved into a three-color one, with a gray-white sensation in addition. Consciousness must be satisfied with only a part of its demands.

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[1] Ebbinghaus, Theorie des Farbensehens. Zeitschrift für Psychol. und Phys. der Sinnesorgane, Bd. V., Heft 3, 1893.

[2] C. L. Franklin, On Theories of Light-Sensation, Mind, Vol. 2, N. S. No. 8, 1893.

The experiments described in this paper are equally fatal to both blue and yellow as fundamental sensations, but can be considered hostile to this theory only as regards the blue. Mrs. Franklin might then go one step farther and assume the Young-Helmholtz color-sensations by the substitution of violet for blue.

The Young-Helmholtz Theory.

The experiments we have described are plainly seen to harmonize completely with the fundamental color processes of the Young-Helmholtz Theory. Red, green and violet being these fundamental sen-

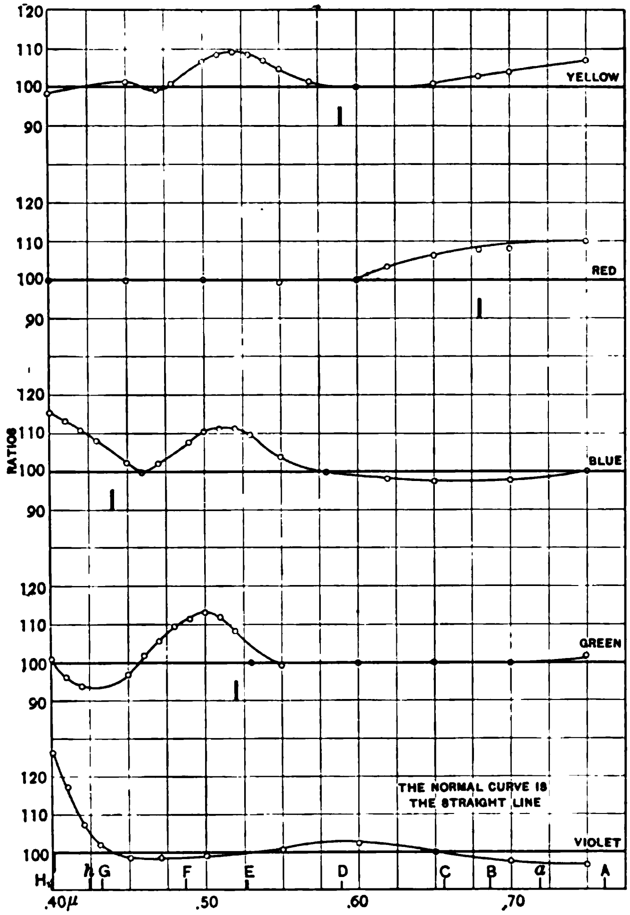

Fig. 12.

sations give characteristic results in an abnormal condition. Yellow and blue are both compounded of two other sensations, and experiment shows the theory verified. Rest in the dark makes all sensations more sensitive, which is shown by the curve obtained under this condition. The curve obtained for the action of white light, however, cannot be disposed of so easily, and, as before, we must consider it as evidence in support of a white sensation. There are many other color theories; but these we have considered may be considered typical of all.

At the suggestion of Dr. Nichols these curves have been drawn in another way which shows the deviations of the abnormal from the normal curves in a much more striking manner. All parts of the normal curve are reduced to the same ordinate, making that curve a straight line parallel to the axis of abscissæ. The corresponding values of the abnormal curves are then reduced in the same ratio and these ratios plotted. For convenience the ordinate of the normal curve is taken as 100. The variations from the normal, discussed in the preceding pages, are much more evident especially in the parts corresponding to the blue and violet regions of the spectrum. It is evident that this is a representation of what would actually have been obtained, had all parts of the spectrum possessed the same luminosity.

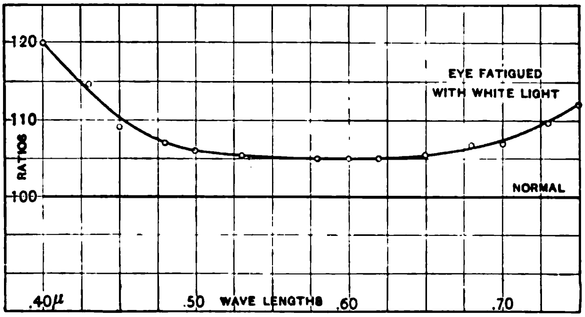

Fig. 13.

Only those curves obtained when the eye was fatigued with various colors and with white light are so reduced, as the others do not show their characteristics any better by this method. The five color curves are plotted together for comparison in Fig. 12, the white in Fig. 13. In addition the same curves are plotted in Fig. 14, with circle readings as abscissæ instead of wave lengths, which

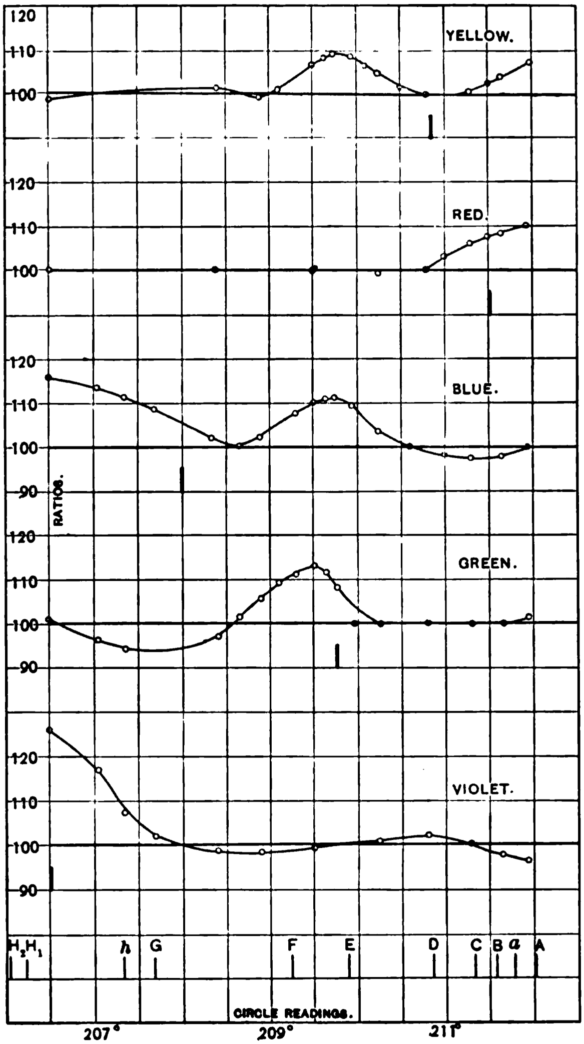

Fig. 14.

represents the conditions of experiment more accurately, as the observations were made on a prismatic spectrum.

In these last diagrams the part of the spectrum used for fatiguing the eye is marked by means of a vertical line under the respective curves.

EXAMINATIONS OF NATURALLY COLOR-BLIND PERSONS.

S. W. F., a student in Cornell University who was known to be color-blind, kindly consented to allow his eyes to be tested. Being accustomed to make scientific measurements, his observations were made with care and accuracy. The limits of the spectrum were the same to him as to the normal eye. The apparatus did not permit his neutral point to be determined accurately, but he indicated it to be at about $\lambda = .480\ \mu$. As is usual with red-blind persons, he called all on the red side of the neutral point yellow, and the other blue. When tested with the Holmgren worsteds he made the usual matches of the red-blind. In addition, however, he indicated blindness of some other part of the retina, as shown by his matches of light yellows and greens. Measurements were made *for both eyes* in the manner described for obtaining normal curves.

The following table gives the persistence of color impressions for the right eye:

TABLE XI.

S. W. F. (right eye).

Wave length.	**Persistence.**	**Wave length.**	**Persistence.**
.410 μ	.0297 sec.	.533 μ	.0133 sec.
.438 "	.0246 "	.550 "	.0128 "
.455 "	.0214 "	.573 "	.0133 "
.465 "	.0202 "	.596 "	.0124 "
.475 "	.0184 "	.620 "	.0128 "
.484 "	.0170 "	.678 "	.0161 "
.493 "	.0162 "	.715 "	.0191 "
.512 "	.0148 "	.758 "	.0224 "

The data are plotted in Fig. 15 in comparison with the third normal curve of the writer. These values show a very great increase in the duration of the impressions in the red; the curve then approaches nearly to the normal in the yellow, then rises in the yellowish-green forming a curious little bunch or elevation, thence continuing parallel to the normal through the blue and violet. It will be noticed that the whole curve is displaced upward, which indicates that the whole spectrum was not as bright to the color-blind eye as to the normal. This bunch in the yellow-green is not accidental nor is it due to an erroneous measurement, since it was confirmed by experiments several weeks apart. It is also confirmed in the curve for the left eye of the same subject, shown in figure 16 in comparison with a new normal curve of the writer, the determination of which was rendered necessary by a change in the pressure of the acetylene and consequently a change in the luminosity of the spectrum.

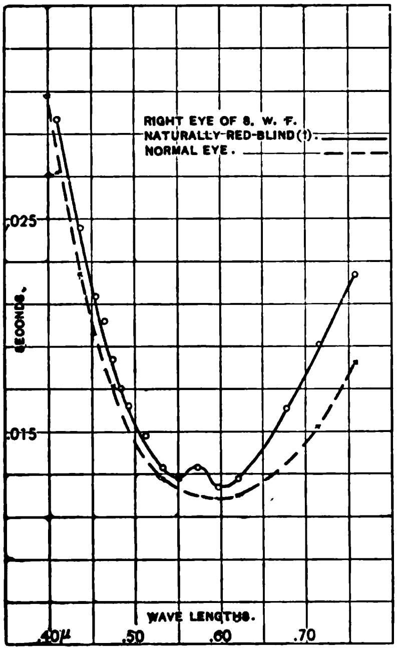

Fig. 15.

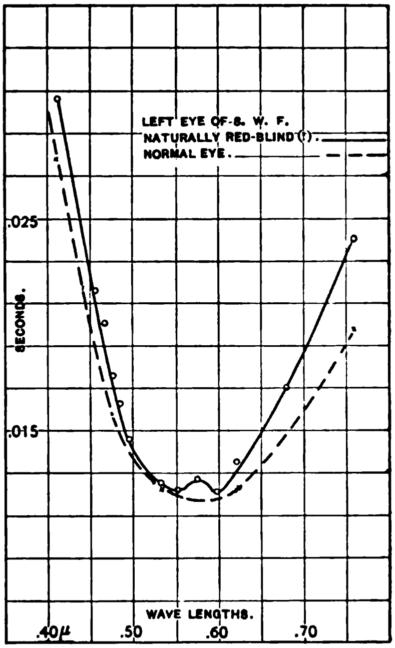

Fig. 16.

The form of the curves is the same for both eyes, both showing the same peculiarities. It may be possible that this is the curve of the typical red-blind subject. Ferry, however, in the article cited, found curves slightly different from these for red-blind persons. In his curves there were no elevations in the yellow-green part of the spectrum. The curve for the normal eye (Fig. 6), fatigued with red, shows the increased persistence of the red though not to the same extent as the naturally color-blind subject. No disturbance of the curve occurs in any other part.

Table XII gives the data for the left eye.

TABLE XII.

S. W. F. (*left eye*).

Wave length.	Persistence.	Wave length.	Persistence.
.410 μ	.0307 sec.	.550 μ	.0122 sec.
.455 "	.0216 "	.573 "	.0127 "
.465 "	.0201 "	.596 "	.0121 "
.475 "	.0176 "	.620 "	.0135 "
.484 "	.0162 "	.678 "	.0170 "
.493 "	.0146 "	.758 "	.0240 "
.533 "	.0125 "		

Mr. J. E. T., of the Chemistry department of the University, informed the writer that he had some difficulty in observing red lines in spectroscopic analysis, and kindly consented to make the necessary observations for his persistency curve. When tested with the Holmgren worsteds he matched the proper colors with the confusion samples, and seemed as far as that is concerned to have normal color vision. But finally being tested with other samples he made matches which indicated a partial red-blindness at least, and also a disturbance of the violet. He stated also that some difficulty was experienced with violet lines in spectroscopic work.

The values of the persistence are given in the following table:

TABLE XIII.

J. E. T.

Wave length.	Persistence.	Wave length.	Persistence.
.400 μ	.0428 sec.	.574 μ	.0112 sec.
.415 "	.0296 "	.596 "	.0114 "
.438 "	.0214 "	.620 "	.0119 "
.475 "	.0157 "	.678 "	.0144 "
.500 "	.0136 "	.715 "	.0184 "
.533 "	.0117 "	.758 "	.0224 "
.550 "	.0114 "		

These values, for the right eye, will be found in Fig. 17 in comparison with a normal curve of the writer. Both eyes, the right and left, gave approximately the same curve, only one of which is here plotted. It will be noticed that the part of the curve corresponding to red shows a characteristic elevation above the normal, indicating red-blindness. Through the yellow-green the points on the curve were obtained quite close together to see if any elevation occurred as in the previous case. No deviation from a smooth curve was, however, obtained. The persistence of green is shown to be, in this case, less than in the normal eye, indicating that, to the former, green was relatively much brighter. This is quite in agreement with other cases of color-blindness studied by various observers. Comparison of the two curves also shows that the lowest point on that obtained for the color-blind eye does not correspond to the same part of the spectrum as the similar point on the normal, but is displaced towards the blue end of the spectrum. This indicates

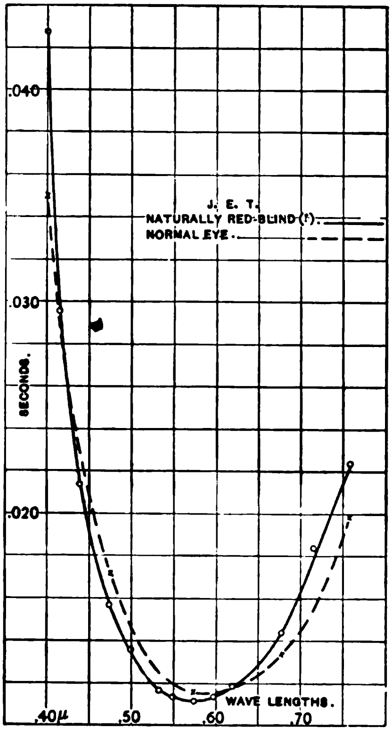

Fig. 17.

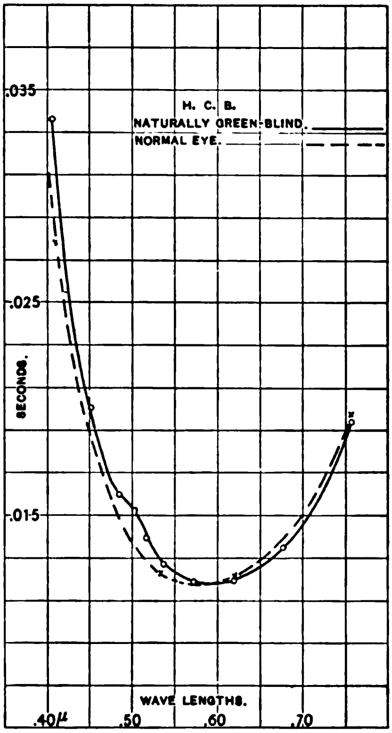

Fig. 18.

that the point of maximum brilliancy is shifted in that direction, a result also commonly observed. The curve also shows a persistence greater than normal in the extreme violet. Though there is greater difficulty in making observations in this region than in any other, yet since all Mr. T's readings here were greater than for the normal eye, it must indicate some abnormal condition of the retina as regards its appreciation of violet. It is not possible to say conclusively, without further investigation, whether this is a case of violet color blindness or not.

Another student of the University, Mr. H. C. B., also consented to determine his persistency curve. Upon examination with the worsteds he indicated green-blindness. Persistency curves for both eyes were obtained, but their close agreement renders only one necessary. The following values for the left eye were obtained:

TABLE XIV.

H. C. B. (*left eye*).

Wave length.	Persistence.	Wave length.	Persistence.
.405 μ	.0336 sec.	.538 μ	.0127 sec.
.454 "	.0201 "	.574 "	.0119 "
.487 "	.0160 "	.620 "	.0120 "
.502 "	.0152 "	.678 "	.0135 "
.517 "	.0139 "	.758 "	.0194 "

These values will be found plotted in Fig. 18 in comparison with a curve for the normal eye. The characteristic elevation in the green is well shown, though the persistence is not increased to quite the extent anticipated. It may indicate only a partial green-blindness. No other part of the curve differs from the normal. This curve, and the one preceding, show that an eye may be color-blind to red or green without necessarily affecting any other sensation, a conclusion which does not agree with the Hering theory of color blindness.

From these three curves for color blindness, it is not possible to draw many definite conclusions; but there is evidently reason to believe that the study of many cases of color blindness by this method will throw a new light upon this interesting subject, and may lead to important evidence as to the real nature of color-vision itself. Preparations for such an investigation, on the part of the writer, are being made.

SUMMARY.

The results arrived at in this investigation may be conveniently summed up as follows:

1. The persistence of all color impressions reaches a maximum when the eye has been kept open in the dark, the increased duration being unequal for rays of different refrangibilities. The increase is least in the yellow, and grows larger towards both the red and violet. The curve obtained under such conditions may be called the *extraordinary* normal curve.

2. The persistence of all color impressions is a minimum when the eye is under the influence of diffused daylight. The curve thus obtained has been termed in this paper the *ordinary* normal curve.

3. When the eye has been blindfolded the values of the persistence in all cases are intermediate between the extraordinary and ordinary normal curves, showing, however, unequal increases as in the extraordinary curve.

4. When the retina has been exposed to strong red, green or violet light, the persistence of impressions of the same color as the fatiguing light is increased, forming a characteristic elevation or bunch in the part of the persistency curve corresponding to that color.

5. When the retina has been fatigued with yellow or blue light there is no increase in the persistence of impressions of those colors, but the characteristic elevations are produced in the parts of the curves corresponding to red and green or green and violet respectively.

6. These phenomena strongly confirm the fundamental color sensations of the Young-Helmholtz theory.

7. When the retina has been fatigued with white light, the persistences of all color impressions are unequally increased. This apparently does not harmonize with the Young-Helmholtz theory, but indicates a partially independent white sensation.

8. Both rest and fatigue of the retina cause an increase in the

persistence of all colors. No explanation of this fact has yet been found.

9. In the case of naturally color-blind eyes, the persistence of color impressions to which the retina is blind is abnormally increased, thus affording a method of determining the wave lengths to which the retina is incapable of responding normally.

10. In addition to the loss of one fundamental color sense, the naturally color-blind retina may also lack the power of responding to a group of waves affecting another sensation. The further study of cases of color blindness in this way would probably throw new light upon this interesting and important subject.

In conclusion, I wish to gratefully acknowledge my indebtedness to Professor E. L. Nichols, who suggested the subject and method, and also to Mr. J. S. Shearer, of the department of Physics, for their many valuable suggestions regarding both apparatus and the experimental results of the entire investigation.

Physical Laboratory,
Cornell University, May, 1900.

GIBBS THERMODYNAMICAL MODEL FOR A SUBSTANCE FOLLOWING VAN DER WAALS' EQUATION.

By W. P. Boynton.

VAN DER WAALS' equation is commonly written

$$(1) \qquad (p + a/v^2)(v-b) = RT.$$

For the purposes of a general discussion it is often convenient to suppress the three constants a, b, R, by taking the critical pressure volume and temperature as the units of pressure, volume, and temperature respectively. If these be called p_c, v_c, T_c, and the so-called *reduced* pressure, volume, and temperature be indicated by

$$\pi = \frac{p}{p_c}, \quad \varphi = \frac{v}{v_c}, \quad \vartheta = \frac{T}{T_c}.$$

It can be shown that the equation will reduce to the form

$$(2) \qquad \left(\pi + \frac{3}{\varphi^2}\right)(3\varphi - 1) = 8\vartheta,$$

which *appears* to have no empirical constants.

The form of the isothermals, with whose general properties we are familiar, is shown in Fig. 1. In this figure we notice certain peculiarities. These we will discuss a little later.

In connection with questions raised in a previous article[1] it seemed to me desirable to find the form of a Gibbs' Thermodynamical Model for a substance whose properties in two states were represented by a comparatively simple analytical expression. A substance following van der Waals' equation is as much an ideal substance as is a perfect gas, yet it exhibits many properties common to actual substances. For a substance following equation (1) the first law of Thermodynamics

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[1] Phys. Rev., 10, p. 228, 1900.

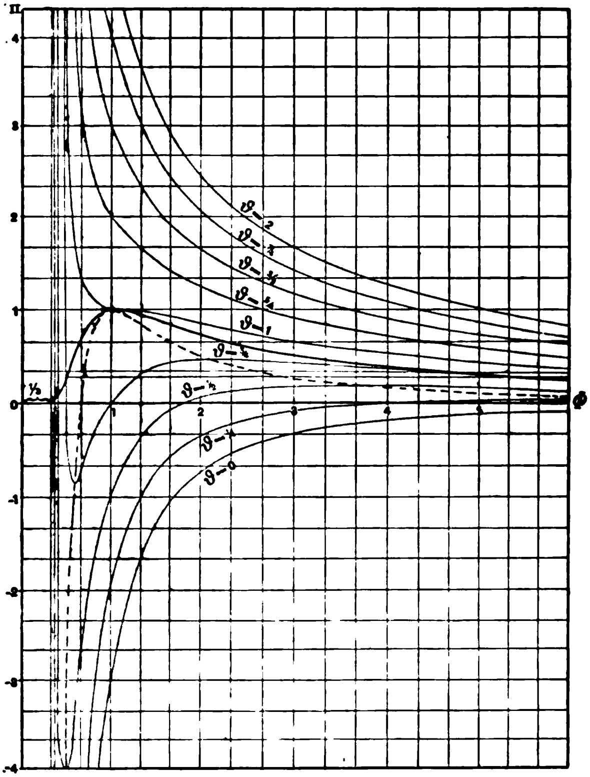

Fig. 1.

$$dQ = dU + pdv.$$

becomes

(3) $$dQ = C_v dT + (p + a/v^2)dv$$

from which it readily appears that

$$dU = C_v dT + a/v^2 dv$$

and if we regard C_v as a constant, an assumption entirely in harmony with, and even suggested by, those ideas of the Kinetic Theory which gives us the original equation, we have

(4) $$U = \int_0^T C_v dT + \int_b^v \frac{a}{v^2} dv = C_v T + \frac{a}{b} - \frac{a}{v}.$$

The coördinates of the surface we wish are the energy, volume and entropy. We may introduce the latter and eliminate T by comparing (3) with

$$dQ = TdS$$

from which we can deduce

$$(5) \qquad S = C_v \log T + R \log (v - b) + \text{Const}$$

$$T = K' e^{\frac{S}{C_v}} (v - b)^{-\frac{R}{C_v}}$$

and substituting in (4),

$$(6) \qquad U = K e^{\frac{S}{C_v}} (v - b)^{-\frac{R}{C_v}} + a/b - a/v$$

where

$$K = C_v K'.$$

R/C_v is a constant, equal, according to the Kinetic Theory, to two-thirds of the ratio of the kinetic energy of translation of the molecules to their total energy, or to $\frac{C_p}{C_v} - 1$ very nearly, that is, in ideal cases such as we are considering, to 2/3, 2/5, 2/6, 2/7, etc. This then is the equation of the model, and we see immediately that it satisfies the differential equations

$$T = \frac{\partial U}{\partial S} = K e^{\frac{S}{C_v}} (v - b)^{-\frac{R}{C}} / C_v = K' e^{\frac{S}{C_v}} (v - b)^{-\frac{R}{C_v}}$$

$$p = -\frac{\partial U}{\partial v} = \frac{R}{C_v} K e^{\frac{S}{C_v}} (v - b)^{-1-\frac{R}{C_v}} - \frac{a}{v^2}$$

$$= \frac{RT}{v - b} - \frac{a}{v^2}.$$

For a general discussion it is desirable to have the equation in terms of the reduced coördinates. At the critical state equation (5) becomes

$$S_c = C_v \log T_c + R \log (v_c - b) + \text{Const}$$

and hence

$$S - S_c = C_v \log \frac{T}{T_c} + R \log \frac{v - b}{v_c - b}$$

or remembering that

$$T/T_c = \vartheta, \quad b = v_c/3, \quad v/v_c = \varphi$$

(7) $$S - S_c = C_v \log \vartheta + R \log \frac{3\varphi - 1}{2}$$

whence

(8) $$\vartheta = e^{(S-S_c)/C_v} \left(\frac{3\varphi - 1}{2}\right)^{-R/C_v}$$

Equation (4) $$U = C_v T + a/b - a/v$$

gives, substituting for T and v their values

$$T = \vartheta T_c = \frac{8a}{27bR}\vartheta$$

$$v = \varphi v_c = 3b\varphi$$

$$U = \frac{8aC_v}{27bR}\vartheta + \frac{a}{b} - \frac{a}{3b\varphi}$$

$$= \frac{a}{b}\left(\frac{8C_v}{27R}\vartheta + \frac{3\varphi - 1}{3\varphi}\right)$$

and substituting the value of ϑ just found,

$$U = \frac{a}{b}\left[\frac{8C_v}{27R} e^{(S-S_c)/C_v} \left(\frac{3\varphi - 1}{2}\right)^{-R/C_v} + \frac{3\varphi - 1}{3\varphi}\right]$$

which contains no arbitrary constant except S_c, but does contain the factor a/b which depends upon the properties of the substance. At the critical state

$$U_c = \frac{a}{b}\left[\frac{8C_v}{27R} + \frac{2}{3}\right]$$

which enables us to express U entirely in terms of the properties of the critical state.

$$\frac{U}{U_c} = \frac{\frac{8}{27}\frac{C_v}{R} e^{\frac{S-S_c}{C_v}} \left(\frac{3\varphi - 1}{2}\right)^{-\frac{R}{C_v}} + \frac{3\varphi - 1}{3\varphi}}{\frac{8}{27}\frac{C_v}{R} + \frac{2}{3}}$$

or letting

$$\varphi = x$$

$$(S - S_c)/C_v = y$$

$$U/U_c = z$$

$$z=\frac{\frac{8}{27}\frac{C_v}{R}e^y\left(\frac{3x-1}{2}\right)^{-\frac{R}{C_v}}+\frac{3x-1}{3x}}{\frac{8}{27}\frac{C_v}{R}+\frac{2}{3}}.$$

The general properties of this equation are very much the same for the different values of the ratio $\frac{R}{C_v}$. For my computations I have selected the value 2/7, suggested by the Kinetic Theory for such triatomic gases as H_2O and CO_2. For these it becomes.

$$(9) \qquad z=\frac{14}{23}e^y\left(\frac{3x-1}{2}\right)^{-\frac{2}{7}}+\frac{9}{46}\frac{3x-1}{x}.$$

The general form of the surface is shown by Fig. 2, sections by planes of constant entropy, $y=$ const.

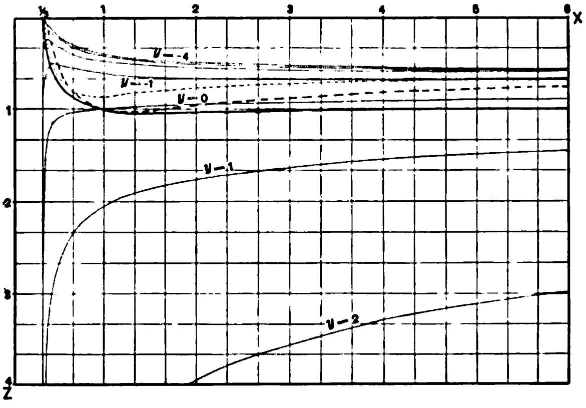

Fig. 2.

ISOTHERMALS.

Equation (7) becomes, dividing both members by C_v and substituting the new variables,

$$(10) \qquad y=\log\vartheta+\frac{R}{C_v}\log\frac{3x-1}{2}$$

$$=\log\vartheta+\frac{2}{7}\log\frac{3x-1}{2}.$$

These curves are shown in Fig. 3 for the same values of ϑ which were employed in Fig. 1.

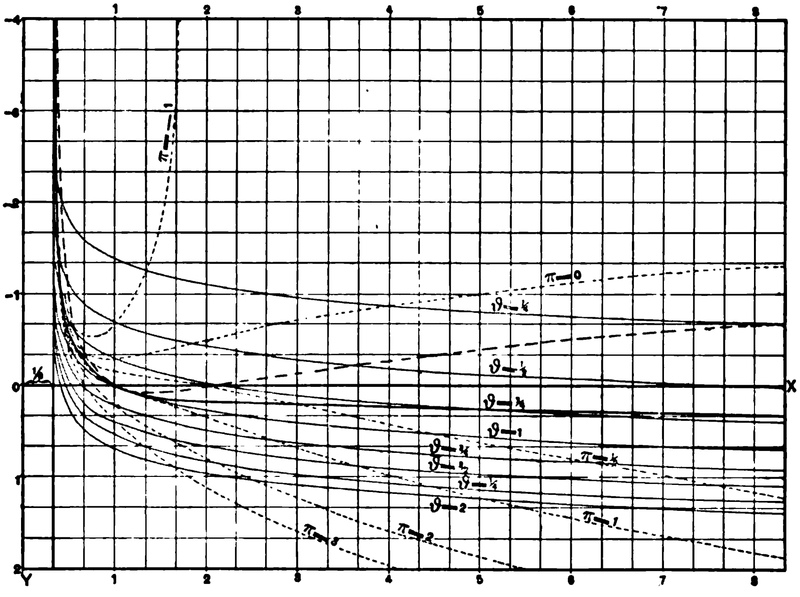

Fig. 3.

ISOPIESTICS.

Similarly substituting in (2) the value of ϑ in (8), and changing the coördinates, we obtain as the equation of the isopiestics,

$$(11) \qquad y = \log\left(\pi + \frac{3}{x^2}\right) + \frac{9}{7}\log\frac{3x-1}{2} - \log 4.$$

These curves also are shown in Fig. 3.

LINE OF INSTABILITY.

The unstable and explosive condition corresponding to certain states of a substance following van der Waals' equation is characterized by the fact that when the temperature is kept constant an increase of volume is accompanied by an increase of pressure, and vice versa, or analytically

$$\frac{\partial p}{\partial v}_{(T=\text{Const})} > 0.$$

The curve limiting this unstable region will then have the differential equation

$$\frac{\partial \pi}{\partial \varphi}_{(\vartheta)} = 0.$$

It is most convenient to obtain this directly from (2), writing

$$\pi = \frac{8\vartheta}{3\varphi - 1} - \frac{3}{\varphi^2},$$

$$\frac{\partial \pi}{\partial \varphi} = -\frac{3 \cdot 8\vartheta}{(3\varphi - 1)^2} + \frac{6}{\varphi^3} = 0,$$

$$4\vartheta = \frac{(3\varphi - 1)^2}{\varphi^3}.$$

Eliminating ϑ between this equation and (2) we have

$$\pi = \frac{2(3\varphi - 1)}{\varphi^3} - \frac{3}{\varphi^2}, \tag{12}$$

the equation of the broken line in Fig. 1. Eliminating instead with (8), and making the same substitutions as before, we have

$$\left.\begin{aligned} e^{y} &= \frac{1}{x^3}\left(\frac{3x - 1}{2}\right)^{\frac{16}{7}} \\ y &= \frac{16}{7} \log \frac{3x - 1}{2} - 3 \log x, \end{aligned}\right. \tag{13}$$

or eliminating y by the equation of the surface, (9)

$$z = \frac{14}{23} \frac{1}{x^3} \left(\frac{3x - 1}{2}\right)^2 + \frac{9}{46} \frac{3x - 1}{4}, \tag{14}$$

whose projections are given by the broken lines in Figs. 2 and 3, respectively.

REGION OF TWO PHASES.

It is a familiar fact that while van der Waals' Equation, apparently applying to both the liquid and vapor phases, suggests only such transformations from one phase to the other as shall keep the substance considered in one homogeneous phase, the actual transformations are usually effected by the discontinuous process of vaporization, during which the substance is non-homogeneous, being present in both the liquid and vapor phases at the same time. The actual isothermal of this process is represented in figure 1 for the temperature $\vartheta = 3/4$ by a horizontal straight line, intersecting the ideal isothermal in three points. In Gibbs' Model, as Maxwell has shown,[1] such an isothermal is given by a line connecting the two points of tangency of a plane resting upon the model. We may then attempt to locate the boundary line separating the regions representing a single phase from those referring to the co-existence of two phases by either of two methods.

If the area under the actual and under the ideal isothermals is to be the same, if we call the volume of the liquid just ready to vaporize φ_1, and that of the saturated vapor φ_2, both at the temperature ϑ and the pressure Π, then

$$\int_{\phi_1}^{\phi_2}\Pi d\varphi=\int_{\phi_1}^{\phi_2}\pi d\varphi=\int_{\phi_1}^{\phi_2}\left(\frac{8\vartheta}{3\varphi-1}-\frac{3}{\varphi^2}\right)d\varphi$$

or integrating,

$$\Pi(\varphi_2-\varphi_1)=\frac{8}{3}\vartheta\log\frac{3\varphi_2-1}{3\varphi_1-1}+3\left(\frac{1}{\varphi_2}-\frac{1}{\varphi_1}\right)$$

and since van der Waals' equation itself still applies to the saturated vapor and liquid

$$\Pi=\frac{8\vartheta}{3\varphi_2-1}-\frac{3}{\varphi_2^2}$$

$$\Pi=\frac{8\vartheta}{3\varphi_1-1}-\frac{3}{\varphi_1^2}$$

Three equations which are sufficient to give the relations between Π, φ_1, φ_2 and ϑ if we can solve them. If we do obtain the solution, we can plot the locus of φ_1 and φ_2 in Fig. 1, and by algebraic transformations find the corresponding locus in Fig. 2, and Fig. 3.

Or, we may attempt the direct solution. The equation of the surface of our model (9), is of the form

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[1] Theory of Heat, pp. 195-205.

$$z = me^{y}\left(\frac{3x - 1}{2}\right)^{-r} + n\frac{3x - 1}{x}$$

The equation of a plane is

$$z = ax + by + c.$$

If now the plane touch the surface at two points x', y', z', x'', y'', z'', the coördinates of these points must satisfy both equations, giving us

$$z' = me^{y'}\left(\frac{3x' - 1}{2}\right)^{-r} + n\frac{3x' - 1}{x'},$$

$$z'' = me^{y''}\left(\frac{3x'' - 1}{2}\right)^{-r} + n\frac{3x'' - 1}{x''},$$

$$z' = ax' + by' + c,$$
$$z'' = ax'' + by'' + c.$$

The fact that the plane and surface are mutually tangent makes the slopes in the principal directions the same, or

$$\frac{\partial z'}{\partial x'} = a = -\frac{3}{2}mre^{y'}\left(\frac{3x' - 1}{2}\right)^{-1-r} + n\frac{3x' - 1}{x'},$$

$$\frac{\partial z''}{\partial x''} = a = -\frac{3}{2}mre^{y''}\left(\frac{3x'' - 1}{2}\right)^{-1-r} + n\frac{3x'' - 1}{x''},$$

$$\frac{\partial z'}{\partial y'} = b = me^{y'}\left(\frac{3x' - 1}{2}\right)^{-r},$$

$$\frac{\partial z''}{\partial y''} = b = me^{y''}\left(\frac{3x'' - 1}{2}\right)^{-r}.$$

A point xyz which lies on the line connecting the two points of tangency will satisfy the two equations

$$\frac{y - y'}{x - x'} = \frac{y' - y''}{x' - x''},$$

$$\frac{z - z'}{x - x'} = \frac{z' - z''}{x' - x''},$$

which gives us ten equations between which we may eliminate the six coördinates, x', y', z', x'', y'', z'' and the three constants a, b, c, still leaving an equation involving x, y, z, which is the equation of the developable surface generated by a plane rolling on the model. The intersection of this surface with our model will be the line desired, separating the regions relating to one phase from that relating to two phases.

These two analytical methods of solution are here given for the sake of completeness. A direct and complete solution by algebraic methods does not seem possible, and lack of time has prevented my attempting approximate or graphical methods of solution. The horizontal portion of the isothermal $\vartheta = \frac{3}{4}$, in Fig. 1, was actually drawn by the method of trial, so as to make the areas included between it and the two segments of the ideal isothermal equal. This

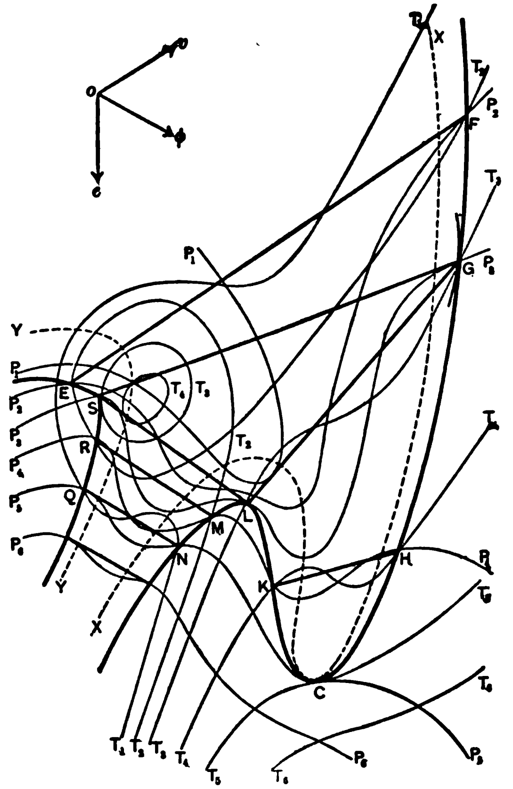

Fig. 4.

enabled the line of separation to be drawn of approximately the right position and form.

DISCUSSION.

The lines of the various figures form a network which is to be considered as spread out over the surface of either model. These lines are lines of constant volume, constant pressure, constant temperature, and constant entropy. Fig. 1 is slightly less complex than Fig. 3, lacking the system of isentropics. Except for this there must be a one to one correspondence between the two net-works, since they represent identically the same properties, thus the same lines pass through the critical point in each, and in each the same lines have points of tangency or intersection for the same values of x.

In the more specific discussion of the lines of Fig. 3, it is well to compare it with Fig. 4, which is taken from Maxwell.[1] An important difference which must be taken into account is that Fig. 3, refers only to the liquid and gaseous states, while Fig. 4 has also a region referring to the solid state.

The *isothermals* are indicated in Fig. 3 by the letter ϑ, in Fig. 4 by the letter T. An adiabatic or isentropic increase of volume is, so far as I know, under all circumstances accompanied by a fall of temperature. This property appears in Fig. 3 in which the isothermals slope downward to the right, but not in Fig. 4, in which they slope in general upward to the right, but occasionally have singular forms.

The *isopiestics* are indicated in Fig. 3 by the letter π, in Fig. 4 by the letter P. The general form of a line of constant negative pressure it given by the curve $\pi = -1$. It has either two or no intersections with an isothermal, being tangent in one special case for each isopiestic. This property appears from Fig. 1. Each negative isopiestic passes off to infinity becoming asymptotic to a vertical line. The one shown has for its asymptote the line $x = \sqrt{3}$. The limiting case is the line $\pi = 0$. Like all the other isopiestics it is asymptotic to the line $x = 1/3$. It cuts all the isothermals twice or not at all, being tangent to one particular isothermal, and approaches parallelism to the isothermals, after cutting above them all, for $x = \infty$. This corresponds to the fact that in Fig. 1 the axis $\pi = 0$ is an asymptote to all the isothermals. Another limiting case is the line $\pi = 1$, which is tangent to the isothermal $\vartheta = 1$ at the critical point, and cuts every other isothermal once only. An example of an intermediate case is the isopiestic $\pi = 1/2$, which cuts every isothermal either once or three times. The other case is illustrated by the lines $\pi = 2$ and $\pi = 3$ which cut every isothermal once only. All these properties appear from figure 1, and the fact that they also appear in Fig. 3, may be taken as a check upon the accuracy of the drawing. In general, the isopiestics for positive pressures slope downward to the right, although for very small pressures less than $\pi = 1/2$ they may in the middle of the field slope up to the right. This does not appear in Fig. 4, which also does not exhibit the bunching of lines asymptotic to $x = 1/3$. The line P_1, which represents a negative pressure, is not entirely unlike the line $\pi = -1$.

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[1] Theory of Heat, p. 207.

The *Line of Instability*, represented by the broken line in Fig. 3, and by the dotted line in Fig. 4, has in the two cases not dissimilar properties, but in the former it dips below the level of the critical point, and passes off to infinity *horizontal.*

The line dividing the regions corresponding to a single phase from that corresponding to two co-existant phases is represented in both figures by a heavy line, which passes through the critical point and is there tangent to the critical isothermal, critical isopiestic, and the line of instability. It cuts all the isothermals below the critical twice, and all the positive isopiestics below the critical twice. In figure 3 it lies entirely between the isopiestics $\pi = 0$ and $\pi = 1$. At the critical point it passes below the level of the critical point, and passes off to infinity horizontal, as do the isothermals. It appears immediately from this diagram that the adiabatic expansion of a saturated vapor would result in partial condensation. Whether this line would in other regions at lower temperatures have such a form that adiabatic expansion would superheat a saturated vapor could only be told if we were able to obtain an analytic solution for the line, or plot it point by point. The matter is not of great importance, as we are dealing only with an equation, and not an actual substance.

The lines of Fig. 3 have all been plotted from their equations, with the exception of the one last discussed, which, however, we have been able to draw with fair accuracy from our knowledge of its relations to the other lines of the figure. If the axes of coordinates of Fig. 4 are to be considered as similarly situated, the discrepancy between the two is very considerable, and very troublesome. If, however, the axes are to be taken as obliquely as they are drawn, v corresponding to x and φ to y, or if we regard Fig. 4 as a perspective drawing rather than a projection on one of the principal planes, the discrepancies, while by no means disappearing, become much less.

UNIVERSITY OF CALIFORNIA,
BERKELEY, CALIFORNIA, July 2, 1900.

ON THE FORMS OF CURVES PRESENTED BY THE MICHELSON INTERFEROMETER.

By John Cutler Shedd.

Preliminary.

THE theory of the interference phenomena produced by this instrument has been entered into somewhat in detail by Professor A. A. Michelson.[1] In the present paper it is proposed, first to follow the ground plan presented by Professor Michelson and then to give a point of view which is believed to be novel.

PART I.

In Fig. 1 is given a plan view of the usual form of interferometer.

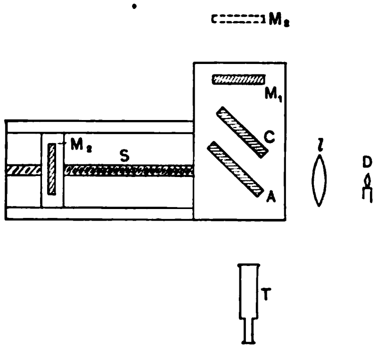

Fig. 1.

$M_1M_2 \equiv$ heavily silvered mirrors.

$A \equiv$ plate, half silvered on side toward M_1.

$C \equiv$ plate of same thickness as A and placed parallel to it. Plate C is called the compensator.

M_2 is on a movable carriage controlled by screw S.

The interference pattern is viewed by the eye, or telescope, placed at T.

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[1] Phil. Mag. [5], 13, 1882, pp. 236–242.

It is evident that the mirror M_2 may be regarded as occupying a position behind M_1, as shown in the dotted lines. Regarding M_2 as so placed, we may, as has been shown by Professor Michelson, suppress everything excepting M_1 and M_2.

Let *de* be drawn parallel to the line of intersection of the planes of M_1 and M_2 and let the angle of intersection be φ.

The whole phenomenon may now be discussed on the basis of a source of light S placed in the plane of M_1 and of its image in M_2.

In this discussion the following notation will be used:

$t_0 \equiv$ Perpendicular distance between M_1 and M_2. On the instrument this is controlled by the screw S (Fig. 1).

$\varphi \equiv$ Angle between M_1 and M_2. This is controlled by adjusting screws at the back of M_1.

$P \equiv$ Perpendicular distance from M_1 to the plane containing the interference pattern. This plane of maximum distinctness is called the focal plane.

$\theta \equiv$ Angle subtended by the source of light at point P on the focal plane, in a plane perpendicular to the plane of φ.

$i \equiv$ Same in plane parallel to plane of φ.

$\Delta \equiv$ Difference of path traversed by the two rays producing interference at P.

Professor Michelson has shown that the value of Δ is given by the equation

$$\Delta = 2\frac{t_0 + P \tan i \tan \varphi}{\sqrt{1 + \tan^2 i + \tan^2 \varphi}}. \qquad (1)$$

In general Δ has all values but the value which will give the most distinct fringes is given by the conditions

$$\frac{\partial \Delta}{\partial \theta} = 0$$

and

$$\frac{\partial \Delta}{\partial i} = 0.$$

This gives the value of P for which the phenomenon of interference is most distinct. The value is

$$P = \frac{t_0}{\tan \varphi} \tan i. \qquad (2)$$

It follows from equation (2) that if $t_0 = 0$ or $i = 0$ then $P = 0$ and the interference fringes are on the surface of M_1. If $t_0 = 0$ and $\varphi = 0$, or $i = 0$ and $\varphi = 0$ then P is indeterminate. If $\varphi = 0$ then t_0 is everywhere uniform and $P = \infty$. If φ and i have same sign P is positive and the fringes are in front of M_1. If φ and i have opposite signs P is negative and the fringes are behind M_1.

If M_2 is behind M_1, then φ is regarded as positive and t_0 as positive. If M_2 is in front of M_1 φ and t_0 are both regarded as negative. The plane containing the interference pattern is called the *Focal* plane, and upon this plane the eye or telescope must be focused in order to get the most distinct view of the fringes. The Focal plane is parallel to the plane of M_1 and at a distance P from it.

Any point on the Focal plane is determined by the equations

$$\left.\begin{aligned} x &= P \tan i \\ y &= P \tan \theta \end{aligned}\right\} \qquad (3)$$

and the form of curve produced in the interference pattern is found by substituting equations (3) in equation (1). This gives the following equation of the interference curves:

$$\Delta^2 y^2 = (4P^2 \tan^2 \varphi - \Delta^2)x^2 + 8t_0P^2 \tan \varphi x + P^2(4t_0^2 - \Delta^2). \quad (4)$$

The further discussion of the subject is an analysis of equation (4), and it is very essential to its good understanding that every term involved be thoroughly apprehended. We therefore repeat:

$2t_0 \equiv$ Perpendicular distance between M_1 and its image in M_2, optically considered [this reduces to the value of Δ when i and θ both are zero].

$\varphi \equiv$ angle that M_1M_2 make with each other [physically the angle by which they differ from 90°].

$P \equiv$ Perpendicular distance of the focal plane from M_1.

$i, \theta \equiv$ Angles of incidence upon the focal plane, of the interfering beams: i, being measured in the plane parallel to and θ in the plane normal to the plane of φ. The focal plane is normal both to the plane of i and of θ.

$\Delta =$ Difference of path of the interfering beams.

In the article already referred to Professor Michelson expresses equation (4) in the equivalent form

$$\Delta^2 y^2 = (4S^2K^2 - \Delta^2)x^2 + 8TSKDx + (4T^2 - \Delta^2)D^2 \qquad (5)$$

and then points out what curves are represented by equation (5) and the values of the constants necessary in each case.

It would seem to be equally as simple and more satisfactory to refer directly to equation (4). The following are the principal cases that arise.

(1) $\Delta = 0$; Equation (4) becomes

$$x = \frac{t_0}{\tan\varphi} \qquad (6)$$

representing a straight line.

(2) $\varphi = 0$; then

$$x^2 + y^2 = \frac{P^2}{\Delta^2}(4t_0^2 - \Delta^2) \qquad (7)$$

giving circular fringes.

(3) $\Delta = 2P\tan\varphi$. In this case equation (4) becomes

$$y^2 = \frac{4Pt_0}{\Delta}x_1. \qquad (8)$$

In equation (8) this axis of y has been shifted so that

$$x_1 = x - P\frac{4t_0^2 - \Delta^2}{4t_0\Delta}$$

thus giving the central equation. Equation (8) is that of a parabola

(4) $\Delta > 2P\tan\varphi$; Equation (4) takes the form

$$\frac{x^2}{A^2} + \frac{y^2}{B^2} = 1 \qquad (9)$$

where

$$A^2 = \frac{\Delta^2P^2(4t_0^2 - \Delta^2)}{\Delta^2 - 4P^2\tan^2\varphi}$$

and

$$B^2 = P^2(4t_0^2 - \Delta^2)$$

thus giving the semi-axes of the ellipse

(5) $\Delta < 2P\tan\varphi$. In this case the hyperbola

$$\frac{x^2}{A^2} - \frac{y^2}{B^2} = 1 \qquad (10)$$

is obtained where

$$A^2 = \frac{\Delta^2 P^2(\Delta^2 - 4t_0^2)}{\Delta^2 - 4P^2 \tan^2 \varphi}$$

and

$$B^2 = P^2(\Delta^2 - 4t_0^2).$$

A more detailed consideration of these cases is reserved for the second part of the paper.

PART II.

The curves just considered may also be very readily developed by considering their eccentricity. The general expression for the eccentricity of equation (4) is

$$e = \sqrt{1 + A/B}$$

where A is the coefficient of x^2 and B of y^2. In the present case this becomes the very simple expression

$$e = \frac{2P \tan \varphi}{\Delta}. \qquad (11)$$

Also since in equation (2)

$$P = \frac{t_0}{\tan \varphi} \tan i$$

we get

$$e = \frac{2t_0 \tan i}{\Delta}. \qquad (12)$$

Finally substituting the value of Δ given in equation (1) we get

$$e = \sin i \cos i \sqrt{1 + \tan^2 i + \tan^2 \theta} \qquad (13)$$

or

$$e = \sin i \sqrt{1 + \tan^2 \theta \cos^2 i}. \qquad (14)$$

The various cases that rise can be readily covered by equations (11) to (14). The following will be considered.

Case 1. $\Delta = 0$; then by equation (11) or (12)

$$e = \infty \qquad (15)$$

and the curves are straight lines. If, however, t_0 be also zero, then by equations (11) or (12) e is indeterminate. The meaning of this is that mirrors M_1 and M_2 coincide and hence there is but a single source of light. The conditions therefore are not favorable for interference.

However, if $t_0 = 0$ and $\varphi > 0$ then $P = 0$ and it is also apparent that both i and θ are zero. If i and θ are both zero it is apparent by equation (13) that $e = 0$, and the curves if existent, would be circles. This point will be returned to later.

Case 2. $t_0 = 0$ and $\varphi > 0$; then $P = 0$, and the central band will be a straight line (corresponding to $\Delta = 0$ and $e = \infty$) flanked on either side by bands of opposite curvature ($\Delta > 0$ $e < \infty$). The fringes on one side are behind the mirror (P negative) while those on the other side are in front of the mirror (P positive).

Referring to equations (1) and (11) viz.:

$$\Delta = 2\frac{t_0 + P \tan i \tan \varphi}{\sqrt{1 + \tan^2 i + \tan \theta}} \tag{1}$$

and

$$e = \frac{2P \tan \varphi}{\Delta} \tag{11}$$

we see that since P is small, being zero for the central band, the values of i and θ rapidly increase, hence (t_0 also being very small) the value of Δ is small (Eq. 1) and the value of e (Eq. 11) will be very large. The larger the value of φ the more rapidly does i change (i being in the plane of φ) and the more slowly does the value of e fall. Furthermore the central straight band lies parallel to the intersection of M_1 and M_2 and hence the more rapidly i changes (*i. e.*, the larger the value of φ) the closer together will the fringes be. Hence as φ is made to increase the field becomes covered with narrow fringes that are practically straight lines. If now M_2 be made to recede, so as to increase t_0, the fringes soon appear curved. Again increasing the value of φ tends to make the fringes straight, but a limit is soon reached where they are too narrow to be distinguishable.

For work over a limited range of t_0, it is frequently convenient to use straight fringes. It is also apparent that under these conditions P is practically zero and hence the telescope is to be focused on mirror M.

This point is very well illustrated in the work of Morley and Rogers in their research on the thermal expansion of metals.[1]

Case 3. $\varphi = 0$. In this case $e = 0$ and the fringes are circles. If $\varphi = 0$ then $P = \infty$ (Eq. 2) and the telescope must be focused for parallel light. Regarding equation (7) we see that the radius of each circle is

$$R = \frac{P}{\varDelta}\sqrt{(2t_0 + \varDelta)(2t_0 - \varDelta)}.$$

But $2t_0 - \varDelta = n\lambda$ and $2t_0 + \varDelta = 2\varDelta$ (approximately), so that

$$R = P\sqrt{\frac{n\lambda}{t_0}} \quad \text{(approximately)}.$$

$\therefore R \propto \sqrt{n}$ showing that the circular fringes follow the same law as do Newton's rings.

Case 4. $\varphi > 0$, but small. We obtain by equation (6), if we put $2t_0 = \varDelta$, and from equation (12)

$$e = \tan i. \tag{15}$$

For small values of i the fringes are circular, becoming elliptical as i increases. Thus for values of $i < 5°$, $e < 0.1$.

It is easy to so adjust t_0 and φ that circular fringes shall be seen in the center of the field of view, surrounded by increasingly elliptical fringes as i increases in value.

In this connection a very interesting case arises. For large values of t_0, φ and i being of opposite sign, the focal plain is behind M_1. Under these conditions t_0 and φ are positive, i and P negative. By equation (2) $\varDelta$ is negative and e, of course, positive. Also for large values of t_0 the value of P given by equation (3) is large, and the value of i is necessarily very small, hence e is small (Eq. (12)), and the fringes are circles. If now M_2 be advanced until $t_0 = 0$ then $P = 0$ (Eq. (2)) and the focal plane is on M_1, or, more correctly, the focal plane intersects M_1 in the field of vision. At this point $e = \tan i$ (practically) and the fringes are still circles. Since $t_0 = 0$ at this point circular chromatic fringes may be obtained. These fringes are beautifully distinct and brilliant.

---

[1] PHYSICAL REVIEW, Vol. IV, p. 7, 1896.

If M_1 be still further advanced so as to bring M_2 in front of M_1, then t_0 and φ change sign, becoming negative; i and P become positive, while Δ remains negative.[1]

The value of Δ then becomes

$$\Delta = 2\frac{-t_0 + P \tan i \tan \varphi}{\sqrt{1 + \tan^2 i + \tan^2 \varphi}}$$

and $t_0 > P \tan i \tan \varphi$, so that Δ is negative. As M_1 is still further advanced t_0 increases in value and so also P. For a time the value of Δ will diminish and the value of e (Eqs. 11 or 12) increase. The result is that the curves rapidly change from circles ($e = 0$) t_0 ellipses ($e > 0$ but < 1) then to the parabola ($e = 1$) becoming hyperbolæ ($e > 1$) and finally giving the straight line[2] ($e = \infty$).

If after the minimum value of Δ is reached t_0 be still further increased in value, the increased value of P is more than counterbalanced by the diminished value of tan i, so that the term P tan i tan φ diminishes in value. The value of Δ therefore increases and the value of e diminishes. The result is that when M_2 is further advanced the curves change curvature (the conjugate hyperbolæ coming into view) and pass through the hyperbolic, parabolic, elliptic forms and finally again appear as circles.

The foregoing case furnishes a very ready method of finding the chromatic fringes (zero position of the instrument) without any previous acquaintance with the instrument. The scheme is as follows: Having secured fringes with the sodium flame in the usual manner, run M_2 out 6 or 7 mm. and adjust M_2 so that the parallax is a minimum. This will give a small value of φ. Now advance M_2 through the zero position until the curves are hyperbolæ. Finally reverse the motion and let M_2 recede until the fringes just begin to become circular. If, now, white light be substituted for the sodium flame, and M_2 be made to slowly recede by means of the tangent screw, chromatic fringes quickly appear. So distinct are they that the diffused light of the room is sufficient to give them very clearly.

---

[1] The negative sign before Δ indicates that the increment of Δ, as i increases, is a diminishing increment. The result is that the focal plane becomes a curved surface concave to the observer for negative values of P (P diminishing as i increases in value). This can be detected by the eye, the fringes appearing to lie on a concave surface.

[2] This will depend upon whether Δ becomes zero or merely reaches a minimum value and then increases again. On the instrument Δ may be controlled, and the fringes made to be practically straight lines.

Case 5. $\Delta = 2P \tan \varphi$ or $\Delta = 2t_0 \tan \varphi$.

In this case $e = 1$. (16)

Putting this value in equation (13) we obtain

$$1 = \sin i \cos i \sqrt{1 + \tan^2 i + \tan^2},$$

which may be reduced to the form

$$\sin \theta = \frac{1}{\sqrt{1 + \sin^2 i}}. \qquad (17)$$

The limiting values of θ and i are

$$i = \pm 90^\circ, \text{ and } \theta = 90^\circ \pm 45^\circ.$$

It is thus evident that the parabola cannot be seen unless $\theta > 45^\circ$; and it is equally evident that θ cannot have this value unless the focal plane is quite close to M_1. This explains why the parabola is only seen for small values of t_0; or large values of φ, giving small values for P.

Case 6. $\Delta < 2P \tan \varphi$ or $\Delta < 2t_0 \tan i$.

Then $e > 1$, and the curves are hyperbolæ. What has been said under Case 5, as to the value of t_0 and φ holds equally for this case, showing that the hyperbolic curves are only observed when the value of t_0 is very small.

Case 7. The Equilateral Hyperbola.

In this case $e = \sqrt{2}$ and equation (14) reduces to

$$\tan \theta = \sqrt{\frac{2}{\sin^2 i} + \sin^2 i - 1}. \qquad (18)$$

The following table based upon equation (18) shows the relation of i and θ.

If i and θ be interchanged the conjugate system of hyperbolæ is obtained. In the general case of the hyperbola the minimum value of θ was found to be 45°. In the equilateral hyperbola the minimum value is 54°, showing that in this case the range of t_0 is still more restricted than in the general case.

TABLE I.

sin i	i	tan θ	θ	P
0	0°	∞	90°	$\frac{t_0}{\tan \phi} \times 0 = 0$
0.5	30°	2.7	70°–	$\frac{t_0}{\tan \varphi} \times 0.58$
0.707	45°	1.87	62°–	$\frac{t_0}{\tan \phi} \times 1$
0.877	60°	1.58	58°–	$\frac{t_0}{\tan \varphi} \times 1.73$
1	90°	1.4	54°40′	$\frac{t_0}{\tan \phi} \infty = \infty$

Case 7. Rotation of the Compensator. It is very instructive, for a given setting of the mirror M_2, to slowly rotate the compensating plate (C, Fig. 1). The effect of such rotation is to change the length of path through the glass, and hence to alter the value of t_0. If the value of t_0 be already large, this change will not appreciably effect either the value of P (Eq. 2) or that of $\varDelta$ (Eq. 1), and hence the value of e remains sensibly unaltered.

If, however, t_0 be small the rotation of the compensator will produce a large relative change in t_0 and hence a large change in both $\varDelta$ and P, resulting in a rapid change in the value of e.

For example, if the value of t_0 be small, and the compensating plate be parallel to plate A; then on rotation such as to diminish the optical path through the glass, the circular fringes are seen to change to the elliptical form, the value of e becoming greater than zero.

As t_0 becomes still smaller, due to further rotation of the compensator, e increases in value, and, corresponding to this the fringes pass through the Parabolic, into the hyperbolic form. Soon the conjugate system of hyperbolæ appear in the field of view, presenting a very pretty sight. If the compensator be rotated until it becomes parallel to M_1, the behavior of the fringes changes. As the parallel position is passed the direction of motion of the fringes reverses, and the value of t_0 begins to increase. The reversal in the movement of the fringes furnishes a very sensitive test for the parallelism of plates C and M_1 (Fig. 1). This test for parallelism might, from its sensitiveness, be made of more than passing value in other lines of work.

Case 8. $\varphi = 0$; $t_0 = 0$. Under these conditions P is indeterminate (Eq. 2). Therefore Δ is indeterminate (Eq. 1), and so also e by equations (11) or (12). If $t_0 = 0$ it is apparent that M_1 and M_2 coincide, and, as has already been pointed out, there should be no interference and hence no interference pattern.

This however will depend upon whether the two beams of light are exactly in phase. Now since one beam suffers internal and the other external reflection at plate A, if a difference of phase other than $\frac{1}{2}\lambda$ takes place interference would result and an interference pattern would be seen. Since M_1 and M_2 coincide the values of i and θ would seem to be 0°, and the value of e by equation (13) would then be

$$\begin{aligned} e &= \sin i \cos i \sqrt{1 + \tan^2 i + \tan\theta} \\ e &= 0 \end{aligned} \qquad (19)$$

and hence the fringes would be circles. Or we may consider that a difference of phase between the two beams, due to any cause, would be equivalent to a finite value for t_0. Then since $\varphi = 0$, e would be zero by equation (11).

In the case of white light, in addition to the difference of phase due to reflection, the compensating plate C introduces a *dispersion*[1] effect (excepting when C is strictly parallel to A) thus (in general) adding to the phase difference.

The result is that very brilliant chromatic *circular* fringes are to be seen even when $\varphi = 0$ and $t_0 = 0$. The presence of fringes furnishes a test for the presence of a phase difference produced by reflection. Under these conditions as many as 147 circular chromatic fringes have been counted.

Equation (13), perhaps, sums up all of the preceding cases in as concise a manner as is possible. When t_0 is large it is evident that i and θ must both be very small and hence $e = 0$ (practically). As t_0 becomes smaller the possible values of i and θ are larger giving the successively larger values for e.

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[1] This is the "chromatism" investigated by Conn, C. R. t, xciii, 1881 and by Michelson, Phil. May [5] 13, 236, 1882.

It is fascinating to observe the transformation of the interference fringes as they pass from one form of curve to another. Not only are the phenomena very beautiful in themselves but there is no better way of becoming thoroughly conversant with this admirable and many-sided instrument than is furnished by the study of the manifold curves presented by the interferometer. For the verification of the deductions given in this paper, I wish to thank Mr. A. C. Scott, of the staff of the Agricultural College, Kingston, R. I., who during the past summer collaborated with me in the physical laboratory of the University of Wisconsin.

Physical Laboratory, Colorado College.

NEW BOOKS.

The Strength of Materials. By J. A. EWING. Cambridge University Press; New York, The Macmillan Co., 1899. Pp. xii+245.

The work above referred to was printed at the University press, Cambridge, in 1899, and as we learn from the preface, is intended principally to set forth briefly a lecture room treatment of the subject which is to be supplemented by laboratory and drawing office work.

The book contains twelve chapters and an appendix and constitutes a most excellent treatise on the subject. So far as it relates to the application of mathematics to the strength of materials it is decidedly the best work published, for the student. Throughout the entire book the distinction between stress and strain is made prominent, and although the terms are used correctly they are used in an entirely different sense from that of some American writers on the subject. Stress is defined as the mutual action between two bodies or between two parts of a body whereby each of the two exerts a force upon the other. Strain is defined as a change of shape produced by stress. Many of the American writers use the word *strain* in the sense of stress as defined above. The distinction made by Professor Ewing is to be commended, as it is believed that the theory relating to the strength of materials would be much simplified if all writers would adopt these terms and use them in this manner.

The theoretical discussions are stated with admirable clearness and the book covers the broad mathematical field relating to strength of materials and its application to frames and simple trusses thoroughly. The method of treatment makes it well adapted for use as a text-book.

A chapter is devoted to the Testing of Materials in which considerable information is given regarding testing machines. This chapter is the only one which is open to criticism from an American standpoint. All the testing machines illustrated are either of an antiquated type or are those manufactured by English builders. The forms are, from an American standpoint, unmechanical, unwieldy and awkward to use although doubtless giving accurate results. It would seem that a text-book intended for general use should present information pertaining to other forms of testing machines, especially when it is considered that the forms shown have only a very limited use and are confined to a country which at the present time is manufacturing only a very small fraction of the steel used for structural purposes by the world, and moreover this fraction is constantly diminishing.

On the whole the book is a valuable one as it presents in a small compass all the important principles and the results of the principal investigations made on this important subject.

R. C. Carpenter.

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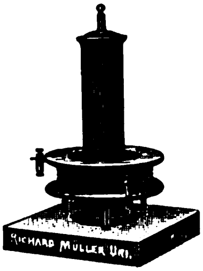

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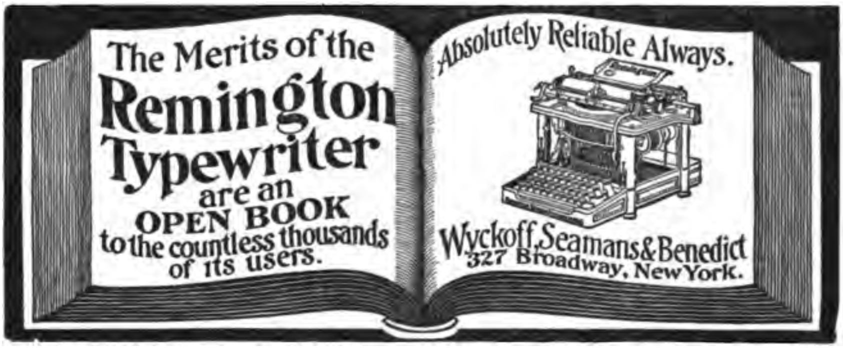

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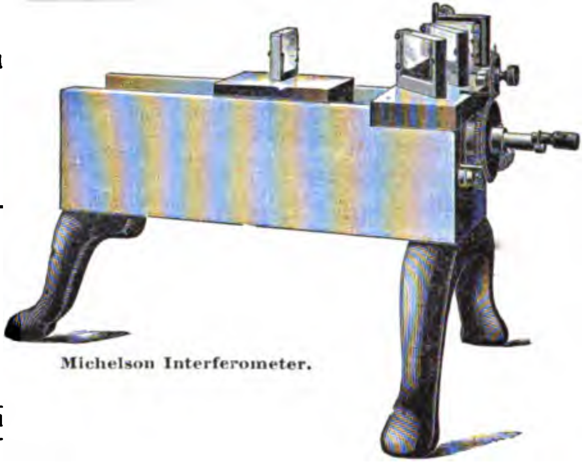

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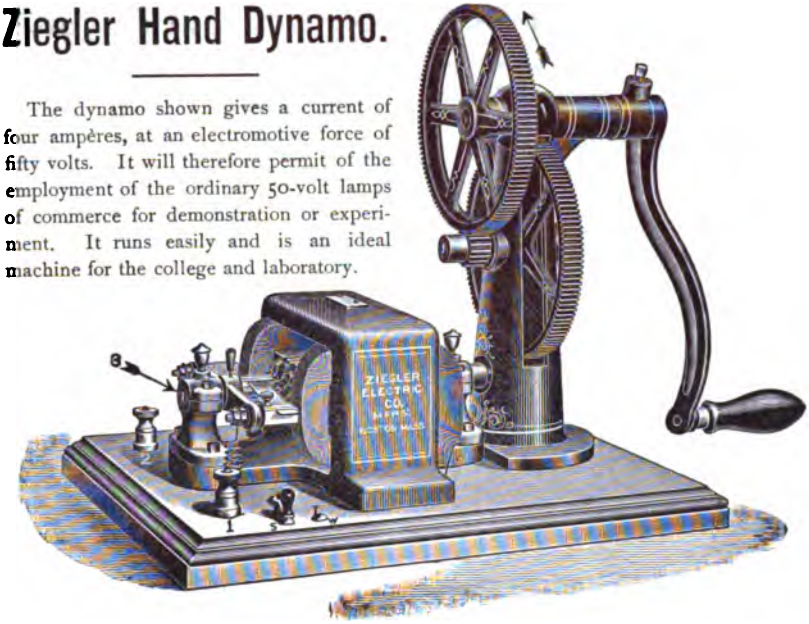

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